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On the cavitation problem and the influence of the surface energy in a viscoplastic medium

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Abstract

In this Note, the solution for spherically symmetric cavitation in a viscoplastic material is analysed. To ensure of the reality of the physical behaviour of the material, the problem is studied by considering a hollow sphere whose matrix obeys to a modified Bodner and Partom model. This local phenomenon is understood in the sense of the rapid growth of a pre-existing void and a particular attention is made to understand the influence of the surface energy on the critical dilative stress. ***To cite this article: F. Zaïri et al., C. R. Mecanique 333 (2005).***

Résumé

Sur le problème de la cavitation et de l'influence de l'énergie de surface dans un milieu viscoplastique. Dans ce travail, la solution pour la cavitation à symétrie sphérique dans un matériau viscoplastique est analysée. Afin de s'assurer de la réalité physique du comportement du matériau, le problème est étudié en considérant une sphère creuse dont la matrice obéit à un modèle de Bodner et Partom modifié. Ce phénomène local est assimilé à la croissance rapide d'un vide préexistant et une attention particulière est portée sur l'influence de l'énergie de surface sur la contrainte critique de dilatation. ***Pour citer cet article : F. Zaïri et al., C. R. Mecanique 333 (2005).***

Keywords: Computational solid mechanics; Viscoplasticity; Nonlinear behaviour; Cavitation; Void growth; Surface tension

Mots-clés : Mécanique des solides numérique ; Viscoplasticité ; Comportement non-linéaire ; Cavitation ; Croissance de vide ; Tension de surface

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Nous proposons dans cette Note d'analyser la solution décrivant l'inflation d'un vide ou encore la cavitation dans un matériau viscoplastique présentant un comportement fortement non-linéaire. L'effet de la vitesse, de la tension de surface et de la taille initiale du vide est également mis en évidence.

Le phénomène particulier de la cavitation, dans les matériaux élastiques ou à écoulement, a fait l'objet de nombreux travaux depuis 45 ans [1–4,8–10,14]. Une brève revue de la littérature conduit à la conclusion que le choix du comportement de la matrice est de première importance sur les résultats et sur la réalité physique du comportement du matériau.

Nous présentons brièvement, en guise de première partie, un modèle viscoplastique développé initialement pour les métaux [11] et que nous avons modifié dans un précédent travail [12,13] pour représenter le comportement fortement non-linéaire de polymères vitreux : Éqs. (2) et (3). En utilisant un formalisme à variables internes simulant la résistance à l'écoulement moléculaire, une variable interne Z_1, représentant le durcissement, est superposée à une autre variable Z_2 représentant l'adoucissement. Les performances de ce modèle modifié ont pu être vérifiées sur différents polymères vitreux [12,13] : La forme complexe du chemin contrainte-déformation et la sensibilité à la vitesse de déformation se sont révélées être bien capturées par ce modèle.

Dans une deuxième partie, on aborde la cavitation, dans le cas statique, en réalisant une connexion entre ce phénomène local et le comportement du matériau. Le problème est étudié en considérant un vide sphérique, interprété comme un défaut, centré dans une sphère homogène, isotrope et viscoplastique soumise à une traction isotrope sur sa surface externe et à une tension de surface sur sa surface interne (la pression associée étant donnée en (5)). L'équation d'équilibre pour ce problème est donnée dans la direction radiale en (4). La contrainte équivalente de von Mises, dérivant du modèle viscoplastique modifié (Éq. (2)) et de l'écriture classique des composantes de la déformation en coordonnées sphériques, est donnée en (6). L'injection de la contrainte équivalente (Éq. (6)) et des variables internes (Éq. (7)) dans l'Éq. (4), permet d'aboutir à l'Éq. (9), qui est reformulée dans (12) à partir de (10) et (11). Dans cette étude, la solution approximative de la relation (12) a été analysée. Il est à remarquer que la solution exacte, donnée dans [14], peut être retrouvée en considérant uniquement l'instant de l'écoulement et une surface interne de la sphère creuse libre de tout effort.

On présente à la Fig. 1, une étude de sensibilité montrant l'influence du paramètre de reprise du durcissement α. On remarque que la résistance à la dilatation croit avec la diminution de la valeur de α. L'influence de la vitesse de dilatation macroscopique et de la fraction volumique de vide initiale sur la contrainte critique peut être clairement vue à la Fig. 2. Une attention toute particulière est portée sur l'influence de la tension de surface sur le processus de cavitation et cette prise en compte constitue l'une des principales originalités de ce travail. La Fig. 3 montre que la tension de surface a pour effet d'augmenter sensiblement la contrainte critique et n'agit pratiquement plus pour des cavités de taille supérieure au micromètre.

1. Introduction

The sudden formation of a void in a solid material, phenomenon named cavitation, has been of major interest in the mechanic of solids and has been studied by many works since 45 years. Experimentally, Gent and Lindley [1] have observed many spherical cavities in a rubber subjected to tension loading. The theoretical solution of the cavitation problem in a neo-Hookean sphere, subjected to uniform radial tension at the outer surface, was given by Ball [2] and was found to be in good agreement with the experiments of Gent and Lindley [1]. The cavitation in the rubbery phase of glassy polymers blended with rubber particles has received recently particular attention [3,4]. The influence of the surface energy in this analytical modelling was the topic of particular interests and was found as an active parameter for cavitation phenomenon. These studies are based on a linear elastic analysis and assume that rubber particles cavitation occurs before plasticity (shear bands) formation. However, for some rubber blended polymers the sequence of these toughening deformation mechanisms was found reversed [5,6]. Furthermore, it is now commonly established that the craze initiation in homogeneous ductile glassy polymers starts at a local plastic deformation [7]. In the literature, the cavitation problem has been also investigated in a plastic material by Tvergaard [8] or in a viscoplastic material by Badea and Predeleanu [9] using the Perzyna viscoplastic model. The cavitation in a nonlinear elastic solid [10] or in a plastic or viscoplastic solid [9] is often seen as the rapid growth of a pre-existing void.

It is commonly known that the deformation of ductile materials (plastic or viscoplastic) is dominated by an intrinsic post-yield nonlinear behaviour controlling the void growth process until fracture. Therefore, the choice of a stress–strain relation is a primary importance in the problem.

In this Note, constitutive equations, used to predict the highly nonlinear behaviour and strain rate sensitivity of glassy polymers, are briefly presented. Next, we analyze an approximate solution describing spherical symmetric cavitation in this viscoplastic medium. Furthermore, the influence of the surface tension on the critical stress is also studied. In the last section, the main remarks are given.

2. A viscoplastic model

One of the more popular viscoplastic model is this proposed by Bodner and Partom [11]. Although the macromolecular structure of polymers differ of that of metals, in a previous papers [12,13] we have modified this model developed initially for metals, to predict the stress–strain polymers response. The viscoplastic strain rate is an explicit function of both the deviatoric stress $\boldsymbol{\sigma}'$, the von Mises equivalent stress σ_{eq} and the accumulated viscoplastic strain rate $\dot{p}$:

$$\mathbf{d}^p = \frac{3}{2}\dot{p}\frac{\boldsymbol{\sigma}'}{\sigma_{\mathrm{eq}}} \tag{1}$$

The accumulated viscoplastic strain rate is expressed as:

$$\dot{p} = \frac{2}{\sqrt{3}}D_0\left(\frac{\sigma_{\mathrm{eq}}}{Z_1+Z_2}\right)^{2n} \tag{2}$$

where Z_2 and Z_1 are two internal variables representing the resistance to molecular flow and the intrinsic strain softening and rehardening behaviour, respectively. Z_1 and Z_2 are governed by the following differential equations:

$$\dot{Z}_1 = \frac{m}{Z_{10}}\big(Z_1-(1-\alpha)Z_{10}\big)\dot{p} \quad \text{and} \quad \dot{Z}_2 = h\left(1-\frac{Z_2}{Z_{2s}}\right)\dot{p} \tag{3}$$

It is important to note that in the evolution equation (3) of the internal state variables, the accumulated viscoplastic strain is used instead of the viscoplastic work. The material constants include: D_0, n, Z_{10}, m, h, Z_{2s} and α. D_0 is the limiting shear strain rate, n is the strain rate sensitivity parameter, Z_{10} is the initial value of Z_1, Z_{2s} is the saturation value of Z_2, m, h and α are the nonlinear parameters. The initial value of Z_2 is taken equal to zero.

The complex shape of the stress–strain curves as well as the strain rate dependent of homogeneous glassy polymers are captured by this model [12,13]. So, in order to ensure that the physical behaviour of the material is realistic, the constitutive model is used to study the yield behaviour of a hollow sphere under hydrostatic loading in the outer surface and a surface energy in the inner surface.

3. Spherically symmetric cavitation

The hollow sphere, of inner radius a and outer radius b, is assumed to be homogeneous, isotropic and incompressible. The sphere is subjected to hydrostatic stresses and obeys constitutive model presented in Section 2. The spherical coordinates (R,Θ,Φ) and (r,θ,φ), respectively, in the undeformed and deformed configuration are used. The effect of inertia on the growth rate of voids is neglected. Furthermore, the distortion phenomenon is not accounted for in the analysis. The equilibrium equation for spherically symmetric deformation, in the static case, can be reduced in the radial direction as: $\partial\sigma_{rr}/\partial r - 2\sigma_{\mathrm{eq}}/r = 0$, where r is the radial coordinate in the current configuration and with $\sigma_{\mathrm{eq}} = \sigma_{\theta\theta} - \sigma_{rr}$ (σ_{rr} is the radial stress and $\sigma_{\theta\theta}$ is the hoop stress). The interior cavity surface is subjected to a surface tension and the outer surface of the sphere is subjected to a uniform radial stress σ_m. The one-dimensional problem of spherical void growth in the sphere is given by:

$$\sigma_{rr}(b) - \sigma_{rr}(a) = \sigma_m - P_i = \int_a^b \frac{2}{r}\sigma_{\mathrm{eq}}\,\mathrm{d}r \tag{4}$$

where P_i is the internal hydrostatic stress in the cavity which is expressed, in the current configuration, as:

$$P_i = \frac{2\gamma}{a} = \frac{2\gamma}{a_0}\left(\frac{f_0}{f}\frac{1-f}{1-f_0}\right)^{1/3} \tag{5}$$

In Eq. (5), γ is the surface tension, a_0 is the initial inner radius, $f = V_v/V$ and $f_0 = V_{v0}/V_0$ are the current and initial void volume fraction, respectively, V_v and V_{v0} are the current and initial void volume, respectively, V and V_0 are the current and initial volume, respectively.

By symmetry, the strain components in spherical coordinates can be easily obtained and therefore the von Mises equivalent stress can be given by:

$$\sigma_{\mathrm{eq}} = (Z_1 + Z_2)\left(\frac{\sqrt{3}}{D_0}\frac{a^2\dot{a}}{r^3}\right)^{1/2n} \tag{6}$$

The internal state variables, used to simulate the flow molecular resistance in Eq. (6), can be expressed explicitly by the integration of Eqs. (3):

$$Z_1 = (1-\alpha)Z_{10} + \alpha Z_{10}\exp\left(\frac{m}{Z_{10}}p\right) \quad \text{and} \quad Z_2 = Z_{2s}\left(1 - \exp\left(-\frac{h}{Z_{2s}}p\right)\right) \tag{7}$$

If we interest, as a first approximation, only at the instant of yield, i.e. $h = 0$ and $m = 0$, the solution of Eq. (4) exactly becomes:

$$\frac{\sigma_m}{Z_{10}} = \frac{4n}{3}\left(\frac{\sqrt{3}}{D_0}\frac{\dot{a}}{a}\right)^{1/2n}\left(1 - f^{1/2n}\right) + \frac{P_i}{Z_{10}} \tag{8}$$

We can note that in the case of free inner surface one recovers the solution given in [14]. From a variety of models investigated, it is found that the critical hydrostatic stress or the instability strongly depend of the stress–strain relation.

By substituting the expressions (6) and (7) into (4), we arrive at the formula:

$$\sigma_m - \frac{2\gamma}{a} = \int_a^b 2\left[(1-\alpha)Z_{10} + \alpha Z_{10}\exp\left(\frac{m}{Z_{10}}p\right) + Z_{2s}\left(1 - \exp\left(-\frac{h}{Z_{2s}}p\right)\right)\right]\left(\frac{\sqrt{3}}{D_0}\frac{a^2\dot{a}}{r^3}\right)^{1/2n}\frac{\mathrm{d}r}{r} \tag{9}$$

The accumulated viscoplastic strain can be expressed as:

$$p = \int_0^t \dot{p}\,\mathrm{d}t = 2\ln\left(\frac{r}{R}\right) = 2\ln\left(\frac{r}{(r^3 - \beta^3)^{1/3}}\right) \tag{10}$$

where β is a parameter depending only on time and defined by $\beta^3 = r^3 - R^3$.

We get:

$$u = (R/r)^3 = 1 - (\beta/r)^3 \tag{11}$$

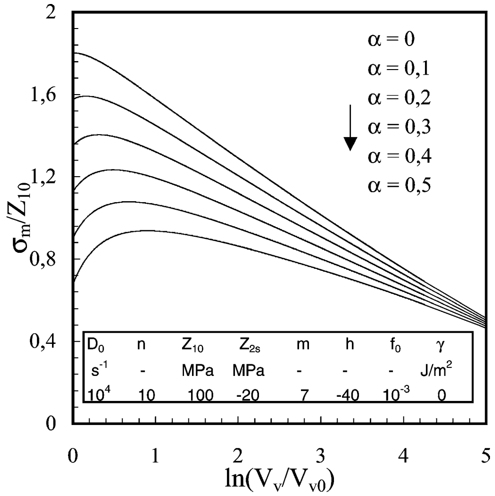

Fig. 1. Normalized hydrostatic stress versus volume strain of the void and effects of the hardening parameter α.

Fig. 1. Contrainte hydrostatique normalisée en fonction de la déformation volumique du vide et effets du paramètre d'écrouissage α.

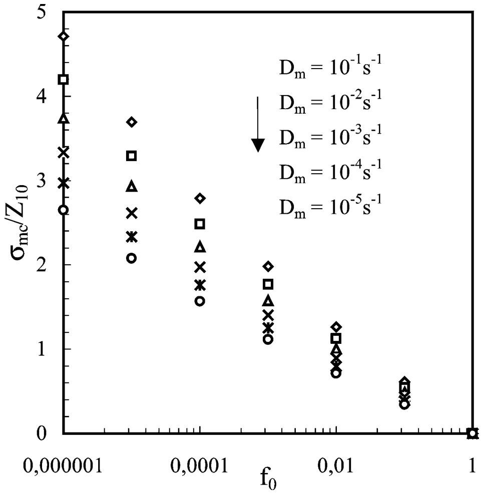

Fig. 2. Critical normalized hydrostatic stress versus initial void volume fraction and effects of the expansion rate.

Fig. 2. Contrainte hydrostatique critique normalisée en fonction de la fraction volumique de vide initiale et effets de la vitesse d'expansion.

By substituting Eqs. (10) and (11) in Eq. (9), this last one can be rewritten as:

$$\sigma_m - \frac{2\gamma}{a} = \int_{V_{v0}/V_v}^{V_0/V} \frac{2}{3}\left[(1-\alpha)Z_{10} + Z_{2s} + \alpha Z_{10}\left(\frac{1}{u}\right)^{2m/3Z_{10}} - Z_{2s}\left(\frac{1}{u}\right)^{-2h/3Z_{2s}}\right] \times \left(\frac{\sqrt{3}}{(1-V_{v0}/V_v)D_0}\frac{\dot{a}}{a}(1-u)\right)^{1/2n} \frac{\mathrm{d}u}{1-u} \tag{12}$$

where $\dot{a}/a$ is related to the mean macroscopic strain rate D_m: $\dot{a}/a = D_m/f$.

The relation between the hydrostatic stress, the surface tension, the material properties, the expansion rate and the void expansion by considering the spherically symmetric cavitation problem can be easily given by the approximate solution of Eq. (12). The influence of the strain softening and the strain hardening on the post-yield behaviour is taken into account in the analysis. In order to obtain realistic material parameters from test data of various materials, an original method was developed [12]. In the next, an approximate value of these parameters is used.

In Fig. 1, the normalized hydrostatic stress versus true volume strain of the void is plotted for five different values of the hardening parameter α when loading is applied by a constant expansion rate $D_m = 10^{-4}\ \mathrm{s}^{-1}$. In order to drive the void expansion, below the critical dilative stress, an increase of the normalized hydrostatic stress is required. Resulting to instability of void growth, the hydrostatic stress decreases monotonically as the void continues to grow. Furthermore, the peak stress to reach instability during void growth increases with the decreasing of the α value.

In Fig. 2, the critical normalized hydrostatic stress is plotted versus the initial void volume fraction for five different expansion rate. The material parameters are those of Fig. 1 and $\alpha = 0.2$. When the initial void volume fraction f_0 decreases, the critical normalized hydrostatic stress σ_{mc}/Z_{10} increases and this effect increases with the increasing of the mean macroscopic strain rate.

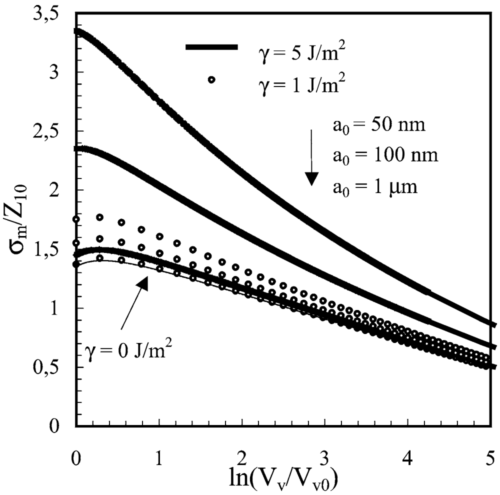

Fig. 3. Effects of the surface tension and the initial void size on the normalized hydrostatic stress.

Fig. 3. Effets de la tension de surface et de la taille initiale du vide sur la contrainte hydrostatique normalisée.

In Fig. 3, with the same material parameters as previously, the strong effect of the surface tension on the normalized hydrostatic stress can be clearly seen for two surface tension values and a constant expansion rate $D_m = 10^{-4}\ s^{-1}$. The surface tension in the cavity increases its growth resistance. This sensitivity analysis also shows the influence of the initial cavity size a_0. For a initial void size greater than 1 µm, the hydrostatic stress does not differ greatly with the no surface tension behaviour.

4. Conclusion

In this work, we have studied a spherical void interpreted as a defect and centered in a spherical viscoplastic matrix under purely hydrostatic loading and an approximate solution was analyzed. In particular, the role of the hardening parameter α, the expansion rate and the initial void volume fraction has been highlighted. An attempt was made to understand the influence of the surface tension in cavitation process. These one contributes to the resistance against growth of the void and does not act any more for initial void size bigger than 1 µm.

However, at the void size scale, the surface energy value is not very well known and constitutes one of the remaining questions. Furthermore, the study of the void initiation process at the nanometer scale with this continuum model can be questioned. At this scale, a molecular approach [15] would be perhaps more appropriated [16] than the formalism used in this study.

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