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Cut-off analysis of coherent vortical structure identification in a three-dimensional external flow

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Abstract

Vortical structure identification has more recently been applied in the study of the transport of vortical structures in low Reynolds number three-dimensional complex geometry flows. An important issue in this identification procedure is to choose an appropriate cut-off value λ_2 which takes into consideration the finite precision vortex interfaces. This cut-off choice is studied in this Note and applied to an external flow around a curved cylinder. The vortex identification technique at different cut-off values is compared to the threshold of the vorticity field showing the efficiency of choosing the optimal tolerance gap. The computations are performed with a fully three-dimensional spectral/hp element method. ***To cite this article: A. Miliou et al., C. R. Mecanique 333 (2005).***

Résumé

Analyse des seuils de coupure pour identifier des structures cohérentes dans un écoulement tridimensionnel externe. Ce travail est consacré à l'identification des structures cohérentes présentes dans l'écoulement tridimensionnel d'un fluide visqueux aux bas nombres de Reynolds avec des géométries complexes. Une des issues importantes de ce processus d'identification est le besoin de spécifier un seuil numérique λ_2 pour tenir compte des limites de précision de ce genre de calcul, en ce qui concerne l'interface des structures identifiées. Dans ce travail, ce seuil a été étudié et appliqué à un écoulement externe autour de tuyaux courbés. Les choix sont ensuite comparés aux résultats de la méthode du seuil de la vorticité mettant en évidence l'importance du choix approprié d'un seuil optimal. Tous les calculs sont effectués par une méthode tridimensionnelle du type 'spectral/hp element'. ***Pour citer cet article : A. Miliou et al., C. R. Mecanique 333 (2005).***

Keywords: Computational fluid mechanics; Vortex identification technique; Spectral/hp element method

Mots-clés : Mécanique des fluides numérique ; Identification des structures cohérentes

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L'identification des structures cohérentes représente un élément important dans la dynamique des écoulements réels. En effet, ces structures sont principalement convectées par l'écoulement, et constituent des entités qui demeurent quasi intactes au cours du temps. Elles transportent de plus une bonne partie de l'enstrophie [13]. Plusieurs stratégies d'identification existent. La plus naturelle consiste en la visualisation des isovaleurs de la vorticité. Si cette première méthode permet de se faire une bonne idée de la localisation des tourbillons, elle n'est toutefois pas suffisamment précise dès qu'il s'agit d'isoler ces tourbillons. Un critère de détection utilisant un seuil de vorticité fixé aussi, a priori, ne donne pas des résultats fiables. En effet, si par définition, il permet d'isoler les parties de l'écoulement dans lesquelles la vorticité est la plus forte (en valeur absolue), ce critére ne permet pas toujours de distinguer les zones cohérentes des zones non cohérentes. Les Figs. 2(e) et 2(f) montrent comment ce type de détection confond des zones cisaillées non-cohérentes avec des structures concentrées, transportées dans l'écoulement. Jeong et Hussain ont alors proposé [5] une méthode plus satisfaisante pour un fluide incompressible, qui consiste à identifier les noyaux tourbillonnaires en fonction du principe de pression minimale dans la rotation d'un fluide parfait. Cela revient à considérer la nature des valeurs propres du tenseur $\boldsymbol{S}^2+\boldsymbol{\Omega}^2$, dont la deuxième plus grande valeur propre doit être négative ($\boldsymbol{S}$ et $\boldsymbol{\Omega}$ correspondent aux composants symétrique et antisymétrique du tenseur de gradient de vitesse). Comparé à l'usage direct des isocontours de vorticité, ce critère a l'avantage d'isoler des structures rotationnelles telles que des tourbillons, de zones non-cohérentes (la couche limite, ...) et irrotationnelles. Pour effectuer cette étude, on utilise une simulation numérique directe de l'écoulement d'un fluide visqueux incompressible autour d'un cylindre courbé (Fig. 1) par une méthode tridimensionnelle du type 'spectral/hp element' [14].

On montre que, même si le critère de Jeong et Hussain ne nécessite aucun seuil théorique donné a priori, il est cependant nécessaire de spécifier un seuil numérique pour sa mise en oeuvre. La détermination de ce paramètre est effectuée en considérant le pourcentage d'enstrophie contenue dans les zones cohérentes détectées par le critère de Hussain en fonction du paramètre λ_2 (Fig. 3). Trois différents comportements sont alors observés : pour des très petites valeurs du seuil, proche de la définition initiale, les résultats sont dominés par le bruit numérique qui rend le critère inutilisable (Figs. 2(a) et 3). Ensuite, on constate l'apparition d'une zone intermédiaire optimale identifiant des structures cohérentes présentes dans l'écoulement (Fig. 3). Dans cette gamme, plus on accroît la valeur de λ_2, plus la taille des zones rotationnelles identifiées se voit réduite (Figs. 2(b) et 2(c)). On observe enfin, une région asymptotique. Cette région correspond physiquement aux couches de cisaillement contenant de très grandes quantités d'enstrophie sans pour autant correspondre à une structure cohérente (Figs. 2(d) et 3). Ces résultats confirment l'existence d'une région optimale intermédiaire pour le choix de λ_2 en dehors de laquelle on sera confronté soit aux zones rotationnelles mélangeant des structures cohérentes et non-cohérentes, soit à un manque d'informations sur la vorticité. Ce critère de détection, ayant finalement été validé, pourrait être utilisé de façon pertinente afin d'établir certaines propriétés intrinsèques aux tourbillons, en fonction du nombre de Reynolds : nombre et taille des tourbillons, pourcentage d'enstrophie de l'écoulement qu'ils transportent, etc.

1. Introduction

Bluff bodies are defined as those for which the flow separates from a large section of the body's surface [1]. Bluff bodies are encountered in many engineering applications i.e. offshore riser pipes, bridges, heat exchanger tubes etc. The study of the flow past these bodies is of great importance both in the research community and in the industrial world.

Once the boundary layers separate, they then become free shear layers. Above a body-dependent critical Reynolds number, the shear layers interact and roll up, resulting in vortices being shed from alternating sides of the bluff body. This produces the formation of two rows of discrete vortices in a staggered array in the wake commonly known as a von-Kármán vortex street and this process is called vortex shedding.

The shedding of vortices can cause large unsteady forces of high amplitude which under some circumstances can yield structural failure. As a result, the study of vortex shedding and transport has received significant attention from the research communities over the past century.

One of the most helpful techniques to study two- and three-dimensional vortex dynamics is the identification of the Coherent structures. Coherent structures generally refer to the organized and concentrated rotational patterns within the flow and are a useful way to characterize flow evolution in time. Several studies have been performed to identify two-dimensional structures (e.g. [2–4]). In three-dimensional flows different techniques like λ_2 criterion [5], Q criterion [6] and M_z criterion [7], are used to identify a vortex. Their properties have been compared to each other in [8] and [7]. An important feature of coherent structures identification in the λ_2 criterion is the reduction of a vector flow field into scalar quantity. Typically a specific isocontour value, or cut-off, of the scalar quantity is then chosen to identify the coherent structures. Whilst the original definitions [2,5] defined exact values for the cut-off, in numerical practice the cut-off value is varied to eliminate numerical noise.

In this Note, we focus on the choice of cut-off and its effect on the identification procedure extending previous research [4] to a three-dimensional flow: the external flow past a curved circular cylinder which generates a rich vortex shedding pattern. The asymptotical behaviour of the enstrophy values versus the cut-off choice is studied to determine the most optimal cut-off ranges. All computations are performed employing a 3D spectral/hp element method.

2. Numerical method

The mesh generation for all the computational investigations was accomplished with *Felisa* where a modified advancing layers method is employed near the pipe wall regions and a method based on the advancing front technique is employed for the rest of the domain [9].

Computations have been performed using *Nektar*, *SK*, [10], a three-dimensional incompressible Navier–Stokes solver, based on the spectral/hp element method. Spectral/hp methods are high-order discretization methods where one can increase simultaneously the number of elements (h-refinement) in the domain and the order, P, of the polynomial expansions in each element (p-refinement). In the current work this discretization has been used with unstructured and hybrid shape conforming subdomains.

To discretize the incompressible Navier–Stokes equations a high-order stiffly stable splitting-scheme [11] is employed where the non-linear terms are solved explicitly and the linear terms are solved implicitly. This high-order splitting-scheme can be interpreted as propagating the velocity and pressure fields over a time step Δt in three substeps. The first step explicitly advances the non-linear terms. Subsequently a Poisson equation for pressure is solved which imposes the divergence-free condition on the intermediate velocity field from the first step. Finally Helmholtz's systems are solved for each velocity component to impose viscous linear terms. Therefore the full scheme requires the solution of a Helmholtz equation for each component of the velocity fields and a Poisson equation for the pressure field. We note that all of the results presented were produced by the three-dimensional code where the spectral/hp element discretisation was adopted in all three Cartesian directions.

All the flow fields obtained with *Nektar* are solutions to the Navier–Stokes equations using higher order polynomial expansions. Linear finite elements are, however, employed for the post-processing utilities. The integral of ω^2 in the domain was calculated with linear shape functions N_i in three-dimensions by first decomposing each element to a large number of smaller tetrahedra. If the original element is a tetrahedron for example, then it can be decomposed to n^3 similar smaller tetrahedra with $n + 1$ equispaced interpolating points along each edge of the original element. A better resolution of the non-smooth function in space is enabled this way.

3. Identification and cut-off analysis

In order to investigate the wake structures developed by the external flow past a curved cylinder, the λ_2 criterion presented by Jeong and Hussain [5] was used for identifying the vortex cores in the wake. The λ_2 criterion stems from the pressure minimum criterion due to inviscid flow rotation and dictates that the second largest eigenvalue of $\boldsymbol{S}^2 + \boldsymbol{\Omega}^2$ must be negative in order to locate a vortex core. $\boldsymbol{S}$ and $\boldsymbol{\Omega}$ are the symmetric and antisymmetric components of the velocity gradient tensor, i.e.

$$\mathbf{S}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad \boldsymbol{\Omega}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right).$$

This identification is more insightful than the direct use of vorticity isocontours because it isolates rotational structures, such as vortices, from boundary layer and other non-rotational shear features.

To non-dimensionalise λ_2 we define $\lambda_2^\star = \lambda_2 d/U$ where d and U are a characteristic length and a velocity of the problem which for this case are taken as the cylinder diameter and free-stream velocity. The determination of an appropriate cut-off value can be performed numerically and has previously been investigated in two-dimensional flows [4,12]. In this work Creusé and Mortazavi identified an optimal cut-off value for small perturbations in a laminar flow over a dihedral plane. They showed that the optimal cut-off should be large enough to remove the numerical noise of the background flow, and small enough to distinguish the ensemble of vortical structures.

In Fig. 2 we show the $\lambda_2^\star$ coherent structures generated by the three-dimensional flow past a curved cylinder. This geometry is effectively a quarter ring of 12.5 non-dimensional radius of curvature that has an artificial blending applied to it towards the bottom and ends semi-submerged into the ground. The flow is into the page. Fig. 1 shows this geometry. For this case, a shear velocity profile and a viscous wall were applied. Specifically, as we view Fig. 1, the top boundary and the left boundary would be a symmetry boundary condition. The bottom boundary would be a viscous wall [$V = 0\ 0\ 0$]. The right boundary would be a free-stream boundary condition. The inflow and outflow planes are parallel to this page. An exponentially decaying sheared profile was chosen for the planes having a free-stream boundary condition. The sheared profile is: $v = 1 - \mathrm{e}^{-(z+12.5)}$. The velocity is equal to unity at the top of the geometry (expressed as $U_{\infty T} = 1$) and equal to zero at the bottom to conform with the no-slip condition at the wall boundary. The Reynolds number based on $U_{\infty T}$ is 100.

Several values of $\lambda_2^\star$ have been used and we present here the more characteristic features which have been obtained for (a) $\lambda_2^\star = -0.001$, (b) $\lambda_2^\star = -0.1$, (c) $\lambda_2^\star = -0.3$, (d) $\lambda_2^\star = -1$. We note that at very low values close to the analytical definition, as shown in Fig. 2(a), the structures are dominated by numerical noise and no-coherent shear layers and vortex sheets. However, within the range $0.1 \leqslant |\lambda_2^\star| \leqslant 0.6$ we observe a more optimal representation of the coherent structures capturing the three-dimensional vortex shedding and roll-up procedure from the curved

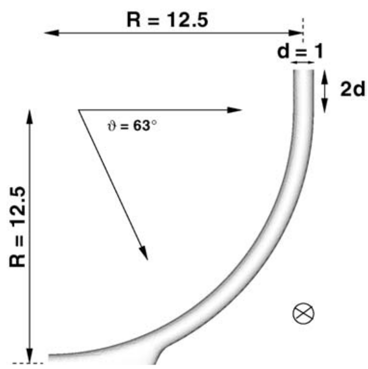


Fig. 1. Definition of body, $R/d = 12.5$. Artificial blend is applied on sections of the body that correspond to angles larger than 63° from the horizontal.

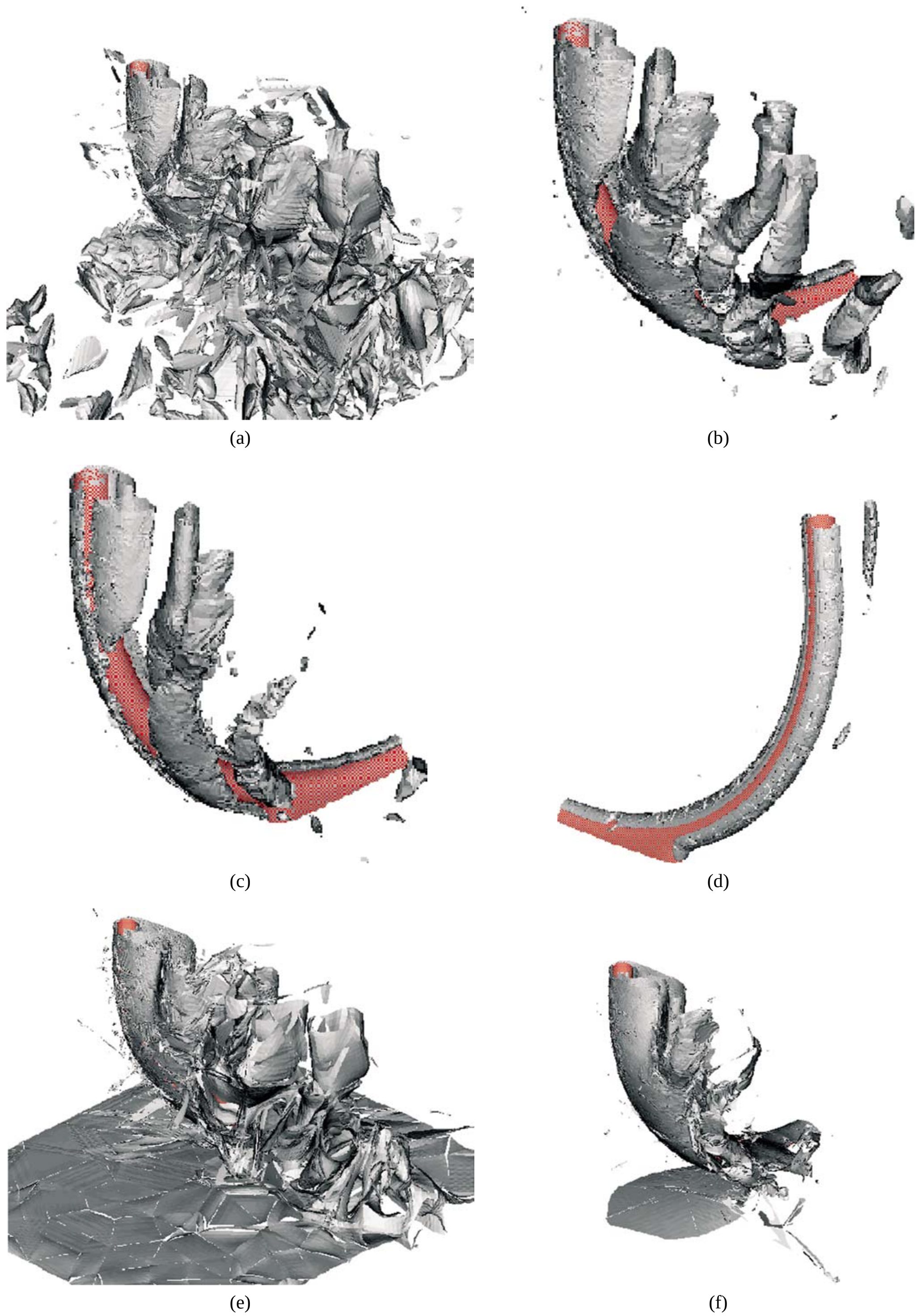
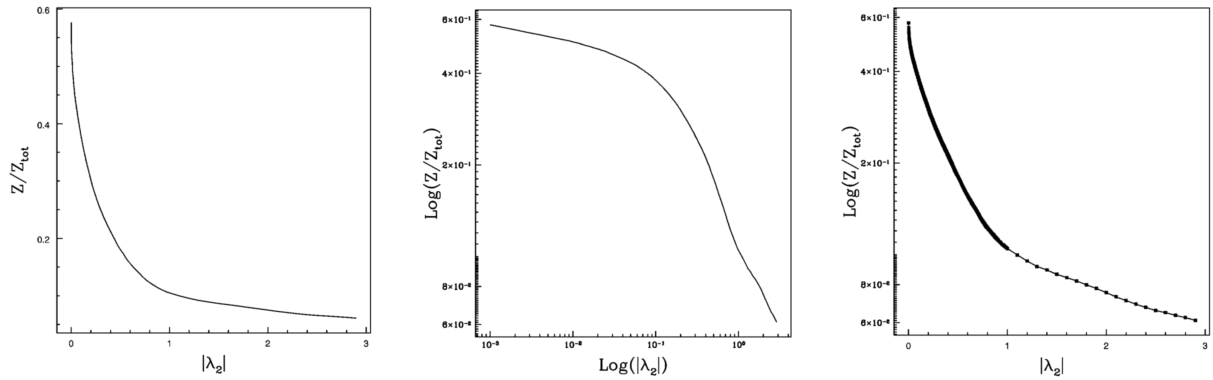


Fig. 2. Isocontours for 4 values of $\lambda_2^\star$ at: (a) $\lambda_2^\star = -0.001$; (b) $\lambda_2^\star = -0.1$; (c) $\lambda_2^\star = -0.3$; (d) -1; and normalised enstrophy isocontours at values of: (e) 0.363; and (f) 1.2.



(a) (b) (c)

Fig. 3. (a) Normalised enstrophy versus λ_2; (b) log–log axis; (c) lin–log axis.

cylinder without being obscured by the numerical noise. For even higher values of $\lambda_2^\star$, as shown in Fig. 2(d), we observe that only the most energetic shear layers are captured by the $\lambda_2^\star$ isocontour (here the criterion neglects a large part of coherent cores and tubes). Furthermore, Figs. 2(e) and 2(f) depict normalised enstrophy isocontours at values of 0.363 (average) and 1.2. The difference between the vortex core identification criterion, that captures coherent structures, and the magnitude of vorticity is thus highlighted. That means that contrary to coherent vortex identification techniques, using normalised enstrophy isocontours does not permit to avoid noisy rotational shear areas in the detection procedure.

To try and quantify the features observed in Fig. 2 we have calculated the proportion of the enstrophy captured within the coherent structures using the $\lambda_2^\star$ definition. This has then been normalised by the total enstrophy within the global flow domain and plotted as a function of the cut-off value as shown in Fig. 3. In this figure we show the same data plotted respectively on log–log and lin–log axis. They confirm basically the previous comments revealing other potential cut-offs. As the figure suggests, three different behaviours are distinguishable. For $|\lambda_2^\star| < 0.1$ there is a small region of high negative slope where a very large percentage of the total enstrophy is captured. This region corresponds to the numerically noisy vorticity fields as shown in Fig. 2(a). Next we observe change in slope of the log–log curve Fig. 3(b) between $0.1 < |\lambda_2^\star| < 0.6$ where the captured enstrophy corresponds to Figs. 2(b) and 2(c). In this range the captured enstrophy decreases with the cut-off magnitude. As the cut-off is increased, only the more energetic near wake structures are resolved, as can be seen in Figs. 2(b) and 2(c), thereby eliminating large regions of relatively low enstrosphy in the far wake structures. Finally, for $|\lambda_2^\star| > 0.6$, a third region is evident with a different slope that corresponds to the very energetic shear layers near the cylinder body as shown in Fig. 2(d). This implies that increasing $\lambda_2^\star$ above an optimal range of cut-off values generates an abrupt loss of information affecting all structures outside of the shear layers which contain the highest enstrophy.

4. Conclusions

In this work a three-dimensional spectral/*hp* element method was used to simulate an external flow around a three-dimensional curved cylinder. This 3D low Reynolds number flow generates complex vortical structures and to investigate the wake structures the $\lambda_2^\star$ criterion of Jeong and Hussain [5] has been applied. As noted by Jeong and Hussain, this technique is preferred to the vorticity magnitude which is unable to distinguish between boundary layers or vortex sheets and coherent vortical structures.

The main part of this work was focused on the choice of a cut-off range of the λ_2 coherent vortical structure identification. As a procedure for choosing an appropriate cut-off the proportion of the enstrophy captured within the λ_2 coherent structure to the total enstrophy within the global flow domain was studied. Three different behaviours were distinguished relating the coherent structures to the cut-off values. At very low values of the cut-off, close to the idealised definition, the results are dominated by numerical noise thereby making the criterion inapplicable. An optimal intermediate region was observed which captured a decreasing amount of the wake structures and vortex tubes as the cut-off value was increased. Finally, an asymptotic region was observed where the captured enstrophy varied relatively slowly with the cut-off value. This region physically corresponds to capturing only the shear layers containing the highest enstrophy of the flow almost concentrated in the vicinity of solid walls. In conclusion, we propose that numerical application of the λ_2 coherent structure identification criterion should be used in conjunction with an enstrophy capture analysis to determine a valid range of cut-off values.

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