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Large-eddy simulation of transition to turbulence in a heated annular channel

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Abstract

One carries out three-dimensional large-eddy simulations of natural convection in a horizontal annulus using Smagorinsky's dynamic subgrid model. The onset of transition to turbulence and turbulent regimes are analyzed. The characteristics of unstable flows and their influence on the heat-transfer process are studied. ***To cite this article: E.L.M. Padilla, A. Silveira-Neto, C. R. Mecanique 333 (2005).***

Résumé

Simulation des grandes échelles de la transition à la turbulence dans un canal annulaire chauffé. On développe des simulation des grandes échelles tridimensionnelles de la convection naturelle dans un anneau horizontal, grâce au modèle sous-maille de Smagorinsky dynamique. L'apparition de la transition à la turbulence et des régimes turbulents sont analysés. On étudie aussi les charactéristiques des écoulements instables et leur influence sur les mécanismes d'échange thermique. ***Pour citer cet article : E.L.M. Padilla, A. Silveira-Neto, C. R. Mecanique 333 (2005).***

Keywords: Heat transfer; Computational fluid dynamics; Natural convection; Transition to turbulence; Large-eddy simulation; Dynamic subgrid model; Chaos

Mots-clés : Transferts thermiques ; Dynamique des fluides numérique ; Convection naturelle ; Transition à la turbulence ; Simulation des grandes échelles ; Modèle sous-maille dynamique ; Chaos

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1. Introduction

The study of natural convection between concentric cylinders has been the focus of several numerical and experimental investigations since the 1930s, due to the enormous number of practical and technological applications. In the recent years, many numerical works have been performed, some of them in three dimensions. The natural convection between concentric horizontal cylinders was studied by Beckmann [1]. In several later works [2–4], the effects of changes in the control parameters upon the local and global heat transfer were investigated. The flow destabilization due to natural convection in annular geometries was studied by [3] and [4]. Kuehn and Goldstein [3] have looked at cylindrical concentric geometries filled with pressurized nitrogen over a wide range of Rayleigh number (Ra), corresponding to $2.2 \times 10^2 \leqslant Ra \leqslant 7.7 \times 10^7$ and radii ratio of 2.6. Results have shown that the flow becomes first unstable in the plume region, close to $Ra = 2 \times 10^5$, fully-developing to turbulence as Ra grows. McLeod and Bishop [4] studied the problem using helium at cryogenic temperatures, changing the Ra from 8×10^6 up to 2×10^9. They have graphically represented the dramatic changes in the flow pattern when Ra was increased, and new flow structures at the upper part of the cavity have been found.

From a numerical point of view, two-dimensional [5,6] and three-dimensional [7] studies using the κ–ε model have been performed assuming vertical symmetry. Information about the unstable behavior of turbulent natural convection in periodic horizontal annuli can be found in [8,9]. Fukuda et al. [8] have tackled the problem using direct numerical simulations up to $Ra = 5 \times 10^5$.

In the present Note, unstationary numerical studies related to transition inside horizontal annuli submitted to natural convection are presented, assuming periodicity in the axial direction. We use the large-eddy simulation methodology (LES) with a dynamic subgrid model.

2. Physical problem and numerical method

An incompressible flow of a Newtonian fluid (air) with constant physical properties is considered. The buoyancy term is modeled through Boussinesq approximation. We work in an annular cavity formed by two horizontal concentric cylinders of radii R_i (inner) and R_o. The cylinders are isothermic, with inner and outer surface at temperature T_i and T_o respectively, with $T_i > T_o$. The gap between cylinders is labeled L and the axial length L_{ax}. The geometric characteristics are defined by the dimensionless parameters: radii ratio $\eta = R_o/R_i$ and aspect ratio $\Gamma = L_{\mathrm{ax}}/L$.

The problem presented in this Note is governed by the Navier–Stokes and the energy equations, in which the filtering process are applied according to the LES methodology [10], allowing us to separate the subgrid scales from the large scales. This filtering process gives rise to subgrid-stress tensor and subgrid heat turbulent flux. The filtered equations are non-dimensionalized [11] using T_i, T_o, L and viscosity ν. They are written as:

$$\nabla \cdot \vec{v} = 0 \tag{1}$$

$$\partial \vec{v}/\partial t = -\nabla \cdot (\vec{v}\vec{v}) - \nabla p - Gr\, T\vec{k} + \nabla \cdot \left[(1+\nu_t)\left(\nabla \vec{v} + \nabla \vec{v}^T\right)\right] \tag{2}$$

$$\partial T/\partial t = -\nabla \cdot (\vec{v}T) + \nabla \cdot \left[(1/Pr + \nu_t/Pr_t)\nabla T\right] \tag{3}$$

where $Gr = g\beta(T_i - T_o)L^3/\nu^2$ is the Grashof number and $\vec{k}$ a unit vector in the direction of gravity acceleration. The dimensionless eddy viscosity ν_t is evaluated using the dynamical subgrid scale model according to the expression presented by Lilly [12]. The turbulent thermal diffusivity is estimated using the turbulent Prandtl number $Pr_t = 0.6$ [13].

A finite-volume method is employed on a staggered grid in order to discretize the differential equations, with second-order Adams–Bashforth and centered differences schemes for time and space respectively. The fractional step method [14] has been used for the pressure-velocity coupling. The pressure correction field is evaluated by solving a Poisson equation, using the strongly implicit procedure method. A non-uniform mesh is employed in the radial direction, which is more refined near the walls. A uniform mesh is used in the angular and axial directions.

3. Results

Simulations corresponding to the stable and unstable flows in the horizontal annuli have been carried out in the range $4 \times 10^4 \leqslant Ra \leqslant 7.5 \times 10^5$, with $\eta = 2$ and $\Gamma = 2.8$. The mesh has $16 \times 72 \times 24$ cells in the radial, angular and axial directions, respectively.

According to Bishop et al. [15], the instability onset that appears at the upper part of the cavities consists of oscillations. The oscillations amplitude increases as the Rayleigh number increases. In the present work, it was possible to capture theses instabilities precisely with the aid of a numerical probe placed at $r = 1.5$, $\theta = 90°$ and $z = 1.4$, as illustrated by Fig. 1(a). Small periodic oscillations in the velocity and temperature are observed for $Ra \geqslant 4.7 \times 10^4$. For higher *Ra* values, these oscillations increase and change dramatically, as can be seen on Fig. 1(b). In this figure, the radial velocity component is depicted for two different *Ra*. The fluctuations for $Ra = 5.0 \times 10^4$ are very small and periodic. As *Ra* increases, oscillations lose their periodic behavior, with increased amplitude and temporal frequency. For $Ra > 1.5 \times 10^5$ oscillations display a chaotic evolution. At $Ra = 7.5 \times 10^5$ they present a strongly irregular pattern of high amplitude. One observes also the presence of a wide range of frequencies, characterizing a turbulent behavior.

The effect of instability upon the temperature field manifests initially in the plume structure, and after in other regions of the cavity, as can be seen in Figs. 2 and 3. In these figures, instantaneous dimensionless isothermal surfaces for three different *Ra* are shown. One observes in Fig. 2 that, as *Ra* increases, instabilities multiply and intensify, the more affected region being the upper part of the cavity. In the flow corresponding to $Ra = 1 \times 10^5$, the thermal plume oscillates axially, with small amplitudes and, also, small movements along the θ direction. For higher *Ra*, the oscillation is subject to an intense three-dimensionalization. For $Ra = 5.8 \times 10^5$ one observes a chaotic pattern with hot mass transport in both sides, showing turbulent characteristics. The oscillations around 0° and 180° are very intense. In the lower part of the cavity the flow is unstable, but turbulence is less intense.

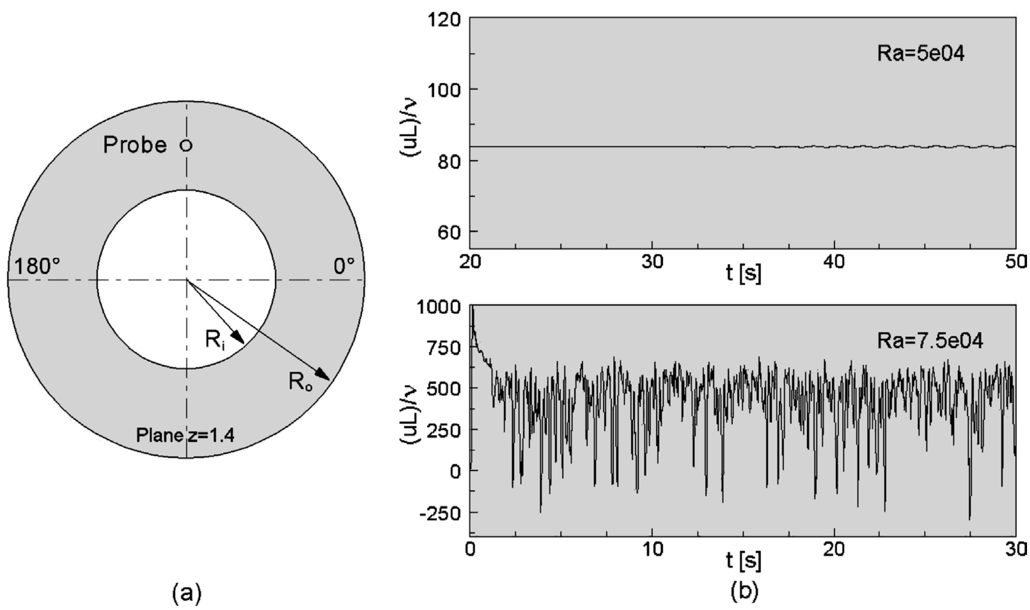

Fig. 1. (a) Geometric characteristics; (b) time distribution of radial velocity.

Fig. 1. (a) Caractéristiques géométriques ; (b) distribution temporelle de la vitesse radiale.

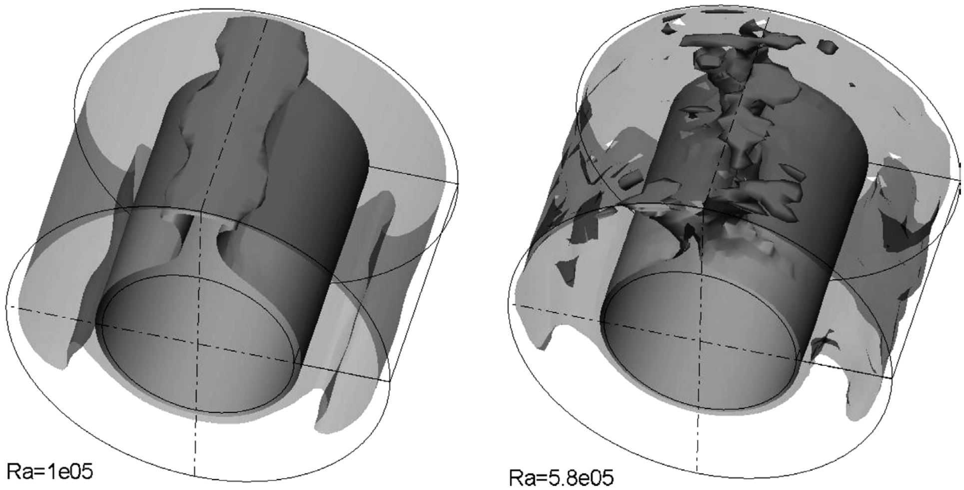

Fig. 2. Instantaneous temperature isosurfaces, thresholds 0.25 (transparent) and 0.65.

Fig. 2. Isosurfaces de température instantanée, seuils 0,25 (transparent) et 0,65.

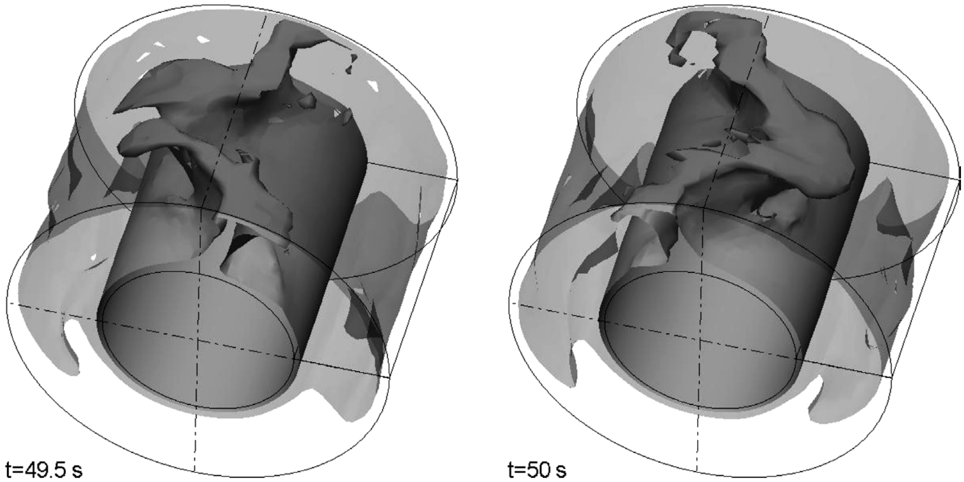

Fig. 3. Temporal evolution of temperature isosurfaces for $Ra = 1.5 \times 10^5$.

Fig. 3. Evolution temporelle de l'isosurfaces de température pour $Ra = 1,5 \times 10^5$.

In Fig. 3, the flow at $Ra = 1.5 \times 10^5$ is depicted for two different instants: 49.5 s and 50 s. The transitional flow presents three-dimensional oscillations. The plume oscillates with vigorous amplitude, moving from the left (49.5 s) to the right (50 s) and back, as experimentally observed in [15] and [4].

We have compared satisfactorily some statistical properties with experimental results of Itoh et al. [16] and [8].

Time-frequency energy spectra of the radial velocity component fluctuation for two values of Ra are presented in Fig. 4. The velocity distribution was taken at the probe of Fig. 1(a). A line with a $-5/3$ slope corresponding to Kolgomorov law has been drawn. One observes that the 'inertial range' (if our temporal spectrum may be associated to a spatial spectrum thanks to some local Taylor hypothesis) increases with Ra. The spectrum at $Ra = 1.5 \times 10^5$ is steeper than $-5/3$, demonstrating an energy concentration in the large scales.

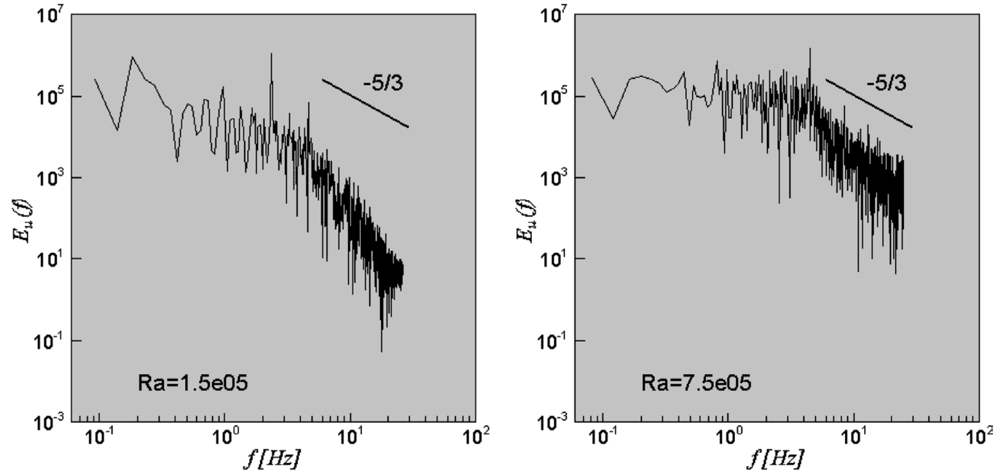

Fig. 4. Energy spectra of the radial velocity fluctuation.

Fig. 4. Spectres d'énergie de la fluctuation de vitesse radiale.

4. Conclusion

Three-dimensional unstationary numerical simulations of the transitional flow submitted to natural convection in an annular cavity were successful performed, employing an LES methodology with dynamic subgrid scale model. We have been able to characterize the onset of the first instabilities. As the Rayleigh number increases, the flow becomes fully irregular and chaotic when a turbulent regime is reached. It was possible to capture the flow dynamic characteristics, as well as the plume transition. The results compare very well with experimental and numerical results from other authors.

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