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Proper Orthogonal Decomposition of the mixing layer flow into coherent structures and turbulent Gaussian fluctuations

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Abstract

The Proper Orthogonal Decomposition is used to decompose fluctuating turbulent flows into a coherent non-Gaussian component and background fluctuations. An application is performed from 2D experimental data of a turbulent plane mixing layer flow. The analyses of the energy spectra and the Probability Density Function of the velocity field show that POD extracts an incoherent part approaching the quasi-Gaussian distribution properties. The background fluctuations are homogeneous with small amplitude. New future applications are then conceivable like the modeling of the incoherent part for particular inflow condition generation methodology and the analysis of the cyclic velocity field variabilities in Internal Combustion engine flow. ***To cite this article: Ph. Druault et al., C. R. Mecanique 333 (2005).***

Résumé

Décomposition des champs de vitesse turbulents d'une couche de mélange en une partie cohérente et une partie turbulente aléatoire. La Décomposition Orthogonale aux valeurs Propres est utilisée pour décomposer l'écoulement de couche de mélange plane turbulente en une partie cohérente issue des premiers modes POD et une partie résiduelle incohérente déduit du résidu de ces modes. A partir d'analyses spectrales et des Fonctions Densité de Probabilité du champ de vitesse, on montre que les fluctuations incohérentes sont de nature quasi-gaussienne. Cette décomposition permet d'entrevoir l'amélioration de certaines procédures de génération de conditions d'entrée des simulations numériques et l'analyse des fluctuations cycliques du champ de vitesse dans les écoulements des moteurs à combustion interne. ***Pour citer cet article : Ph. Druault et al., C. R. Mecanique 333 (2005).***

Keywords: Turbulence; Proper Orthogonal Decomposition; Coherent structures; Gaussian fluctuations; Plane mixing layer

Mots-clés : Turbulence ; Décomposition Orthogonale aux valeurs Propres ; Structures cohérentes ; Fluctuations gaussiennes ; Couche de mélange plane

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1. Introduction

Experimental investigations of turbulent flow as well as numerical ones usually use the fact that instantaneous velocity fields of turbulent flow can be decomposed into three parts: the time average mean part (in the sense of the conventional Reynolds Decomposition), the coherent part and the incoherent turbulent part. The main difficulty relies on the separation between the coherent part of the velocity field and the turbulent background fluctuation. Indeed, coherent structures are embedded inside a randomly distributed field and their extraction still remains complex. Due to the great influence of the large scale coherent structures on mixing processes, noise emission, heat transfer processes, etc., several structure eduction methodologies (see e.g. [1]) based on different flow structure concepts have been now well established for extracting the coherent structures of turbulent flow. For instance, the decomposition of experimental or numerical turbulent flow data can be performed from (i) phase averaging methods, (ii) turbulence filterings [2], (iii) wavelet decomposition [3], (iv) Proper Orthogonal Decomposition (POD) [4], etc. These methods have generally been used to characterize only the coherent part of the flow field. On the other hand, apart from Direct Numerical Simulation, the other simulations methodologies (Large Eddy Simulation, Semi-Deterministic Methodology [5], Unsteady RANS simulations) use other procedures for extracting the part of the velocity field for which Navier Stokes solver is used. The turbulent flow remainder is then modeled according to several assumptions.

In this sense, the objective of this Note is to present a methodology for decoupling the Gaussian background turbulent fluctuations and its relative coherent part. This methodology leads to gain some more information about the turbulent flow dynamics by analyzing not only the coherent part of the velocity field but also the incoherent part and will allow some new investigations for the incoherent turbulent flow modeling. A similar decomposition has already been performed with wavelet decomposition. Indeed, Farge et al. [3] built a Coherent Vortex Simulation (CVS) based on a partition of the flow into a randomly distributed background and organized (in the rest of the field) turbulence. Here, we focus on POD filtering technique for its ability to quantify energy content in the spatial domain and to extract energetic events. The Proper Orthogonal Decomposition first introduced for turbulent flow applications by Lumley [6] provides a mathematically rigorous procedure for extracting the most energetic modes from a fluctuating velocity field that define the coherent structure of the flow.

First, we describe the POD procedure and the POD filtering approach. Then after describing hot-wire experimental database obtained in a plane mixing layer flow, POD filtering is applied to 2D turbulent velocity fields. From the analysis of spectral and Probability Density Function (PDF) of the velocity field, we comment the results and compare them to previous ones obtained from other filtering approaches.

2. Proper Orthogonal Decomposition

Before introducing the Proper Orthogonal Decomposition mathematical concept, the triple decomposition of the *total* instantaneous velocity field $u_i(X,t)$ (X and t are the space and time variables respectively) is recalled: $u_i(X,t) = \overline{U}_i(X) + \tilde{u}_i(X,t)$ with $\tilde{u}_i(X,t) = \hat{u}_i(X,t) + u'_i(X,t)$ where $\overline{U}_i$ is the conventional time average, $\hat{u}_i$ and u'_i are respectively the coherent and incoherent fluctuations.

2.1. Mathematical POD approach

According to Lumley [6], a coherent structure is the structure that has the largest mean square projection of the velocity field. He then proposed to mathematically decompose the instantaneous fluctuating velocity field into a basis set of optimal (in the mean square sense) orthogonal eigenfunctions $\Phi^{(n)}$, which can be deduced from this integral eigenvalue problem: $\int_D R_{ij}(X,X')\Phi_j^{(n)}(X')\,\mathrm{d}X' = \lambda^{(n)}\Phi_i^{(n)}(X)$ where $\Phi_i^{(n)}$ and $\lambda^{(n)}$ are respectively the discrete eigenfunctions and eigenvalues. $R_{ij}(X,X')$ is the two point spatial correlation tensor: $\overline{\tilde{u}_i(X)\tilde{u}_j(X')}$. The projection of the instantaneous velocity fields onto the POD eigenfunctions provides the POD temporal coefficients $a^{(n)}(t)$. POD allows the decoupling between time and space variables and instantaneous fluctuating velocity component $\tilde{u}_i(X,t)$ is then reconstructed with the following equation: $\tilde{u}_i(X,t) = \sum_{n=1}^{N_{\mathrm{mod}}} a^{(n)}(t)\Phi_i^{(n)}(X)$, where N_{mod} is the total number of the POD.

2.2. POD filtering approach

The first step consists in performing a POD on the full available data. N_{mod} POD temporal coefficients and eigenfunctions are then obtained. The second step consists in choosing the number M (cut off POD number) separating the contribution of both coherent and incoherent velocity fields. Then, each fluctuating velocity field is reconstructed with the following equations:

$$\hat{u}_i(X,t) = \sum_{n=1}^{M} a^{(n)}(t)\Phi_i^{(n)}(X), \quad \text{and} \quad u'_i(X,t) = \sum_{n=M+1}^{N_{\text{mod}}} a^{(n)}(t)\Phi_i^{(n)}(X) \tag{1}$$

An interesting feature of the POD decomposition is that coherent and incoherent velocity fields are uncorrelated. Furthermore, due to the matching between the multiscale character of turbulence and the POD representation, POD provides a satisfactory decomposition by extracting most of the localized coherent structures containing the main part of the kinetic energy and represented with the first M modes. To select this cut-off number, spectral analyses and PDF analyses of the incoherent velocity field are performed in order to approach the spectra of a quasi-homogeneous equilibrium turbulence and the quasi-normal PDF of the velocity.

3. POD filtering application

POD filtering is applied to experimental measurements obtained in a two-dimensional plane turbulent mixing layer by means of a hot-wire rake [1,7].

3.1. Experimental data

The two streams, at $U_a = 42.2$ m/s and $U_b = 25.2$ m/s, merge at the trailing edge of a splitter plate with a velocity ratio $r = U_b/U_a = 0.59$. The two boundary layers at the trailing edge are fully turbulent. The measurements were taken in the self similar zone at $x = 600$ mm downstream of the trailing edge of the splitting plate, where the vorticity thickness δ_{ω_0} was 27.6 mm. The velocity vector field is determined by using a rake of 12 equally spaced X-wires uniformly distributed according to the transverse y-direction of the flow [1,7]. The streamwise u_1 and transverse u_2 instantaneous velocity components are obtained at 12 locations. The probes are placed symmetrically around the mixing layer axis and the distance between two probes is 6 mm. To perform the following statistical analyses, 820 000 samples of each velocity taken at 10 kHz are used.

3.2. POD analysis

Details of the POD analysis of this plane mixing flow configuration can be found in Delville et al. [4,7]. Spatial correlation tensor $R_{ij}(y, y')$ is computed for the 12 y-locations for both velocity components. Then a vectorial POD is performed resulting in 24 POD eigenfunctions. The POD energy convergence is given on Fig. 1 where the accumulated eigenvalues $\lambda^{(n)}/\sum_{i=1}^{N_{\text{mod}}}\lambda^{(i)}$ are represented. More than 35% of the total kinetic energy is contained in the first POD mode. Moreover, the four first POD modes ($\sim$17% of the total POD modes) contain 70% of the fluctuating kinetic energy. An illustration of the projected instantaneous velocity vector fields onto the first POD mode is given on Fig. 1 where it is compared to the reference velocity field, which confirms that large scale coherent structures can be extracted from the first POD mode.

Due to the weak number of available POD eigenfunctions, all possibilities for the choice of M have been tested. Finally, the cut-off POD number, M is fixed at 4. The coherent velocity field is then reconstructed from the first 4 POD modes and the remaining POD modes is used to provide the incoherent background velocity field. Note that the coherent and incoherent velocity fields are orthogonalized, so that there is no correlation between these fields. An investigation of the spectra and the PDF of both velocity fields is now performed.

3.3. Spectral analysis of coherent and incoherent flow field

The centerline power spectra deduced from the original, coherent and background fields are superimposed on Fig. 2. The coherent flow field has the quasi-same spectral distribution as the original (total) flow until the frequency 500 Hz

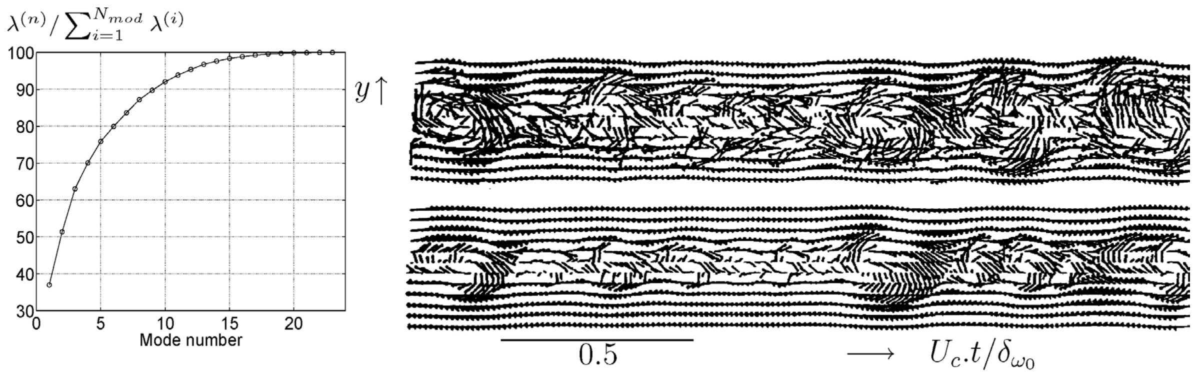

Fig. 1. Left-hand side: POD energy convergence. Right-hand side: Instantaneous plot of velocity vectors represented in the convection frame using Taylor hypothesis with the convection velocity $U_c = (U_a + U_b)/2$. Top: Original velocity vectors; Below: Velocity vectors projected onto the first POD mode.

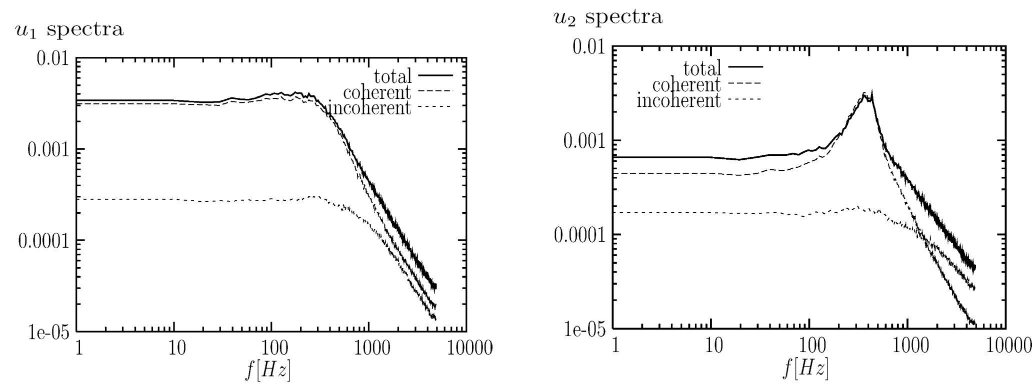

Fig. 2. Original measured total u_1 and u_2 spectrum in the mixing layer centre, — contribution of the first four POD modes to this spectrum (coherent part) and ... the spectrum of the remaining modes (incoherent part).

while the incoherent flow is uncorrelated and exhibit a flat energy spectrum in this frequency domain. We then see that the POD coherent transverse component capture the strongest frequency peak lying around 400 Hz, associated to the Kelvin–Helmholtz two-dimensional instability. Note that although wavelet decomposition performed on the same database [8] and turbulence filter applied to a flat plate boundary layer flow [2] provided similar results concerning the capture of the strongest frequency peak, it appears that none of them provides a flat energy spectrum in low frequency domain. At high frequencies, for both velocity components (u_1 and u_2), the spectral slope obtained for the incoherent spectrum is similar to the one deduced from the original spectrum while the coherent spectrum exhibits a different slope. The spectral content of the small scales has then been well restituted with the POD incoherent field. Finally, the background fluctuations exhibit a spectrum corresponding to a quasi-homogeneous equilibrium turbulence. These figures also show that POD coherent structures are not localized in the spectral domain. They correspond to multiple scales and differ from usual Fourier filtering in time or space approaches.

3.4. PDFs of the velocity fields

A PDF of the total, coherent and incoherent velocity fields are superimposed on Fig. 3 in a log-scale representation, for both velocity components obtained in the mixing layer centre. The incoherent velocity component u_1 shows a statistical behavior quasi-similar to a Gaussian normal distribution. On the other hand, for u_2 spectra representation, a Gaussian distribution is only satisfied for u_2 values in $[-10:10]$ domain. Outside this domain, some discrepancies are observed. The coherent and total velocity fields exhibit a similar behavior corresponding to a non-Gaussian PDF behavior. Note that in the outer zone of the mixing layer, we do not observe a quasi-normal PDF for the incoherent flow field confirming previous results based on wavelet decomposition applied to the same database [8].

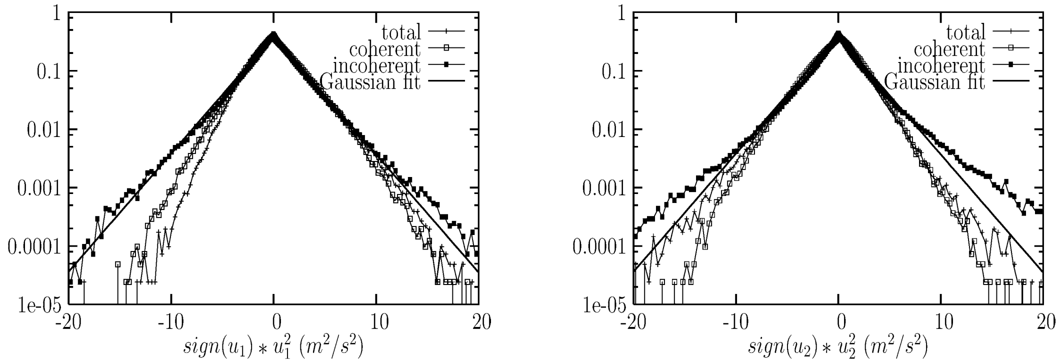

Fig. 3. Comparative analysis of the PDF of all velocity fields in the mixing layer centre. Streamwise component (right) and transverse component (left).

4. Discussion and conclusion

The use of a POD filtering applied to a plane mixing layer flow database allows the eduction of energetic coherent events and background fluctuations at the same time. We showed that the coherent and background velocity fields coexist and interact even if their spectral content and their dynamics are quite different.

POD filtering approach is now compared to wavelet decomposition which has already been performed from the same plane mixing layer database [8]. Even though wavelet and POD approaches are quite different, both decompositions allowing a decoupling between organized structures and random part, provide some similar results even if the spectral energy content of small scales is different. On the other hand, previous POD and wavelet filtering approaches have been compared from homogeneous flow configurations [3]. Due to the degeneration of the POD decomposition into Fourier decomposition for statistically homogeneous case, Farge et al. [3] showed that CVS method is more appropriate to extract the coherent vortices than Fourier decomposition. In this note, on top of treating inhomogeneous test-flow case, we can not adopt the same threshold (POD cut-off number) than in their work. Indeed, in their numerical work, Farge et al. [3] retained only 3% of the total POD modes (containing more than 90% of the kinetic energy) for extracting the coherent velocity field, which is relatively different than our test-case (see Section 3.2). In fact, the spatial organization of both numerical homogeneous and experimental inhomogeneous flows is too different to make good comparisons.

Based on the spectra representation of both coherent and background velocity fields, it appears that POD filtering approach greatly differs from LES approach. Indeed, the resulting POD background field contains large scale energy. Finally, POD application leads to a decomposition very close to the one used in SDM [5] computational methods and the background fluctuations can be viewed as a Gaussian noise. Due to the fact that this Gaussian velocity field is deduced from the residue of POD coherent field and also is uncorrelated with it, some new investigations of the incoherent velocity modeling for numerical simulation may be conceivable. First, it may be possible to develop or to adapt numerical sub grid scale models used in Large Eddy Simulations to this POD energy filtering. Second, recently Druault et al. [9] proposed a new methodology for the generation of experimental POD coherent velocity field as an inflow condition for LES. Then, in order to improve this methodology, we have to take into account and to incorporate the corresponding incoherent velocity fields (which in our test-case contains 30% of the fluctuating energy). In this sense, the use of the POD modes remainder providing background fluctuations with a quasi-Gaussian distribution may be a solution to generate the associated incoherent velocity field.

Moreover, one of the problem of the cyclic variability analysis in Internal Combustion engine flow is the velocity field decomposition extracting the background fluctuations and the corresponding cyclic velocity component. In this sense, the use of this POD energy filtering may be a solution. Such investigations are on going.

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