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Theoretical modeling and numerical study of coalescence of cavities in porous ductile viscoplastic solids

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Abstract

One presents a model for coalescence of cavities in porous ductile viscoplastic solids. The representative volume element considered is schematized as a 'sandwich' consisting of a central porous layer surrounded by two external sound layers, the stress and strain rate tensors being considered as homogeneous in each layer. The sound layers obey the classical Norton model and the porous one some specific homogenized model for porous viscoplastic solids accounting for void shape. An important feature is the description of the peculiar evolution of this shape during coalescence. The model predictions are successfully compared to the results of some finite element micromechanical simulations. ***To cite this article: L. Flandi, J.-B. Leblond, C. R. Mecanique 333 (2005).***

Résumé

Modélisation théorique et simulation numérique de la coalescence des cavités dans les matériaux poreux ductiles viscoplastiques. On présente un modèle pour la coalescence des cavités dans les solides poreux ductiles viscoplastiques. Le volume représentatif élémentaire considéré est schématisé par un 'sandwich' comprenant une couche centrale poreuse entourée de deux couches externes saines, les tenseurs de contrainte et de taux de déformation étant considérés comme homogènes dans chaque couche. Les couches saines obéissent au modèle classique de Norton et la couche poreuse à un modèle homogénéisé spécifique pour les matériaux poreux viscoplastiques prenant en compte la forme des cavités. Un élément important est la description de l'évolution particulière de cette forme pendant la coalescence. Les prédictions du modèle sont comparées avec succès aux résultats de simulations micromécaniques par éléments finis. ***Pour citer cet article : L. Flandi, J.-B. Leblond, C. R. Mecanique 333 (2005).***

Keywords: Porous media; Coalescence of cavities; Porous viscoplastic materials; Model 'in layers'; Micromechanical numerical simulations

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1. Introduction

The study of coalescence of cavities in porous plastic or viscoplastic solids is of great importance for understanding and modeling the ductile rupture of metals. Because of the difficulties of a theoretical treatment of the problem, most works devoted to this question up to now were purely numerical, and consisted of performing micromechanical calculations of the behavior of some 'elementary cell' in some porous material using the finite element method. This route was opened by Koplik and Needleman [1] and subsequently followed by many authors who cannot all be cited here. Most of them studied the case of a plastic material but that of a viscoplastic one, which is of more interest here, was also considered by a number of authors, notably Brocks et al. [2], Needleman et al. [3], Herakovich and Baxter [4], Mohan and Brust [5], Garajeu et al. [6] and Klöcker and Tvergaard [7,8].

The elementary cell considered in almost all of these works was a cylinder with circular basis (approximately representing a cylinder with hexagonal basis, which can be duplicated and piled up so as to build a periodic lattice) containing an initially spherical void. The loading was axisymmetric (equal overall principal stresses in the 'horizontal' plane perpendicular to the 'vertical' axis of the cylinder) with predominant axial stress. In the plastic case, the onset of coalescence was marked by a sudden concentration of the strain rate in the horizontal ligaments linking neighboring voids, the horizontal layers separating these ligaments in the vertical direction becoming suddenly rigid. As a result, the overall deformation mode became a vertical extension with no lateral shrinkage, and the decrease of the overall stress and the increase of the porosity became much more rapid. In the viscoplastic case, the same phenomena were qualitatively observed but the transition from the pre-coalescence phase to the coalescence phase was no longer sudden but gradual, because viscoplastic flow could never completely cease in the horizontal layers separating the inter-void ligaments, at least for the Norton law without threshold considered by the majority of authors. This transition was almost instantaneous for very large values of Norton's exponent (corresponding to an almost ideal-plastic matrix), but spread over a large range of values of the overall deformation for small values of this exponent; in the case of a linearly viscous (Newtonian) material, it was so gradual that the very notion of coalescence became meaningless.

From the theoretical point of view, the first, seminal contribution to the subject was that of Thomason [9,10], who considered only the plastic case. This author evaluated the 'critical' vertical stress inducing coalescence in a representative cell similar to those considered in numerical works, by using limit-analysis with a velocity field 'adapted' to coalescence in that it was zero in the horizontal layers separating the inter-void ligaments. His results were subsequently used by Pardoen and Hutchinson [11] and Benzerga [12], in combination with the Gologanu–Leblond–Devaux (GLD) model [13] (which extends the famous Gurson model [14] for porous plastic solids with spherical voids to spheroidal voids), to define analytical models for coalescence. More specifically, they used the GLD model to describe the overall behavior of the cell prior to coalescence, Thomason's results to predict the onset of coalescence, and specific models to account for the peculiar evolution of the voids toward less prolate or more oblate shapes during coalescence, due to necking of the inter-void ligaments. Another route was opened by Perrin [15] and followed by Gologanu et al. [16]. The main feature of the approach proposed was the schematization of the cell as a 'sandwich' made of three horizontal layers, sound/porous/sound, in which the stress and strain rate tensors were considered as homogeneous; the behavior of the sound layers was described using the von Mises model and that of the porous layer using the Gurson model [14] or the GLD model [13]. In this approach, the onset of coalescence corresponded to the brutal transition from a phase where all layers were plastic to another one where the sole central porous layer remained so, the surrounding sound layers becoming suddenly rigid.

Although the basic physical ideas in the works of Perrin [15] and Gologanu et al. [16] were the same as in the works of Thomason [9,10] and followers [11,12], the simplicity of the analytic treatment allowed for greater flexibility and versatility. Extension to the case of a viscoplastic matrix, for instance, would be difficult using

Thomason's approach because it would demand consideration of new, more complex velocity fields not vanishing in the horizontal layers separating the inter-void ligaments. In contrast, extending the approach of Perrin and Gologanu et al. to viscoplasticity is relatively straightforward, provided that a suitable homogenized model for porous viscoplastic solids is available for use in the central porous layer. Such an extension was proposed by Garajeu et al. [6], using such a specific model.

The aim of the present work is to propose a new extension of the coalescence models of Perrin [15] and Gologanu et al. [16] to the case of a viscoplastic matrix. This extension is similar in principle to that proposed by Garajeu et al. [6] but different in detail. One difference is that a more refined homogenized model for porous viscoplastic solids proposed very recently by the authors [17–19] is used in the porous layer. Another difference lies in the definition of some specific evolution equation for the void shape during the coalescence phase, accounting for the tendency of the void to become less prolate or more oblate. This feature was disregarded in Garajeu et al.'s work [6], although it was considered in other ones devoted to the plastic case [11,12,16]. The predictions of the new model will be critically assessed using the results of some micromechanical simulations.

2. The model

2.1. Generalities

The model is based on consideration of a cylindrical cell of semi-height A, radius B, containing a spheroidal void of semi-axes a and b in the vertical and horizontal directions respectively (Fig. 1(a)). This geometry is characterized by three dimensionless parameters, the porosity f, the shape parameter S of the cell and the shape parameter $S^{(\mathrm{p})}$ of the void, defined by

$$f \equiv \frac{2}{3}\frac{ab^2}{AB^2}, \quad S \equiv \ln\frac{A}{B}, \quad S^{(\mathrm{p})} \equiv \ln\frac{a}{b} \tag{1}$$

Following the approach initiated by Perrin [15] and Gologanu et al. [16], we schematize the cell as a 'sandwich' consisting of one central porous layer of thickness $2d$ plus two surrounding sound layers of thickness $A - d$ (Fig. 1(b)). The central layer is to represent both the void and the inter-void ligament where the strain rate will

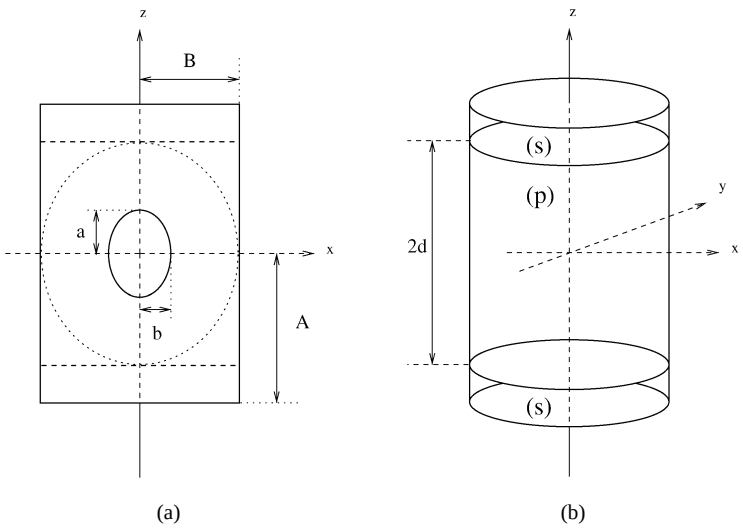

Fig. 1. The elementary cell considered and its schematization by a model composite structure. (a) Elementary cell, (b) Model composite structure.

concentrate during coalescence, and the external layers those zones which will deform less. Quantities pertaining to the sound and porous layers are denoted with upper indices $^{(s)}$, $^{(p)}$ when necessary to avoid a confusion with a similar quantity pertaining to the whole cell; for instance, $\mathbf{D}$, $\mathbf{D}^{(s)}$ and $\mathbf{D}^{(p)}$ denote the overall strain rates in the cell, in the sound layers and in the porous one.

The semi-thickness d of the porous layer is defined as the vertical semi-axis of the largest spheroid enclosed in the cell and *confocal with the void* (Fig. 1(a)):

$$d \equiv \sqrt{B^2 + a^2 - b^2} \tag{2}$$

This definition is logical because the homogenized model which will be used in the central porous layer [17–19] is precisely based on consideration of some spheroidal representative volume element *confocal with the void*. The volume fraction c of the porous layer within the cell and the porosity $f^{(p)}$ within this layer are then given by

$$c \equiv \frac{d}{A} = \mathrm{e}^{-S}\left[1 - \left(\frac{3f}{2}\,\mathrm{e}^{S-S^{(p)}}\right)^{2/3}\left(1 - \mathrm{e}^{2S^{(p)}}\right)\right]^{1/2}, \qquad f^{(p)} \equiv \frac{2}{3}\frac{ab^2}{dB^2} = \frac{f}{c} \tag{3}$$

The cell is subjected to some axisymmetric loading with predominant axial stress; that is, the components of the overall stress tensor $\boldsymbol{\Sigma}$ are zero except $\Sigma_{xx} = \Sigma_{yy} \equiv \Sigma_x$ and $\Sigma_{zz} \equiv \Sigma_z > \Sigma_x$. This type of loading is achieved through the following boundary conditions:

$$\begin{cases} v_\rho = Cst., & \sigma_{\rho z} = 0 \quad \text{on the lateral surface} \\ v_z = 0, & \sigma_{\rho z} = 0 \quad \text{on the bottom} \\ v_z = Cst., & \sigma_{\rho z} = 0 \quad \text{on the top} \end{cases} \tag{4}$$

where $\mathbf{v}$ and $\boldsymbol{\sigma}$ denote the local velocity and stress tensor and cylindrical coordinates (ρ, θ, z) are used. The overall stress and strain rate components are related to their counterparts in the layers through the relations

$$\Sigma_x = (1-c)\Sigma_x^{(s)} + c\Sigma_x^{(p)}, \quad \Sigma_z^{(s)} = \Sigma_z^{(p)} = \Sigma_z; \quad D_x^{(s)} = D_x^{(p)} = D_x, \quad D_z = (1-c)D_z^{(s)} + cD_z^{(p)} \tag{5}$$

2.2. Viscoplastic flow rules in the sound and porous layers

The stress and strain rate tensors are considered as homogeneous in each layer; thus there is no distinction between macroscopic and microscopic quantities within each layer.

In each sound layer, the viscoplastic strain rate $\mathbf{D}^{(s)}$ is assumed to be given by the classical Norton flow rule without threshold:

$$\mathbf{D}^{(s)} = \frac{3}{2}\dot{\epsilon}_0\left(\frac{\Sigma_{eq}^{(s)}}{\sigma_0}\right)^n \frac{\boldsymbol{\Sigma}'^{(s)}}{\Sigma_{eq}^{(s)}} \tag{6}$$

where $\boldsymbol{\Sigma}'^{(s)}$ denotes the stress deviator in the layer, $\Sigma_{eq}^{(s)}$ the corresponding von Mises equivalent stress, and $\dot{\epsilon}_0$, σ_0 and n material constants; n is the Norton exponent.

The porous layer is assumed to obey some homogenized model proposed very recently by the authors [17–19] for porous materials with a Norton matrix. This model provides an implicit expression of the overall stress potential $\Psi(\boldsymbol{\Sigma}^{(p)})$, where $\boldsymbol{\Sigma}^{(p)}$ denotes the overall stress tensor in the layer, through consideration of the *gauge surface* (isopotential surface in the stress space)

$$\mathcal{S} \equiv \left\{\mathbf{S}, \Psi(\mathbf{S}) = \frac{\dot{\epsilon}_0}{(n+1)\sigma_0^n}\right\} \tag{7}$$

First, an approximate equation of this surface is provided in the form $\bar{\Psi}(\mathbf{S}) = 0$, where the *gauge function* $\bar{\Psi}$ reads (for axisymmetric stress tensors):

$$\bar{\Psi}(\mathbf{S}) \equiv C(Q+\eta H)^2 + q(g+1)(g+f^{(\mathrm{p})})\left[F(\kappa H) + \frac{n-1}{n+1}\frac{1}{F(\kappa H)}\right] - (g+1)^2 - \frac{n-1}{n+1}q^2\left(g+f^{(\mathrm{p})}\right)^2 \tag{8}$$

$$F(x) \equiv \left(1+\frac{1}{n}|x|^{(n+1)/n}\right)^n, \quad Q \equiv S_z - S_x, \quad H \equiv 2\alpha_2 S_x + (1-2\alpha_2)S_z \tag{9}$$

The expressions of coefficients C, η, q, g, κ, α_2 here are too long and complex to be recalled. Then, for any stress tensor $\boldsymbol{\Sigma}^{(\mathrm{p})}$, there is a positive scalar $\Lambda(\boldsymbol{\Sigma}^{(\mathrm{p})})$, the *gauge factor*, such that

$$\mathbf{S}(\boldsymbol{\Sigma}^{(\mathrm{p})}) \equiv \frac{\boldsymbol{\Sigma}^{(\mathrm{p})}}{\Lambda(\boldsymbol{\Sigma}^{(\mathrm{p})})} \in \mathcal{S} \quad \Longleftrightarrow \quad \bar{\Psi}\left[\mathbf{S}(\boldsymbol{\Sigma}^{(\mathrm{p})})\right] = 0 \tag{10}$$

Once this gauge factor has been calculated by solving Eq. $(10)_2$, the overall viscoplastic strain rate $\mathbf{D}^{(\mathrm{p})}$ in the porous layer follows from the formula

$$\mathbf{D}^{(\mathrm{p})} = \dot{\epsilon}_0\left[\frac{\Lambda(\boldsymbol{\Sigma}^{(\mathrm{p})})}{\sigma_0}\right]^n \frac{\partial\Lambda}{\partial\boldsymbol{\Sigma}^{(\mathrm{p})}}(\boldsymbol{\Sigma}^{(\mathrm{p})}) = \dot{\epsilon}_0\left[\frac{\Lambda(\boldsymbol{\Sigma}^{(\mathrm{p})})}{\sigma_0}\right]^n \frac{(\partial\bar{\Psi}/\partial\mathbf{S})[\mathbf{S}(\boldsymbol{\Sigma}^{(\mathrm{p})})]}{(\partial\bar{\Psi}/\partial\mathbf{S})[\mathbf{S}(\boldsymbol{\Sigma}^{(\mathrm{p})})] : \mathbf{S}(\boldsymbol{\Sigma}^{(\mathrm{p})})} \tag{11}$$

2.3. Evolution of the internal parameters

The evolution equation of f classically derives from matrix incompressibility, and that of S from $(1)_2$:

$$\dot{f} = (1-f)\operatorname{tr}\mathbf{D}; \qquad \dot{S} = D_z - D_x \tag{12}$$

The evolution equation of $S^{(\mathrm{p})}$ proposed in [17–19] is adopted only in the initial (I) phase preceding coalescence, since this phenomenon was disregarded in these works. It reads

$$\dot{S}_{\mathrm{I}}^{(\mathrm{p})} = h\left(D_z^{(\mathrm{p})} - D_x^{(\mathrm{p})}\right) + \frac{1-f^{(\mathrm{p})}}{f^{(\mathrm{p})}}\frac{(1-3\alpha_1)(1-3\alpha_1')}{1-3\alpha_1+3f^{(\mathrm{p})}(\alpha_1-\alpha_1')}\operatorname{tr}\mathbf{D}^{(\mathrm{p})} \tag{13}$$

where h, α_1 and α_1' are further coefficients the expression of which is again too complex to be given here.

With regard to the evolution of $S^{(\mathrm{p})}$ during the final (F) coalescence phase, the following semi-heuristic expression, accounting for the tendency of the void to grow less prolate or more oblate, was proposed by Gologanu et al. [16] in the plastic case ($n=+\infty$):[2]

$$\dot{S}_{\mathrm{F}}^{(\mathrm{p})(\infty)} = \left[\gamma\left(h + \frac{1-f^{(\mathrm{p})}}{f^{(\mathrm{p})}}\frac{(1-3\alpha_1)(1-3\alpha_1')}{1-3\alpha_1+3f^{(\mathrm{p})}(\alpha_1-\alpha_1')}\right) + \frac{1}{2f^{(\mathrm{p})}}(\gamma-2)\right]D_z^{(\mathrm{p})}, \quad \gamma \approx \frac{2}{3} \tag{14}$$

Also, for $n=1$, linearity implies that the factors connecting $\dot{S}^{(\mathrm{p})}$ to the components of $\mathbf{D}^{(\mathrm{p})}$ must depend on the sole geometric parameters and thus have identical expressions in the initial and final phases; hence the expression of $\dot{S}_{\mathrm{F}}^{(\mathrm{p})(1)}$ must be identical to that of $\dot{S}_{\mathrm{I}}^{(\mathrm{p})(1)}$. Finally, for arbitrary n, the following heuristic 'interpolation' formula is adopted between the values pertaining to the linear and plastic cases:

$$\dot{S}_{\mathrm{F}}^{(\mathrm{p})} \equiv \dot{S}_{\mathrm{F}}^{(\mathrm{p})(n)} = \frac{\dot{S}_{\mathrm{F}}^{(\mathrm{p})(1)}}{n} + \frac{n-1}{n}\dot{S}_{\mathrm{F}}^{(\mathrm{p})(\infty)} = \frac{\dot{S}_{\mathrm{I}}^{(\mathrm{p})(1)}}{n} + \frac{n-1}{n}\dot{S}_{\mathrm{F}}^{(\mathrm{p})(\infty)} \tag{15}$$

Finally, since in the viscoplastic case, there is no clear separation between the initial and final phases, one must describe the continuous transition between the two. The job is done by the following heuristic formula:

$$\dot{S}^{(\mathrm{p})} = X\dot{S}_{\mathrm{I}}^{(\mathrm{p})} + (1-X)\dot{S}_{\mathrm{F}}^{(\mathrm{p})}, \quad X \equiv \left(\frac{2|D_x|}{D_z}\right)^{1/n} \tag{16}$$

[2] In fact Gologanu et al.'s [16] formula is slightly modified here to account for some minor improvement brought into Gologanu et al.'s [13] model by the authors [17–19].

At the beginning of the loading, overall incompressibility almost prevails since the porosity is small, so that $D_x \approx -\frac{1}{2}D_z$, $X \approx 1$ and $\dot{S}^{(\mathrm{p})} \approx \dot{S}_{\mathrm{I}}^{(\mathrm{p})}$. At the end of the loading, $D_x \approx 0$ since the sound layers no longer undergo significant deformation, so that $X \approx 0$ and $\dot{S}^{(\mathrm{p})} \approx \dot{S}_{\mathrm{F}}^{(\mathrm{p})}$. Furthermore, for small values of n, X gradually decreases to zero so that the value of $\dot{S}^{(\mathrm{p})}$ gradually varies from $\dot{S}_{\mathrm{I}}^{(\mathrm{p})}$ to $\dot{S}_{\mathrm{F}}^{(\mathrm{p})}$; but for very large values of n, X remains close to unity as long as the ratio $2|D_x|/D_z$ does not completely vanish, then quickly decreases down to zero when it does, so that the transition between $\dot{S}_{\mathrm{I}}^{(\mathrm{p})}$ and $\dot{S}_{\mathrm{F}}^{(\mathrm{p})}$ is much more abrupt. Thus the predictions of Eq. (16) are qualitatively what one wishes them to be.

2.4. Numerical solution of the equations of the model

The equations of the model are solved in the case where the overall *triaxiality* T (ratio of the overall mean stress $\Sigma_m \equiv \frac{1}{3}\operatorname{tr}\boldsymbol{\Sigma}$ over the overall equivalent stress Σ_{eq}) is fixed to some arbitrary value and the overall axial strain rate D_z is fixed to unity. The evolution equations (12), (16) of f, S and $S^{(\mathrm{p})}$ are integrated using Runge–Kutta's method of order 4. At each step of the calculation, the stress and strain rate components are calculated using a Newton method on the unknown D_x and requiring T to take the value prescribed. This is feasible because the value of D_z being known, if that of D_x is also known, the various stress components follow from the flow rules in the various layers, so that T is a known (albeit complex) function of D_x. Results will be given in the next section.

3. Comparison of model predictions and finite element calculations

Finite element micromechanical simulations are performed for initially spherical and spheroidal voids and various values of the initial porosity, the Norton exponent and the overall triaxiality imposed. For space reasons, we only show the results obtained for an initially spherical void, values of f_0 and T of 0.0104 and 1 respectively, and various values of n.

Fig. 2(a) shows the 'normalized' overall equivalent stress $\Sigma_{\mathrm{eq}}/\sigma_0$ versus the overall equivalent cumulated strain E_{eq}, comparing the results of the finite element computations and the predictions of the model of Section 2. Fig. 2(b) similarly compares the finite element results and the predictions of the authors' model [17–19] not accounting for coalescence. (This model is equivalent to considering that the porous layer fills the whole cell, which

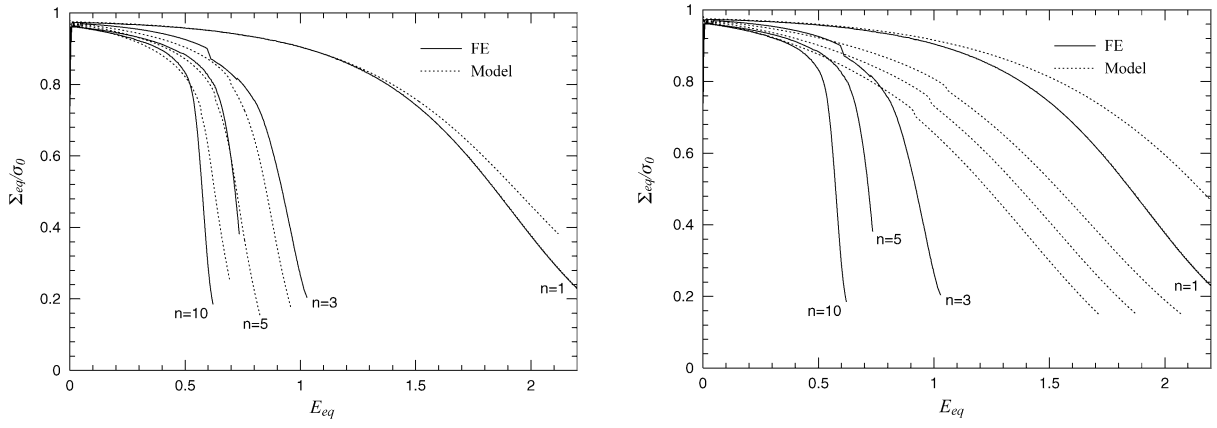

Fig. 2. Evolution of the equivalent stress – Initially spherical void, $f_0 = 0.0104$, $T = 1$. (a) Comparison of FE results with the model accounting for coalescence. (b) Comparison of FE results with the model disregarding coalescence.

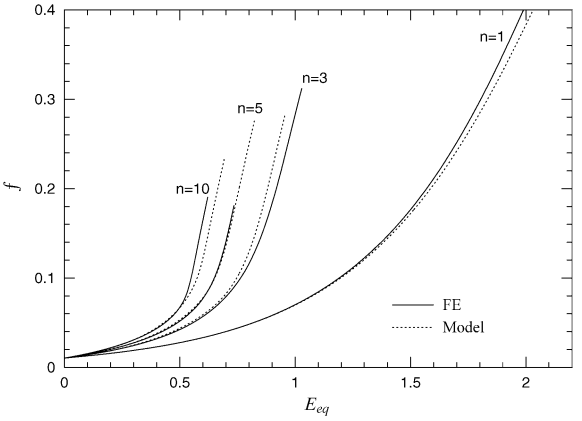

Fig. 3. Evolution of the porosity – Initially spherical void, $f_0 = 0.0104$, $T = 1$.

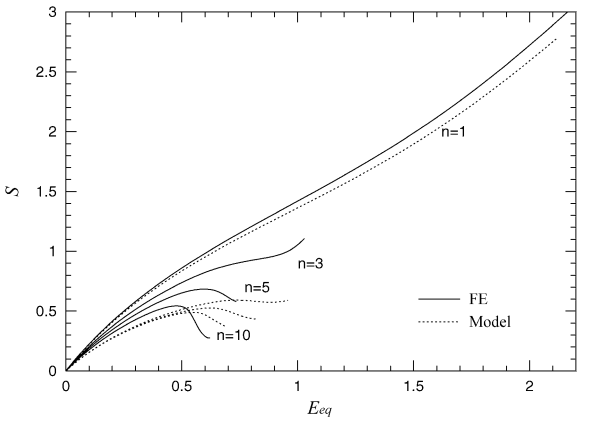

Fig. 4. Evolution of the void shape parameter – Initially spherical void, $f_0 = 0.0104$, $T = 1$.

means dropping equation (2) and taking $d = A$ instead.) Clearly, the model of Section 2 does a much better job than that not incorporating coalescence. It faithfully reproduces the accelerated decrease of the overall stress during coalescence, the quick transition from the pre-coalescence period to the coalescence period for large values of the Norton exponent, and the slower transition for smaller values of this exponent.

Fig. 3 shows the evolution of the porosity in a similar way. Again, the agreement between finite element results and model predictions is quite good; the model correctly captures the accelerated increase of the porosity during coalescence and the dependence of the transition between the two regimes upon the Norton exponent.

Finally Fig. 4 shows the evolution of the void shape parameter. The agreement between finite element results and model predictions is somewhat less satisfactory in this figure; the difficulty of defining a good evolution equation for the void shape parameter has already been emphasized by the authors [19]. The model does capture, however, the general trends of the evolution of the void shape, including, thanks to Eq. (16), the tendency of the void to grow less prolate or more oblate during coalescence.

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