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Computational AeroAcoustics: from acoustic sources modeling to farfield radiated noise prediction

Multiple-scale modelling of acoustic sources in low Mach-number flow

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Abstract

The main difficulty in the calculation of sound generated by fluid flow at low Mach numbers is the occurrence of different scales. The fluid flow is characterized by small spatial structures containing a large amount of energy that may propagate with a small convective velocity, such as small vortices in a turbulent flow. The radiated acoustic waves have small amplitudes and carry a small amount of energy, but have a long wavelength due to their fast propagation velocity. In this paper a perturbation method is used to calculate noise generation and propagation in combination with fluid flow based on the incompressible equations. The idea for the numerical modelling is to introduce a fine grid for the resolution of the fluid flow that is embedded into a larger acoustical domain with a coarse grid adapted to the long wavelength acoustics. To get an appropriate restriction of the acoustic source terms from the fine CFD-grid to the coarse CAA-grid, a multi-scale expansion with one time and two space scales is introduced. ***To cite this article: C.-D. Munz et al., C. R. Mecanique 333 (2005).***

Résumé

Modélisation multi-échelle des sources acoustiques pour un écoulement à faible nombre de Mach. La difficulté principale du calcul du son généré par les écoulements à bas nombre de Mach est l'apparition de différentes échelles. L'écoulement est caractérisé par des petites échelles contenant beaucoup d'énergie qui se propagent avec une vélocité petite comparé aux des ondes acoustiques qui se propagent avec la célérité du son mais qui ne contiennent que peu d'énergie. Dans ce papier une méthode de perturbation est utilisée afin de calculer la génération et la propagation du son d'un écoulement décrit par des équations incompressibles. L'idée pour la modélisation numérique est d'introduire un maillage fin pour l'écoulement qui est plongé dans un domaine acoustique plus grand avec un maillage grossier adapté à l'échelle acoustique. Afin d'obtenir une restriction des termes de source acoustiques du maillage fin de l'écoulement au maillage grossier acoustique, une expansion multi-échelle avec une échelle en temps et deux échelles en espace est introduite. ***Pour citer cet article : C.-D. Munz et al., C. R. Mecanique 333 (2005).***

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1. Governing equations

The compressible Navier–Stokes equations for a perfect gas in primitive variables can be expressed as:

$$\rho_t + u \cdot \nabla\rho + \rho\nabla \cdot u = 0 \tag{1}$$

$$u_t + (u \cdot \nabla)u + \frac{1}{\rho\, M^2}\nabla p = \frac{1}{\rho Re}\nabla \cdot \tau \tag{2}$$

$$p_t + u \cdot \nabla p + \gamma p\nabla \cdot u = (\gamma - 1)\left(\frac{M^2}{Re}\Phi - \frac{\nabla \cdot q}{Pe}\right) \tag{3}$$

Here, ρ, v, p denote the primitive variables: density, velocity and pressure, respectively. The term τ is the viscous stress tensor and Φ is the dissipation function. The Mach number M, the Reynolds number *Re*, and the Peclet number *Pe* appear in the equations as global dimensionless characteristic quantities being measures for compressibility, for viscosity, and for heat conduction, respectively. We remark that the non-dimensionalization of the equations is performed by using the basic reference values for length, density, fluid velocity and the pressure: x_{ref}, ρ_{ref}, u_{ref}, p_{ref}. In the fully compressible regime the non-dimensionalization is usually based on the first three references only. The reference for the pressure is defined to be $\rho_{\text{ref}} u_{\text{ref}}^2$. In this case the sound and the fluid velocity have the same reference values. In the low Mach number regime where the sound velocity tends to infinity compared to the flow velocity and the scales separate this is not longer appropriate. Introducing the basic reference value p_{ref} the sound velocity gets the different reference $\sqrt{p_{\text{ref}}/\rho_{\text{ref}}}$. The quotient of the fluid and sound references is the global Mach number M. For more details see [1] or [2]. Neglecting the viscous effects and the heat flux results in the Euler equations in primitive variables with vanishing right-hand sides in (2) and (3). The factor M^{-2} in front of the pressure gradient in the velocity equations shows the singular behavior of the incompressible limit. The type of the equations changes as the propagation rate of the pressure waves tends to infinity introducing an elliptic constraint. Insight into the limit behavior is obtained by asymptotic considerations in the next section.

2. Mach number zero limit

Klainermann and Majda [3] rigorously showed under assumptions of isentropic flow and with respect to regularity of the initial data that solutions of the incompressible equations are obtained as limit of the solutions of the compressible equations as the Mach number tends to zero. The asymptotic expansions

$$\rho = \rho^{(0)} + M^2\rho^{(2)} + \cdots, \qquad u = u^{(0)} + Mu^{(1)} + \cdots, \qquad p = p^{(0)} + M^2 p^{(2)} + \cdots \tag{4}$$

are inserted into the compressible equations. For brevity, here we restrict ourselves to the inviscid case and give remarks about the case with temperature differences and heat conduction. Under the assumption that there is no compression from the boundary the incompressible Euler equations

$$u_t^{(0)} + (u^{(0)} \cdot \nabla)u^{(0)} + \frac{1}{\rho^{(0)}}\nabla p^{(2)} = 0, \qquad \nabla \cdot u^{(0)} = 0 \tag{5}$$

are obtained as the result of the asymptotic analysis in the limit $M \to 0$. In this case the leading order pressure term in the expansion satisfies $p^{(0)} = \textit{constant}$. It is the background or thermodynamic pressure and is related to the equation of state. The background density $\rho^{(0)}$ is also constant in this case. The couple $u^{(0)}$, $p^{(2)}$ is the incompressible solution, the pressure term $p^{(2)}$ is called hydrodynamic pressure. In the $M = 0$ limit this pressure does not appear in the equation of state or contribute to the total energy.

For flow with variable density, heat conduction and compression from the boundary the limit equations for $M \to 0$ become more subtle. These equations can be written:

$$\rho_t^{(0)} + u^{(0)} \cdot \nabla\rho^{(0)} = -\rho^{(0)}\nabla \cdot u^{(0)} \tag{6}$$

$$u_t^{(0)} + (u^{(0)} \cdot \nabla)u^{(0)} + \frac{1}{\rho^{(0)}}\nabla p^{(2)} = 0 \tag{7}$$

$$\nabla \cdot u^{(0)} = \frac{-p_t^{(0)}}{\gamma p^{(0)}} - \frac{(\gamma - 1)}{\gamma p^{(0)} Pe}\nabla \cdot q^{(0)}, \qquad T^{(0)} = \frac{p^{(0)}}{\rho^{(0)}} \tag{8}$$

The leading order pressure equation gives still a constraint for the divergence of the velocity, but now with a non-zero right-hand side. The divergence of the leading order velocity may vary in space due to the heat flux and due to the compression from the boundary, influencing the leading order pressure $p^{(0)}$ that may now vary in time. The hydrodynamic pressure $p^{(2)}$ again serves as some sort of Lagrangian multiplier to meet this divergence constraint. Due to the non-zero divergence of the velocity, a source term appears in the density equation. The mathematical model for the $M = 0$ limit with heat conduction is described e.g. in [1] in more detail.

3. Single scale perturbation method

Motivated by the asymptotic expansion in the previous section, a perturbation method is introduced. The perturbations ρ', u', p' are compressible corrections of the incompressible flow field:

$$\rho = \rho^{(0)} + M^2\rho^{(2)} + M^2\rho', \quad u = u^{(0)} + Mu^{(1)} + Mu', \quad p = p^{(0)} + M^2 p^{(2)} + M^2 p' \tag{9}$$

where $\rho^{(0)}$ and $p^{(0)}$ denote the background density and pressure, respectively, and $u^{(0)}$ and $p^{(2)}$ is the solution of the incompressible equations. The function $\rho^{(2)}$ is introduced as the change of density by the hydrodynamic pressure and relates to the equation $p^{(2)} - c_0^2\rho^{(2)} = \mathit{constant}$ where c_0^2 denotes the background sound velocity.

This ansatz is inserted into the compressible equations. Neglecting the higher order terms in powers of the Mach number the following linearised perturbation equations are obtained:

$$\rho_t' + u^{(0)} \cdot \nabla\rho' + \frac{\rho^{(0)}}{M}\nabla \cdot u' = -\rho_t^{(2)} - u^{(0)} \cdot \nabla\rho^{(2)} \tag{10}$$

$$u_t' + (u^{(0)} \cdot \nabla)u' + (u' \cdot \nabla)u^{(0)} + \frac{\nabla p'}{\rho^{(0)}\, M} = 0 \tag{11}$$

$$p_t' + u^{(0)} \cdot \nabla p' + \frac{\gamma p^{(0)}}{M}\nabla \cdot u' = -p_t^{(2)} - u^{(0)} \cdot \nabla p^{(2)} \tag{12}$$

The evolution equations on the left-hand side are locally linearised Euler equations. They state the mathematical model for the propagation of the acoustic waves generated by the source terms on the right-hand side. They are determined by the solution of the incompressible equations. This approach may be considered as a scaled and linearised version of the expansion about incompressible flow as proposed by Hardin and Pope [4]. The scaling with respect to the Mach number gives information about the leading order source terms. In [4] is also described how to extend this perturbation method to viscous flow and flow with heat conduction. Here, Hardin and Pope use the usual incompressible solution. The corrections of this solution also contain pressure changes according to the change of the entropy and some sort of heuristic technique is proposed to extract those from the acoustic fluctuations. In the approach considered here this extension is straightforward. The incompressible solution has to be replaced by the solution of the $M = 0$ limit equations with viscosity and heat conduction as listed in the previous section.

4. A multi-scale perturbation method

The considerations in the previous section and the paper [4] do not consider the multi-scale nature of the $M = 0$ limit and the acoustic perturbations. If aeroacoustic noise is considered, then the relevant time scale is defined by the fluid flow. Two space scales appear due to the different propagation velocities. The characteristic acoustic wavelength is $x_{\mathrm{ac}} = x_{\mathrm{ref}}/M$ and is large compared to the characteristic length x_{ref} of the flow when the Mach number M tends to zero. The single aerodynamic scale development loses his validity beyond a distance equal to the characteristic acoustic length scale, as it is clearly demonstrated by Viviand in his multi-scale considerations in [5]. In the following we propose a perturbation method that is based on the multi-scale asymptotic analysis of Klein [2]. For brevity we restrict ourselves once again to basic incompressible flow being homentropic, i.e. heat conduction, viscosity and outer compression are neglected. In this case, the proper multi-scale asymptotic expansion is given by

$$\rho = \rho^{(0)}(\eta, \xi, t) + M^2 \rho^{(2)}(\eta, \xi, t) + \cdots, \qquad u = u^{(0)}(\eta, \xi, t) + M u^{(1)}(\eta, \xi, t) + \cdots \tag{13}$$

$$p = p^{(0)}(\eta, \xi, t) + M^2 p^{(2)}(\eta, \xi, t) + \cdots \tag{14}$$

Here, the independent variable $\eta = x$ denotes the small scale variable connected with the fluid flow while $\xi = Mx$ denotes the large scale acoustic variable. For this ansatz the chain rule $\nabla_x = \nabla_\eta + M\nabla_\xi$ has to be applied. For the homentropic case the basic equations are again given by the incompressible Euler equations as given by (5). All the space derivatives are related to the small space scale flow variable η. The corrections of the incompressible flow field are then given by the perturbation equations for the second order density term $\rho^{(2)}$, the first order velocity term $v^{(1)}$, and the second order pressure term $p^{(2)}$:

$$\rho_t^{(2)} + \nabla_\eta \cdot (\rho^{(0)} u^{(2)}) + \nabla_\eta \cdot (\rho^{(2)} u^{(0)}) + \nabla_\xi \cdot (\rho^{(0)} u^{(1)}) = 0 \tag{15}$$

$$u_t^{(1)} + \nabla_\eta \cdot (u^{(0)} \circ u^{(1)} + u^{(1)} \circ u^{(0)}) + \frac{\nabla_\xi\, p^{(3)}}{\rho^{(0)}} = -\nabla_\xi \cdot (u^{(0)} \circ u^{(0)}) - \frac{\nabla_\eta\, p^{(2)}}{\rho^{(0)}} \tag{16}$$

$$p_t^{(2)} + u^{(0)} \cdot \nabla_\eta\, p^{(2)} + \gamma p^{(0)} \nabla_\eta \cdot u^{(2)} + \gamma p^{(0)} \nabla_\xi \cdot u^{(1)} = 0 \tag{17}$$

In these equations we have small scale as well as large scale derivatives. To get the time evolution of the large scale fluctuations equations (15)–(17) are averaged over the small scale. The average values with bar notation mean $\bar{f} = \frac{1}{|\Omega_{\mathrm{ac}}|} \int_{\Omega_{\mathrm{ac}}} f \,\mathrm{d}\eta$. Assuming sublinear growth of the physical variables, when the Mach number tends to zero, the following set of equations are obtained after the averaging:

$$\bar{\rho}_t^{(2)} + \rho^{(0)} \nabla_\xi \cdot \bar{u}^{(1)} = 0 \tag{18}$$

$$\bar{u}_t^{(1)} + \frac{1}{\rho^{(0)}} \nabla_\xi\, p^{(2)} = -\nabla_\xi \cdot \overline{u^{(0)} \circ u^{(0)}} \tag{19}$$

$$\bar{p}_t^{(2)} + \gamma \rho^{(0)} \nabla_\xi \cdot \bar{u}^{(1)} = 0 \tag{20}$$

We note that Eqs. (18) and (20) lead to the identity $\bar{p}_t^{(2)} = \gamma \bar{\rho}_t^{(2)}$. In these equations we introduce the perturbation ansatz and split the terms into a hydrodynamic and an acoustic part. The ansatz $\rho^{(2)} = \rho_{\mathrm{hyd}}^{(2)} + \rho'$, $p^{(2)} = p_{\mathrm{hyd}}^{(2)} + p'$ and $u^{(1)} = u'$, where the primed quantities denote the fluctuations about the hydrodynamical quantities, is inserted into the equations to get

$$\bar{\rho}'_t + \rho^{(0)} \nabla_\xi \cdot \bar{u}' = -\big(\bar{\rho}_{\mathrm{hyd}}^{(2)}\big)_t \tag{21}$$

$$\bar{u}'_t + \frac{\nabla_\xi \bar{p}'}{\rho^{(0)}} = -\nabla_\xi \cdot \overline{u^{(0)} \circ u^{(0)}} - \frac{1}{\rho^{(0)}} \nabla_\xi\, \bar{p}_{\mathrm{hyd}}^{(2)} \tag{22}$$

$$\bar{p}'_t + \rho^{(0)} \nabla_\xi \cdot \bar{u}^{(1)} = -\big(\bar{p}_{\mathrm{hyd}}^{(2)}\big)_t \tag{23}$$

The influence of fluid convection to the acoustic waves may be taken into account as higher order corrections $M\rho''$, Mu'', Mp'' of the primed acoustic fluctuations as motivated by the multiple-scale asymptotic expansion (13), (14). The corresponding evolution equations are obtained by inserting this extended ansatz into the compressible equations and the usual ordering with respect to powers of the Mach number. After averaging the equations can be expressed as

$$\bar{\rho}''_t + \rho^{(0)}\nabla_\xi \cdot \bar{u}'' = -\nabla_\xi \cdot \overline{(\rho^{(2)}u^{(0)})} \tag{24}$$

$$\bar{u}''_t + \frac{\nabla_\xi \bar{p}''}{\rho^{(0)}} = -\nabla_\xi \cdot \overline{(u^{(0)} \circ u' + u' \circ u^{(0)})} - \frac{1}{\rho^{(0)}}\overline{(\rho^{(2)}u^{(0)})}_t \tag{25}$$

$$\bar{p}''_t + \gamma\rho^{(0)}\nabla_\xi \cdot \bar{u}'' = -\nabla_\xi \cdot \overline{(p^{(2)}u^{(0)})} \tag{26}$$

This approach may be favorable in the regime when the acoustic length scale is much larger than the length scale of the fluid convection. In a first step the acoustic wave generation and propagation without the influence of the convection is calculated. In a second step this influence may be taken into account.

The main purpose of these perturbation methods is the application to the numerical approximation of aerodynamic sound at low Mach numbers. For practical problems the noise propagation into the far field is the field of interest. Perturbation equations with acoustic source terms are used to calculate the noise in the vicinity of the flow field. This is done in the time domain as the flow calculation. While for the flow calculation a fine grid with the small size h has to be used, for the propagation of acoustic waves a coarse grid with step size h/M may be introduced. This is of course necessary, because the acoustic region should be much larger than the flow region. Here, the multiple-scale approach automatically shows the procedure to switch from the fine to the coarse grid. For the flow, either the incompressible equations (5) or the limit equations (6)–(8) for $M \to 0$ are solved on the fine grid. Then the acoustic source terms can be calculated from the flow simulation. They are averaged to the coarse grid as stated in the evolution equations for the long wavelength acoustics.

Furthermore, there is often some discussion about the linearized Euler equations as an appropriate mathematical model for noise propagation. Difficulties may occur, because the linear Euler equations describe besides the acoustic modes also the evolution of vortical and entropy modes. But the time evolution of vortical motion may become unphysical without the nonlinear terms. In the multi-scale long wave length considerations this is avoided. Due to the averaging over the small scale flow structures, the small scale flow phenomena are suppressed. The acoustic wave propagation model obtained does not include vortical motion in the evolution operator on the left-hand side. Taking into account the influence of convection to the acoustic wave propagation needs an additional solution of a pure wave equation. The terms describing the influence with convection is not captured by the evolution operator, but by source terms for the higher order correction.

5. Numerical example

A typical testcase for the validation of a computational aeroacoustics code is the co-rotating vortex pair. It is generated by a pair of two vortices of strength $\kappa = \Gamma/(2\pi)$ each, where Γ is the circulation. They are placed at a distance of $2r_0$ and thus each vortex induces a velocity $q = \Gamma/(4\pi r_0)$ on the other. This causes the vortices to rotate around their common midpoint. For this setting, the exact solution of the potential theory for the incompressible flow field as well as the exact solution of the acoustic far field equations can be determined analytically. The flow generates an acoustic quadrupole. Therefore, this example is commonly chosen for algorithm validation in two space dimensions.

The analytic flow solution is used to calculate the incompressible flow and the acoustic source terms in the perturbation equations. The numerical propagation of the sound waves as obtained by the single scale and the multi-scale perturbation method is then compared to the analytical acoustic solution. The case with a vortex distance of $r_0 = 1.0$ and a rotating Mach number of $M = 0.095$ is chosen. This leads to a rotation period of $T = 2\pi$. The

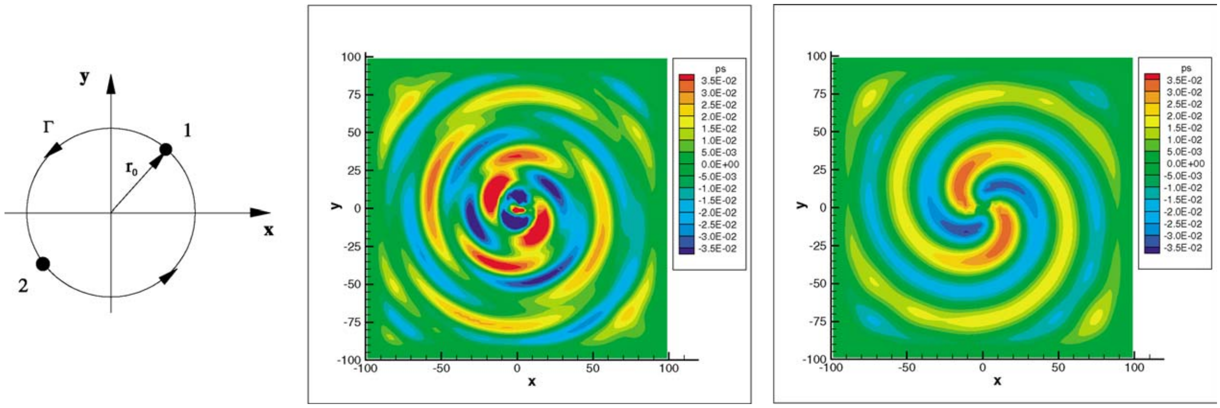

Fig. 1. Configuration and calculation of the co-rotating vortex pair.

Fig. 1. Configuration et simulation d'une paire de tourbillons co-rotatif.

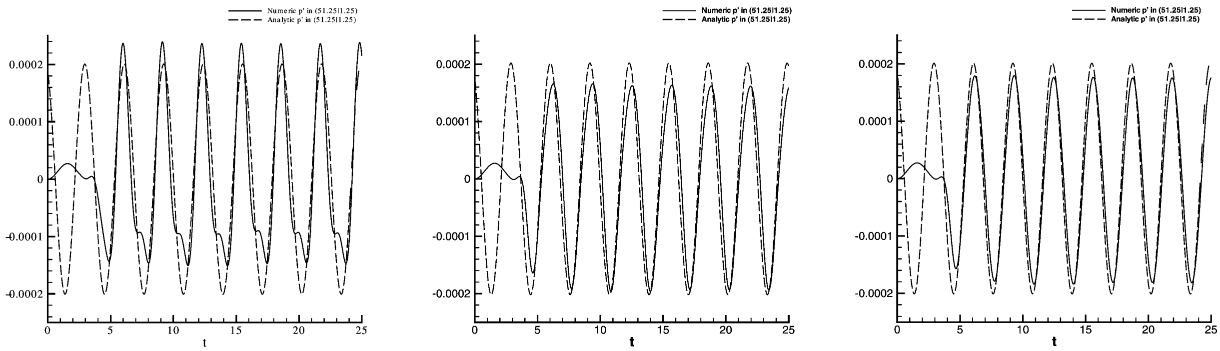

Fig. 2. Comparison of single-scale and multi-scale EIF.

Fig. 2. Comparaison de EIF mono-échelle et multi-échelle.

computational domain is set to be $[-100, 100] \times [-100, 100]$ and is discretized by 80×80 grid points, i.e. with grid spacing of $\Delta x = 2.5$. The acoustic computations were done with a DRP-scheme being fourth order accurate in space and using a fourth order Runge–Kutta method in time.

The left part of Fig. 1 shows the configuration of the co-rotating vortex pair. The figure in the middle shows a calculation with the single scale perturbation method. For the evaluation of the source terms we simply took the point values of the exact solution at the grid points of the acoustic grid. It turned out that we got a strong dependence on the position of the grid points. The worst results are shown in this figure in the middle. Here, the discretization points of the grid are shifted half Δx from the x- and y-axis. The spiral sound waves of the vortex pair are not recovered. The right figure shows a simulation with the multiple-scale model using the same unfavorable grid. Here, the acoustic pressure waves spiral out of the origin and the well known result is achieved.

In Fig. 2 we compare the single-scale and the multi-scale approach against the exact solution of the problem in the point $(51.25, 1.25)$ as a function of time on the shifted grid. The first figure shows the bad results obtained with the single-scale approach. The second and third figure present the results of the multi-scale approach using an averaging over two and six grid-cells, respectively. The best results are obtained with the averaging over six grid-cells being well adopted to the value of the Mach number. With the multi-scale approach quite similar results are also obtained on the non-shifted grid, i.e. the solution becomes much less grid dependent.

6. Conclusions

This paper uses the insight into the incompressible limit of a compressible fluid flow. This motivates a scaling of the expansion about an incompressible flow for small Mach numbers. One advantage of the scaling is that the important terms become obvious, because all the small terms are multiplied by some power of M, e.g., all the products of the fluctuations. Especially in the very low Mach number case the different space scales are accounted by performing a multi scale analysis. The perturbation equations now contain derivatives with respect to the small scale as well as to the large scale variables. The information about the large scales can be extracted via an averaging procedure. This results in a wave equation as propagation model for the acoustic perturbations and source terms on the right-hand side calculated from the flow solution. The influence of fluid convection to the propagation of the acoustic perturbations is represented by higher order perturbations.

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