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C. R. Mecanique 333 (2005) 311–318

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Thermocapillary convection in a three-layer system with the temperature gradient directed along the interfaces

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Received 20 September 2004; accepted after revision 5 January 2005

Presented by Évariste Sanchez-Palencia

Abstract

The paper presents numerical simulations of the Marangoni–Bénard convection in a real symmetric three-layer system. The temperature gradient is directed along the interfaces. Nonlinear regimes of steady and oscillatory convective flows are investigated by means of the finite-difference method. Transitions between the motions with different spatial structures are studied. ***To cite this article: V. Shevtsova et al., C. R. Mecanique 333 (2005).***

Résumé

Étude non-linéaire de la convection thermocapillaire dans un système symétrique à trois couches. Ce travail présente les simulations numériques de la convection de Marangoni–Bénard dans un système réel symétrique à trois couches. Le gradient de température est dirigé le long des interfaces. Les régimes non-linéaires des écoulements convectifs stationnaires et oscillatoires sont étudiés par la méthode des différences finies. Les transitions entre les mouvements à structure spatiale différente sont également étudiées. ***Pour citer cet article : V. Shevtsova et al., C. R. Mecanique 333 (2005).***

Keywords: Heat transfer; Convection; Three-layer system; Interfaces; Instabilities

Mots-clés : Transferts thermiques ; Système à trois couches ; Interfaces ; Instabilités

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doi:10.1016/j.crme.2005.01.001

1. Introduction

It is well known, that the thermocapillary convection plays a dominant role in many technological microgravity experiments. The case when the system has only one interface between different fluids has been studied analytically and numerically in many works (see, for example, the monograph [1]).

Recently a new scientific direction of investigation, convective instabilities in multilayer systems, was conceived. In multilayer systems, containing more than one interface, some new phenomena may arise as a result of the interaction between different interfaces. Numerical investigations of thermocapillary convection in multilayer systems were started in [2–4]. In these papers the linear stability of the mechanical equilibrium state and the nonlinear regimes of convection have been studied. In particular, in [2] and [4] it was shown that a three-layer system may become unstable with respect to the oscillatory disturbances. An analysis of different mechanisms of interaction between Rayleigh and thermocapillary instabilities in real three-layer systems was given in [5].

Prakash and Koster found the analytical solution describing the velocity and the temperature fields for the parallel flow in the core region of a three-layer fluid system under the action of the temperature gradient directed along the interfaces (see [6,7]). The flow field in the end-wall region was analyzed by matching with the core region flow [7,8]. This approach works for long layers in the case of weak nonlinear effects (for small values of Grashof and Marangoni numbers).

The experimental results on Marangoni–Bénard instability under microgravity conditions in three-layer systems have been described in [9–11].

In the present Note we perform the nonlinear simulations of convective flows in a closed cavity filled by a symmetric three-layer system when one fluid is sandwiched between two layers of another fluid. The case where the temperature gradient is directed along the interfaces is considered. Both steady and oscillatory convective regimes are studied.

The paper is organized as follows. The formulation of the problem is given in Section 2. The results of nonlinear simulations are presented in Section 3. Section 4 contains some concluding remarks.

2. General equations and boundary conditions

Let the space between two parallel rigid horizontal plates be filled by three immiscible viscous fluids with different physical properties (see Fig. 1). Even in the absence of gravity, we shall call the layer 1 as the 'top layer' and the layer 3 as the 'bottom layer'. The equilibrium thicknesses of the layers are a_m, $m = 1, 2, 3$. We assume that deformations of interfaces are small, and their influence on the flow and the temperature distribution can be ignored. The surface tension coefficients on the upper and lower interfaces, σ and σ_*, are linear functions of temperature T: $\sigma = \sigma_0 - \alpha T$, $\sigma_* = \sigma_{*0} - \alpha_* T$. We do not take into account buoyancy effects which are negligible in the case of thin layers or under microgravity conditions. The vertical plates are kept at different constant temperatures. The overall temperature drop is θ. Let us use the following notations:

$$\rho = \rho_1/\rho_2, \quad \nu = \nu_1/\nu_2, \quad \eta = \eta_1/\eta_2, \quad \kappa = \kappa_1/\kappa_2, \quad \chi = \chi_1/\chi_2, \quad a = a_2/a_1$$
$$\rho_* = \rho_1/\rho_3, \quad \nu_* = \nu_1/\nu_3, \quad \eta_* = \eta_1/\eta_3, \quad \kappa_* = \kappa_1/\kappa_3, \quad \chi_* = \chi_1/\chi_3, \quad a_* = a_3/a_1$$

Here ρ_m, ν_m, η_m, κ_m, χ_m and a_m are, respectively, the density, the kinematic and the dynamic viscosity, the heat conductivity and the thermal diffusivity of the mth layer ($m = 1, 2, 3$). As units of length, time, velocity, pressure and temperature we use a_1, a_1^2/ν_1, ν_1/a_1, $\rho_1\nu_1^2/a_1^2$ and θ, respectively. The complete nonlinear equations governing the Marangoni convection (see [5]) have the following form:

$$\frac{\partial \vec{v}_m}{\partial t} + (\vec{v}_m \nabla)\vec{v}_m = -e_m \nabla p_m + c_m \Delta \vec{v}_m \tag{1}$$

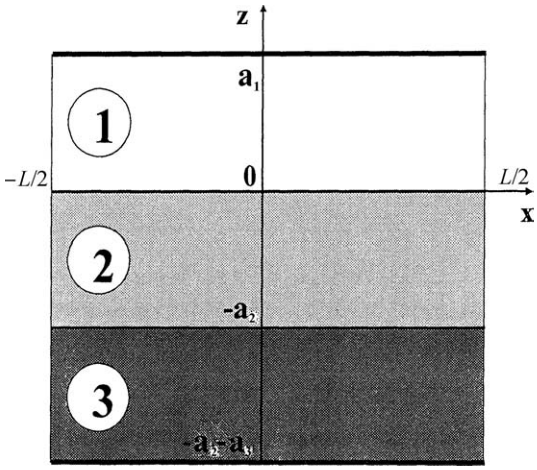

Fig. 1. Geometric configuration of the region and coordinate axes.

$$\frac{\partial T_m}{\partial t} + \vec{v}_m \nabla T_m = \frac{d_m}{P} \Delta T_m \tag{2}$$

$$\nabla \vec{v}_m = 0 \tag{3}$$

Here, $e_1 = c_1 = d_1 = 1$; $e_2 = \rho$, $c_2 = 1/\nu$, $d_2 = 1/\chi$; $e_3 = \rho_*$, $c_3 = 1/\nu_*$, $d_3 = 1/\chi_*$.
The conditions on the rigid horizontal boundaries are:

$$z = 1: \quad \vec{v}_1 = 0; \quad T_1 = 1/2 - x/L \tag{4}$$

$$z = -a - a_*: \quad \vec{v}_3 = 0; \quad T_3 = 1/2 - x/L \tag{5}$$

We consider the following conditions on vertical boundaries ($m = 1, 2, 3$):

$$x = -L/2, L/2: \quad \vec{v}_m = 0; \quad \frac{\partial T_m}{\partial x} = 0 \tag{6}$$

The boundary conditions on the interface $z = 0$ can be written in the form:

$$\eta \frac{\partial v_{1x}}{\partial z} - \frac{\partial v_{2x}}{\partial z} - \frac{\eta M}{P} \frac{\partial T_1}{\partial x} = 0, \quad \eta \frac{\partial v_{1y}}{\partial z} - \frac{\partial v_{2y}}{\partial z} - \frac{\eta M}{P} \frac{\partial T_1}{\partial y} = 0 \tag{7}$$

$$v_{1x} = v_{2x}, \quad v_{1y} = v_{2y}, \quad v_{1z} = v_{2z} = 0 \tag{8}$$

$$T_1 = T_2 \tag{9}$$

$$\kappa \frac{\partial T_1}{\partial z} = \frac{\partial T_2}{\partial z} \tag{10}$$

and at $z = -a$:

$$\eta^{-1} \frac{\partial v_{2x}}{\partial z} - \eta_*^{-1} \frac{\partial v_{3x}}{\partial z} - \frac{M}{P} \frac{\partial T_1}{\partial x} = 0, \quad \eta^{-1} \frac{\partial v_{2y}}{\partial z} - \eta_*^{-1} \frac{\partial v_{3y}}{\partial z} - \frac{M}{P} \frac{\partial T_1}{\partial y} = 0 \tag{11}$$

$$v_{2x} = v_{3x}, \quad v_{2y} = v_{3y}, \quad v_{2z} = v_{3z} = 0 \tag{12}$$

$$T_2 = T_3 \tag{13}$$

$$\kappa^{-1} \frac{\partial T_2}{\partial z} = \kappa_*^{-1} \frac{\partial T_3}{\partial z} \tag{14}$$

Here $P = \nu_1/\chi_1$ is the Prandtl number for the liquid in layer 1 and $M = \alpha\theta a_1/\eta_1\chi_1$ is the Marangoni number.

The boundary value problem formulated above was solved by the finite-difference method. Equations were approximated on a rectangular uniform mesh 84×56 ($L = 3.2$), 112×56 ($L = 16$), 168×56 ($L = 32$) using a second-order approximation for the spatial coordinates. The explicit scheme was applied for solving the evolution equations. The Poisson equations were solved by the iterative Liebman successive overrelaxation method on each time step: the accuracy of the solution was 10^{-5}. The details of the method may be found in [1].

3. Numerical results

Let us consider the real symmetric system silicone oil 47v2–water–silicone oil 47v2 with the following set of parameters: $\eta = 1.7375$, $\nu = 2$, $\kappa = 0.184$, $\chi = 0.778$; $\eta_* = \nu_* = \kappa_* = \chi_* = 1$; $P = 25.7$. We take $a = a_* = 1$. This means that the exterior layers have the same thermophysical properties.

First, let us consider the cavity with the small aspect ratio ($L = 3.2$). Even for small values of the Marangoni number ($M \neq 0$) the mechanical equilibrium state is impossible and a steady motion takes place in the system. The streamlines and isotherms for the definite value of the Marangoni number are presented in Fig. 2. One can see that in the central part of the cavity the flow is nearly parallel. Along the interfaces, the fluids move from the hot wall to the cold wall. In the middle layer the motion consists of two vortices of different signs and has the 'two-store' structure. The flow fields in different layers are coupled by viscous stresses. Near the lateral walls the fluid may move both upwards and downwards. For relatively small values of M the flows are quite symmetric with respect to the vertical axis $x = 0$.

With the increase of the Marangoni number the intensity of the flow near the hot wall becomes higher than that near the cold wall. At the larger values of M the steady motion becomes unstable and the regular oscillations develop in the system. The snapshots of the streamlines for one period of oscillations are presented in Fig. 3. During the oscillatory process the vortices become longer in the horizontal direction (Fig. 3(b)) and than the additional vortices appear in the middle layer (Fig. 3(c)). These new vortices move to the right, than they are reflected by the cold lateral wall and start to move in the opposite direction (Figs. 3(e)–(g)). Finally, the new vortices couple with the main vortices in the middle layer and the oscillatory process is repeated.

Now let us consider the longer cavity with $L = 16$. As in the case of the short cavity, for relatively small values of the Marangoni number ($M \neq 0$) the steady motion takes place in the system. At the larger values of M ($M_* > 130\,000$) the steady state becomes unstable and the oscillations appear in the system. The snapshots of

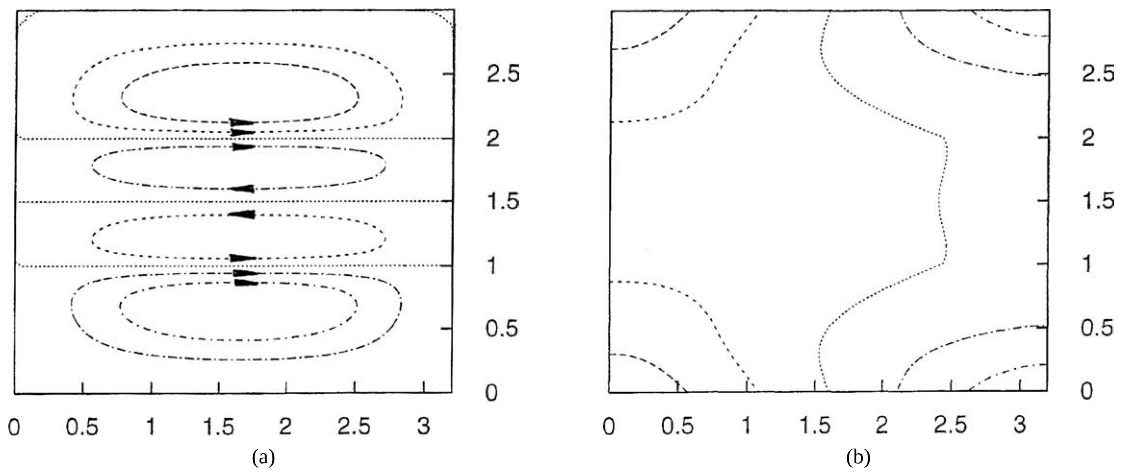

Fig. 2. (a) Streamlines and (b) isotherms for the steady motion at $M = 1000$; $L = 3.2$.

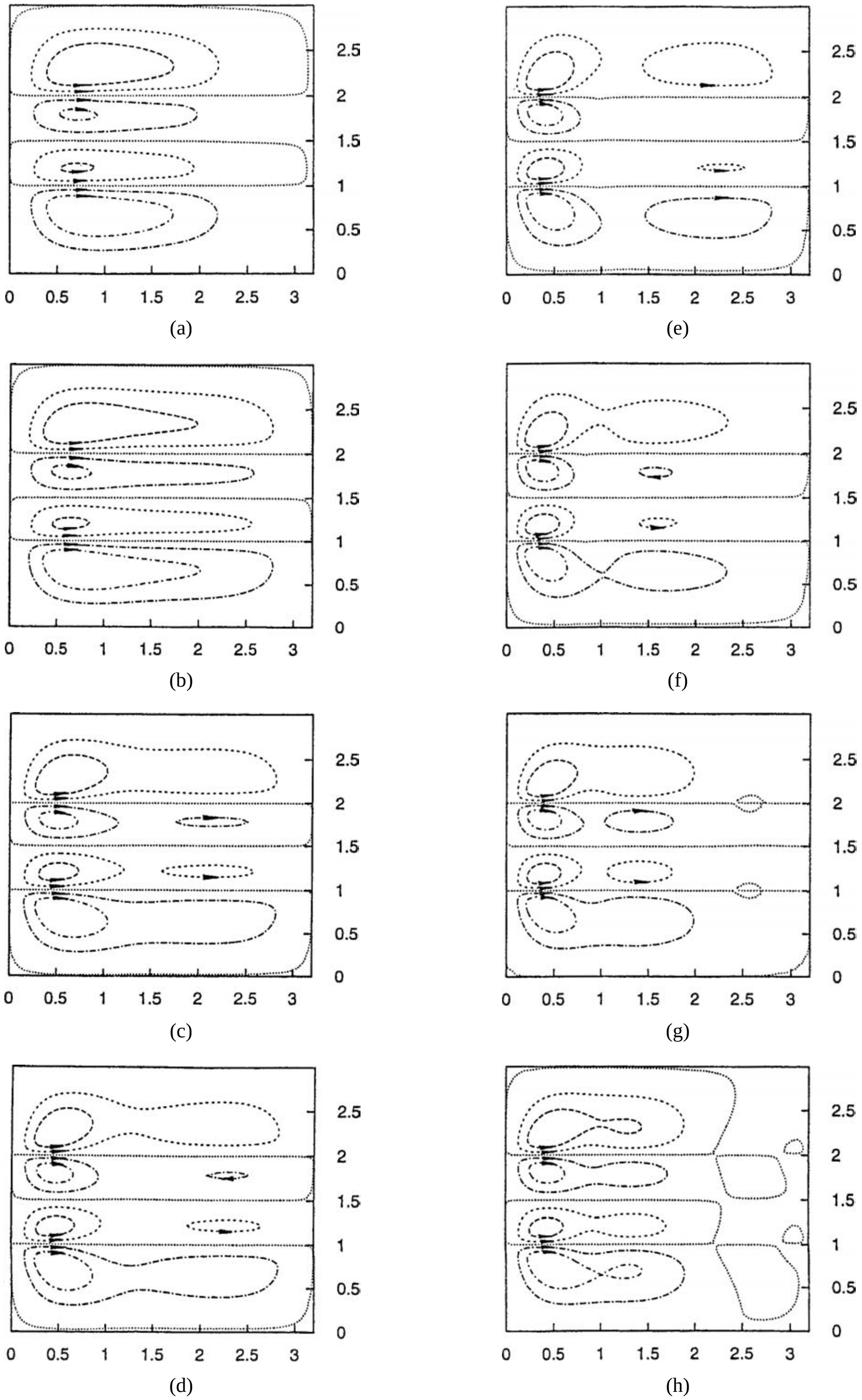

Fig. 3. Snapshots of streamlines for the oscillatory motion during one period at $M = 350\,000$; $L = 3.2$.

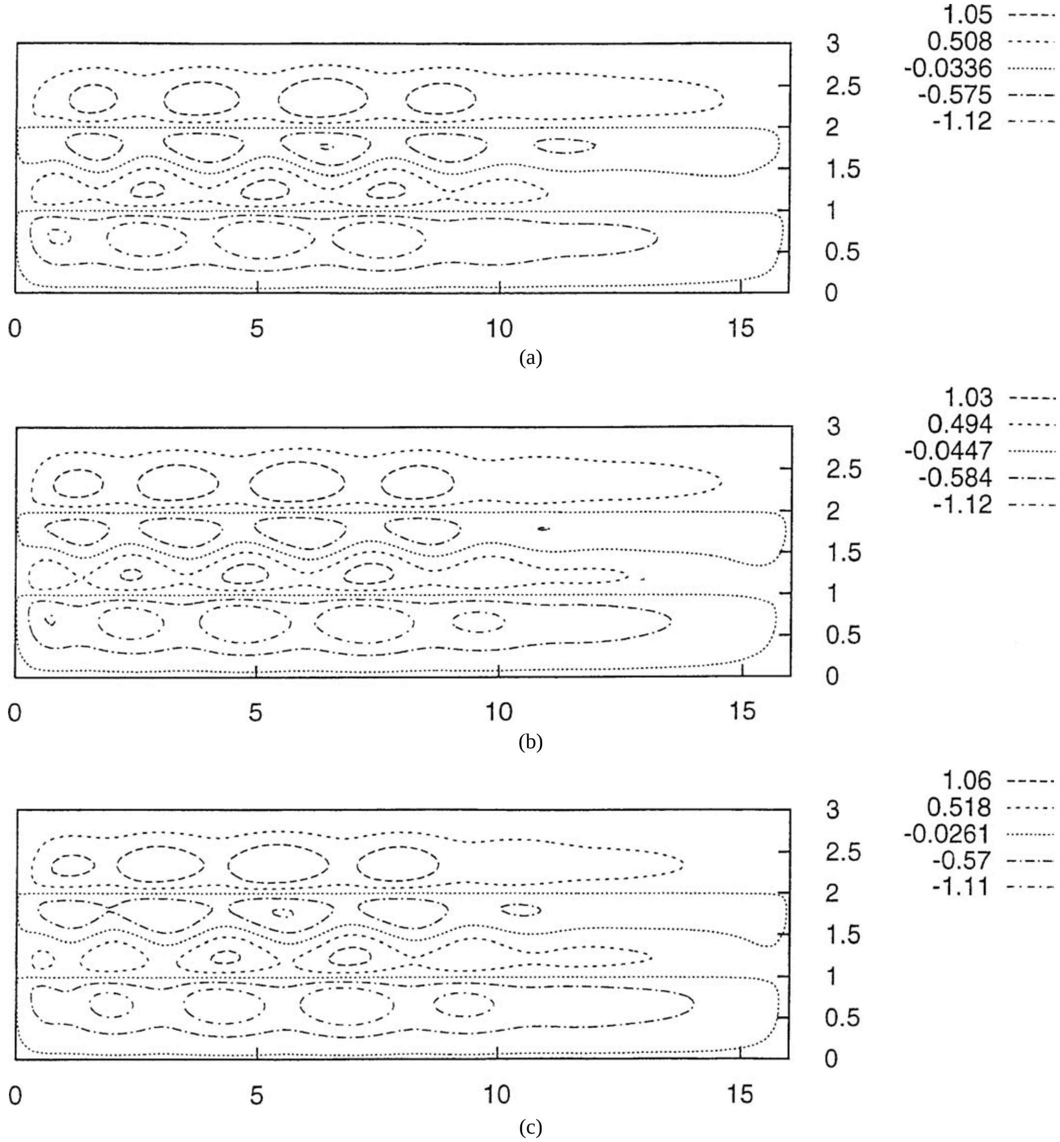

Fig. 4. Snapshots of streamlines for the oscillatory motion during the half of the period at $M = 250\,000$; $L = 16$.

streamlines for the half of the period of oscillations are presented in Fig. 4; the vortices in the middle layer have the chess-order configuration. One can see that during the oscillatory process the number of vortices is changed in the layers. The wave moves to the left, i.e. from the cold end to the hot end. This direction of the motion is characteristic for hydrothermal waves [12].

For the cavities with $L = 32$ the transition from the steady state to the oscillatory flow takes place at $M_* \approx 80\,000$. The snapshots of streamlines for the oscillatory process during one period are shown in Fig. 5. For larger values of M the oscillations become irregular. The full diagram of convective regimes on the plane (L, M) is presented in Fig. 6.

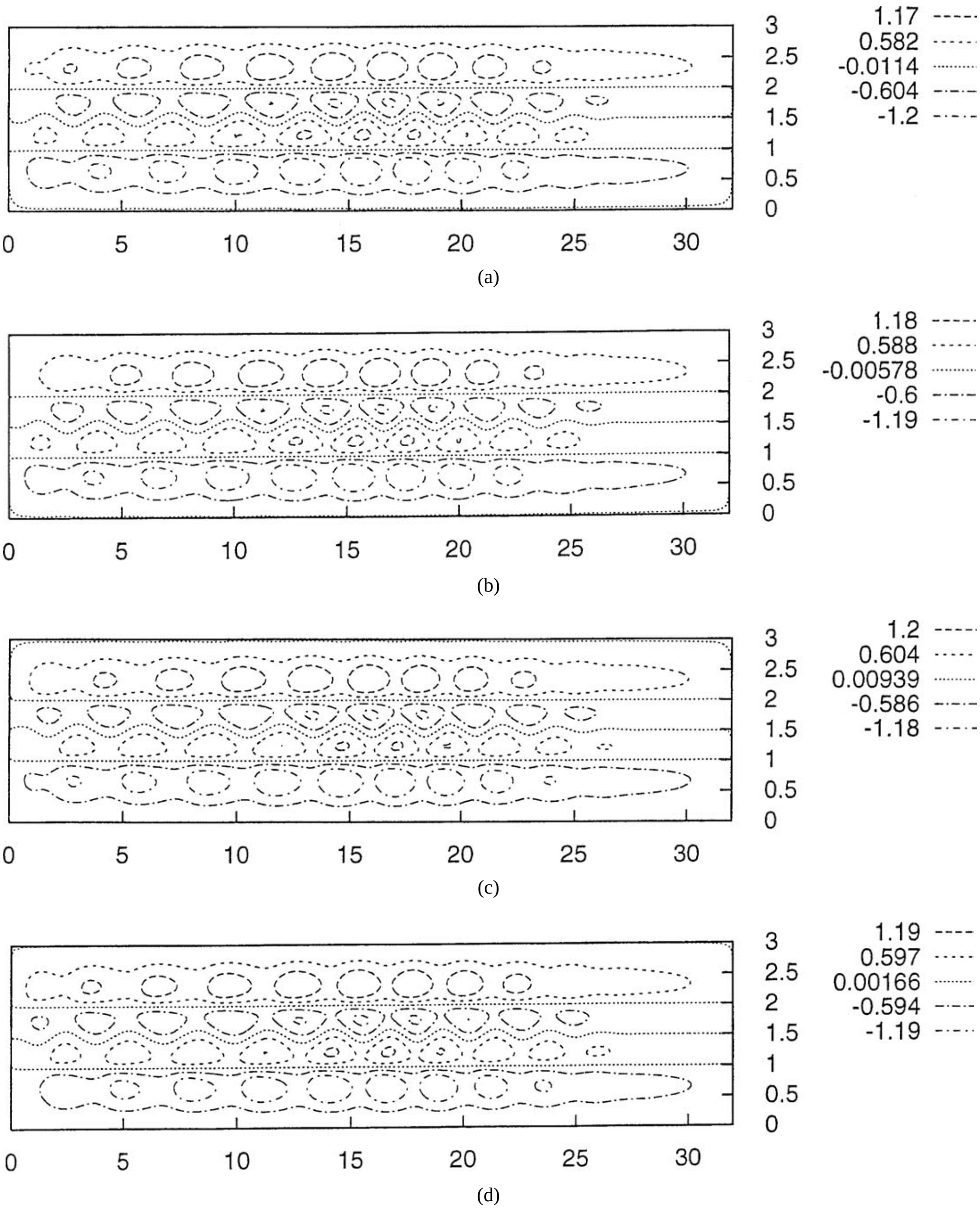

Fig. 5. Snapshots of streamlines for the oscillatory motion during one period at $M = 110\,000$; $L = 32$.

4. Conclusion

Nonlinear regimes of convection in a three-layer symmetric system when the temperature gradient is directed along the interfaces, have been studied. The shape and the amplitude of convective flows are investigated by the finite-difference method. It is found that with the increase of the Marangoni number the steady state becomes

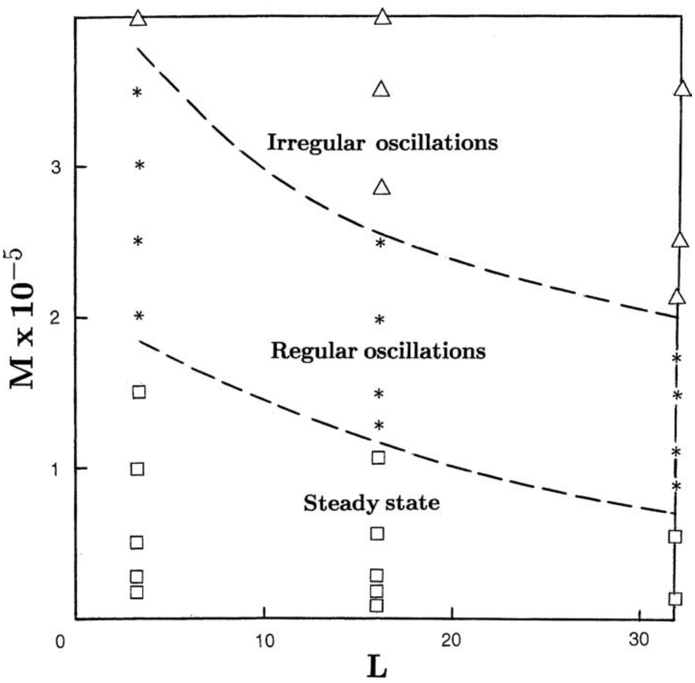

Fig. 6. The diagram of regimes.

unstable and the oscillations develop in the system. In long cavities the hydrothermal waves are developed. The waves move in the direction of the temperature gradient. The diagram of convective regimes is constructed. The presented type of oscillations may be fulfilled in experiments under microgravity conditions.

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