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C. R. Mecanique 333 (2005) 796–803
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A novel implicit method for coastal hydrodynamics modeling: application to the Arcachon lagoon

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Received 19 September 2005; accepted 27 September 2005

Presented by Michel Combarnous

Abstract

The present Note reports on numerical modeling of shallow flows in coastal areas. Successful numerical simulations of such flows should be able to cope with strong irregularities of the bathymetry and to reproduce the covering/uncovering (wetting/drying) of tidal flats due to the tidal oscillations of the free surface. Also, adoption of large time steps is necessary to simulate phenomena which last actually several days or months. In the present study, a new numerical model based on an implicit resolution of the shallow water equations is proposed. A penalty method has been employed for numerical treatment of dry zones emerging during the wetting and drying processes. The capability of the present model has been verified by comparison with standard test cases. Further applications and comparisons have been also carried out to simulate the tidal propagation in the Arcachon lagoon. ***To cite this article: A. Le Dissez et al., C. R. Mecanique 333 (2005).***

Résumé

Une méthode implicite originale pour la modélisation des écoulements côtiers : application au bassin d'Arcachon. La présente Note traite de la modélisation numérique des écoulements en eau peu profonde dans les zones littorales. La simulation numérique de tels écoulements doit pouvoir prendre en compte une bathymétrie fortement irrégulière et reproduire le phénomène d'émergence/immersion des zones intertidales, alternativement couvertes puis découvertes sous l'action de la marée. Utiliser de grands pas de temps est également nécessaire pour simuler des évènements se déroulant en réalité sur plusieurs jours ou plusieurs mois. Dans cette étude, un mouveau modèle est proposé, basé sur une résolution implicite des équations en eau peu profonde. Une méthode de pénalisation est utilisée pour le traitement des zones sèches émergeant au cours du jusant. Les capacités du modèle ont été validées sur des cas tests de la littérature ainsi que lors de son application à la simulation de la propagation de la marée au sein du Bassin d'Arcachon. ***Pour citer cet article : A. Le Dissez et al., C. R. Mecanique 333 (2005).***

Keywords: Computational fluid mechanics; Shallow water equations; Implicit resolution; Augmented Lagrangian; Wetting and drying; Penalty method

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1631-0721/$ – see front matter
doi:10.1016/j.crme.2005.10.003

Mots-clés : Mécanique des fluides numérique ; Équations en eau peu profonde ; Résolution implicite ; Lagrangien augmenté ; Gestion des zones intertidales ; Méthode de pénalisation

1. Introduction

The behavior of waves and currents in mesoscopic coastal environments is often controlled by the water depth that is generally small compared to the characteristical horizontal scale of the flow [1]. Therefore the Shallow Water Equations (SWE) are used to describe nearshore hydrodynamics and investigate the two main types of coastal phenomena: on the one hand, surf zone hydrodynamics (small time and space scales) is studied with explicit shock capturing schemes [2], on the other hand, larger scale coastal flows like tidal currents are dealt with implicit discretizations [3].

In the framework of this last class of problems, a two-dimensional horizontal (2DH) model for coastal hydrodynamics modeling, based on SWE, must satisfy the four following essential requirements:

- (i) robust numerical schemes are necessary to describe the propagation of shocks. Indeed, the hyperbolic nonlinear character of the equations induces the appearance of discontinuous solutions. Moreover, the propagation of waves, such as tide, bore or breaking waves, in the very shallow water areas, implies the presence of strong velocity gradients at the interface between the immersed and emerged zones [4];
- (ii) the model has to be able to work with large time steps;
- (iii) it has to cope with strongly irregular bathymetries;
- (iv) the modeling should achieve an accurate reproduction of the covering/uncovering of tidal flats which are subjected to alternating wetting and drying due to the tidal oscillation of the free surface.

In the present study, a new numerical model is proposed to tackle with difficulties arising from varying bottom, drying and wetting processes as well as large time simulation with reasonable computing time cost. A penalty method has been employed for numerical treatment of dry zones emerging during the wetting and drying processes by adding a Darcy type term in the classical SWE. Governing equations are discretized using an implicit Finite Volume approach on a staggered grid. An augmented Lagrangian method has been used for resolution of the flow field. The present numerical modeling is validated by comparison with several literature test cases. A real coastal application is also considered for simulation of the tidal flow in the Arcachon Lagoon.

2. Physical modeling

The modeling of flooding and dewatering of shallow embayments presents special numerical difficulties, as the domain of simulation must evolve in response to the computed solution itself [3]. Numerous research works have been devoted to this issue, which propose various answers, in Refs. [5–8]. The authors' aim is to build a physical model that intrinsically takes this Wetting And Drying (WAD) process into account. The model is based on the introduction of a Darcy term into the classical three-dimensional Navier–Stokes equations (3D-NS). Initially used to represent the interaction of a fluid with a porous media, this term will enable us to characterize the dry coastal zones. The system of equations is then integrated over the water depth by assuming the hydrostatic distribution of the pressure. This leads to a new formulation of the SWE model, and allows to take into account the dry zones in an original way.

2.1. Interaction between incompressible flow and porous media

The 3D-Navier–Stokes equations including a Darcy term are written as:

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{V}) = 0 \tag{1}$$

$$\partial(\rho\mathbf{V})/\partial t + \nabla\cdot(\rho\mathbf{V}\otimes\mathbf{V}) = -\nabla p + \rho\mathbf{g} + \nabla\cdot\big(\mu(\nabla\mathbf{V} + \nabla^t\mathbf{V})\big) - (\nu/K)\rho\mathbf{V} \tag{2}$$

where $\mathbf{V}$ is the velocity vector, p the pressure, t the time, ρ the density, g the acceleration of gravity, μ the dynamic viscosity, $\nu = \mu/\rho$ the kinematic viscosity and $\mathbf{K}$ the permeability tensor.

The Darcy or volume drag term $-(\nu/k)\rho\mathbf{V}$ allows to take into account the flow of a fluid in mixed fluid-porous system [9]. In fact, these motion equations of Navier–Stokes–Brinkman type represent the flow of a fictitious fluid made of a Newtonian fluid including fixed solid particles. Depending on the value of K, Eq. (2) allows to describe an intermediate situation between the two extreme behaviors of a fluid or a solid.

2.2. New shallow water model including a dry zone term

Assuming the pressure field to be near to the hydrostatic equilibrium, the density constant and the shallow water approximation, the three-dimensional conservative Navier–Stokes–Brinkman (1), (2) are integrated over the water depth. This leads to a novel 2DH shallow water model with a smaller number of unknowns, written as:

$$\partial h/\partial t + \nabla\cdot(h\mathbf{V}) = 0 \tag{3}$$

$$\partial(h\mathbf{V})/\partial t + \nabla\cdot(h\mathbf{V}\otimes\mathbf{V}) = -gh\nabla(h+h_b) + \nabla\cdot\big(\mu h(\nabla\mathbf{V}+\nabla^t\mathbf{V})\big) - C_f h\|\mathbf{V}\|\mathbf{V} - (\nu/K)h\mathbf{V} \tag{4}$$

u and v are the components of the velocity vector $\mathbf{V}$ in the horizontal plane (x, y), h is the water depth, h_b the solid bottom elevation and C_f the friction coefficient.

The new term $-(\nu/k)h\mathbf{V}$ added to SWE takes into account the permeability of the medium and will be helpful for numerical treatment of dry zones, with no fluid flow, as impermeable porous zones characterized by $K = 0$.

3. Numerical methods

From a general point of view, Eqs. (3), (4) are discretized by implicit Finite Volumes on fixed staggered Cartesian grids. In a first approach, the advection terms are approximated by first order upwind schemes and the viscous terms by centered schemes. Our procedure of resolution lays on the following original characteristics:

– An Uzawa minimization algorithm [10] is implemented to solve the coupling between $\mathbf{V}$ and h. An augmented Lagrangian term $gh^{n+1,k}\Delta t\nabla\cdot(\nabla\mathbf{q}^{n+1,k+1})$ has been added to the momentum equations in order to increase the convergence properties of the minimisation procedure [11]. Numerically, an iterative loop is done while $|\mathbf{q}^{n+1,k} - \mathbf{q}^{n+1,k-1}| > \epsilon$:

$$\mathbf{q} = h\mathbf{V} \tag{5}$$

$$\frac{\mathbf{q}^{n+1,k+1}-\mathbf{q}^n}{\Delta t} + \nabla\big(\mathbf{q}^{n+1,k+1}\mathbf{V}^{n+1,k}\big) - gh^{n+1,k}\Delta t\nabla\big(\nabla\mathbf{q}^{n+1,k+1}\big) = -gh^{n+1,k}\nabla\big(h_b + h^{n+1}\big) \tag{6}$$

$$h^{n+1,k+1} = h^{n+1} - \Delta t\nabla\mathbf{q}^{n+1,k} \tag{7}$$

$$\mathbf{V}^{n+1,k+1} = \mathbf{q}^{n+1,k+1}/h^{n+1,k+1} \tag{8}$$

– By analogy with the techniques used in the modeling of the free surface flows [12], the detection of the interface between emerged and immersed zones requires the introduction of a new variable called phase function C. This variable equals 1 in the dry zones and 0 in the water zones. Numerically we have to impose $h = \epsilon$ with $\epsilon(\neq 0) \to 0$ in the emerged zones [2]. The time evolution of the phase function C can be obtained by solving an advection equation $\partial C/\partial t + \mathbf{V}\cdot\nabla C = 0$ with VOF (Volume Of Fluid) methods [12], TVD (Total Variation Diminishing) schemes [13], Front Tracking [14], level set [15], … . More simply, the phase function can be explicitly obtained by $C = (\epsilon/h)^n$ and $n > 1$. The interest of this last method is to be less expansive in computational cost and easier to implement.

– The Darcy term in (4) is used to impose no velocity in the dry zone. This technique is called penalty method [9]. As the topology of this dry zone changes with time, the permeability is updated thanks to the phase function C. Thus, $\mathbf{K} = 10^{-40}$ if $C = 1$ and $\mathbf{K} = 10^{+40}$ if $C < 1$.

Further details about discretization schemes, augmented Lagrangian and penalty methods and iterative solvers are given, for instance, in [16].

4. Validation on 1D test cases

The novel 2DH model for shallow flows has been validated on several test cases. The results of these tests demonstrate the robustness and the accuracy of the model, which does not require any 'stabilizing' terms or artificial viscosity

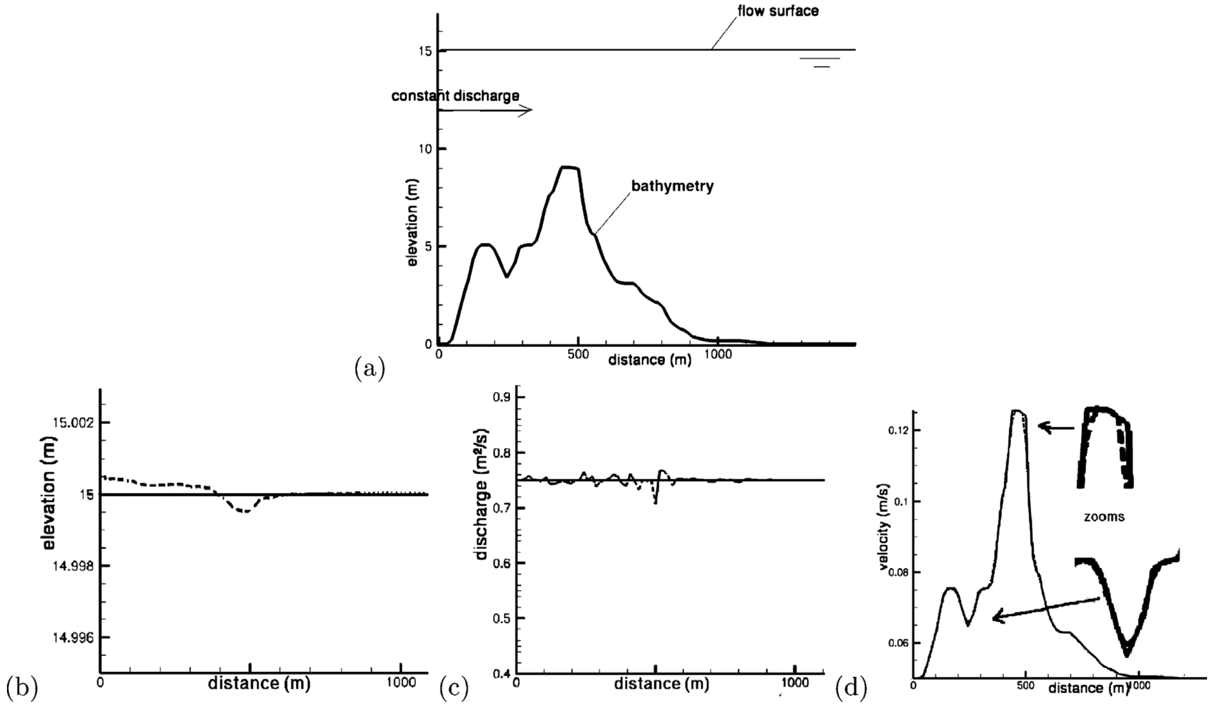

Fig. 1. Flow over an irregular bottom: (a) presentation; (b) numerical (dotted line) and analytical (solid line) water elevations; (c) numerical (dotted line) and analytical (solid line) discharges, (d) numerical (dotted line) and analytical (solid line) depth-averaged velocities.

Fig. 1. Écoulement au dessus d'un fond irrégulier : (a) présentation ; (b) hauteurs d'eau simulée (pointillés) et analytique (trait plein) ; (c) débits simulé (pointillés) et analytique (trait plein) ; (d) vitesses moyennées sur la hauteur d'eau simulée (pointillés) et analytique (trait plein).

to handle correctly the shallow water processes. The following major points are particularly tested: the use of large time steps, the flow over irregular bathymetry and intertidal zones as well as the sensibility of the numerical model to the bottom friction and horizontal turbulent diffusion. Here is presented the validation of the treatments of the strong bottom irregularities and the wetting/drying process [17].

4.1. Irregular bed topography

Following Tseng [18] and Vazquez-Cendon [19], a steady flow over an irregular bed (Fig. 1(a)) is used to demonstrate the ability of our model to implicitly handle strongly irregular bathymetry without any specific numerical treatment. This case provides a good insight into the behavior of numerical errors. Because of the steady state flow conditions, the discharge should be constant at any location over the computational domain.

The channel is 1500 m long. The irregular bed topography and exact results have been extracted from Tseng's paper [18]. At the upstream end, a constant discharge of 0.75 m^3/s is imposed while a zero flux Neuman condition is fixed at $x = 1500$ m. The initial uniform surface elevation is 15 m. By analogy with Tseng's work, the meshgrid is 500 while the time step is 1 s. A quadratic friction law is implemented $C_f \mathbf{V}\|\mathbf{V}\|$ in the momentum equation. The bottom friction coefficient is given by the Strickler law $C_f = g/(K^2 h^{1/3})$ where K is the constant Strickler coefficient. Here, $K = 30$.

Results given in Figs. 1(b)–(d) show the variations in simulated water surface elevation, discharge and velocity. Good agreement is found between numerical and analytical results. The maximum errors are respectively less than: 10^{-3} m, 5×10^{-2} m^2/s and 10^{-3} m/s, corresponding to very low relative values.

4.2. Intertidal flats

The second test case was originally proposed by Balzano [20] to compare the ability of different methods to handle the wetting and drying processes in hypothetical basins governed by tidal flows. A basin (Fig. 2(a)), initially under a

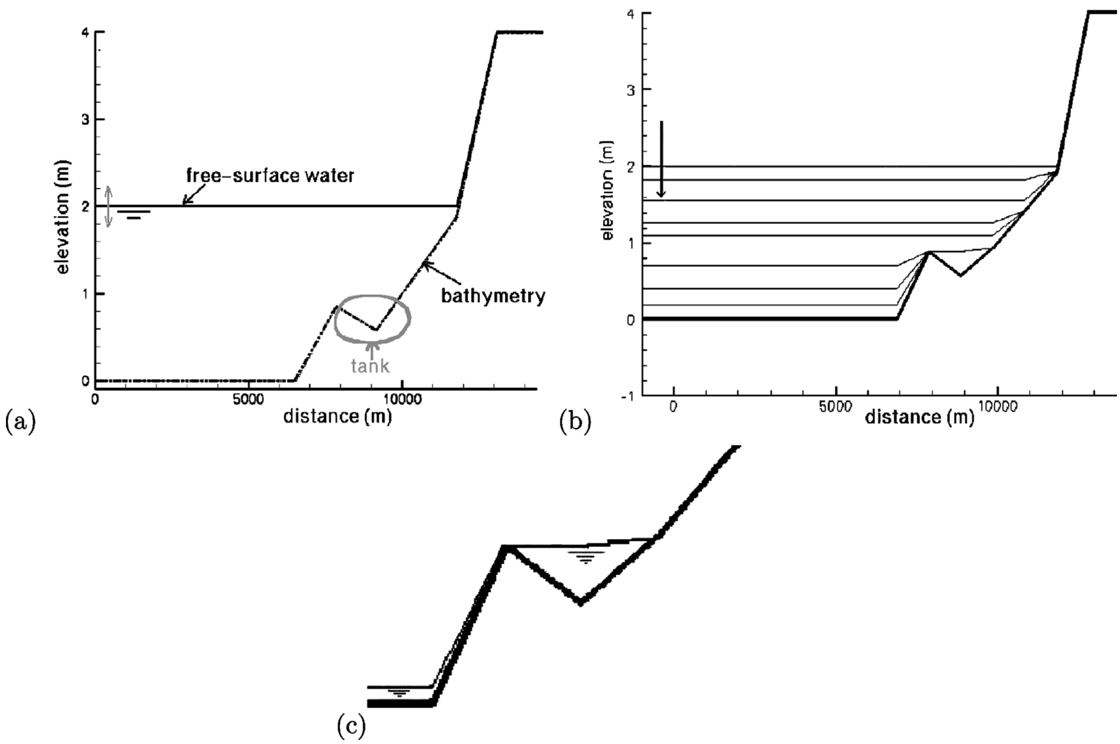

Fig. 2. Drying test simulation in a basin with reservoir at different stages: (a) at $t = 0$ s; (b) water levels during draining; (c) water level near final draining stage.

Fig. 2. Drainage d'un bassin avec réservoir à différents stades : (a) à $t = 0$ s ; (b) niveaux d'eau au cours du drainage ; (c) niveau d'eau au stade final du drainage.

water level of 2 m is emptied in 100 hours by the effect of a sinusoidal depletion of the downstream level. The water level in the tank is expected to reach asymptotically the relative maximum bottom elevation. The modeled domain is 13 800 m long, and the numerical parameters are the same as in Balzano's simulations: the coarse meshgrid contains 12 nodes, and the time step is 300 s. Final state of free surface is presented in Fig. 2(c).

The large time steps are well managed by the model, as well as stiff and complex topographies. Furthermore, the model shows a better treatment of the phenomenon of emergence compared to Balzano's simulations. Indeed, the water level reaches the relative maximum of bathymetry without presenting any physical aberrations (Fig. 2): it should be mentioned that the Alternative Direction Implicit schemes (ADI) used by Balzano are robust, enable large time steps and irregular bed topographies, but show unrealistic water levels below the intertidal zones (Fig. 3).

5. Application to a real tidal coastal site

In this section, a physical validation of the numerical model is considered in order to give a quantitive evaluation of its performances on a real shallow embayment.

5.1. The Arcachon lagoon

Located on the French Atlantic coast, the Arcachon Lagoon is a triangular shaped lagoon of 20 km on a side connected to the ocean through a 3 km-wide inlet [21] (Fig. 4(a)).

This mesotidal embayment [22] presents a strongly irregular bathymetry and an active morphology, in both the tidal inlet and the inner zone of the basin (even if its evolution is slower). Understanding sediment transport and forecasting the evolution of the bathymetry are of a great interest for the Arcachon Lagoon users (sailors, oyster breeders, tourism organisations, ...) and scientists. As the morphodynamics is mainly controlled by the hydrodynamics, modeling sediment transport requires accurate hydrodynamics computation, especially for the intertidal zones, subjected to

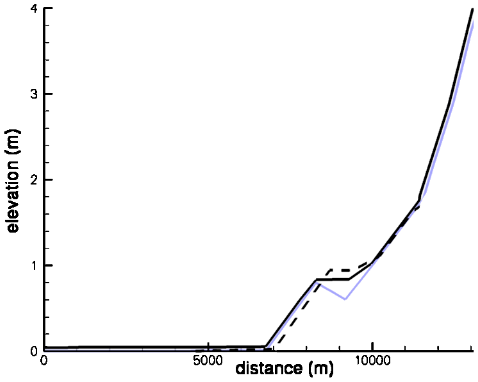

Fig. 3. Comparison between our model (solid line) and a ADI model (dashed line) tested by Balzano: water depth at the end of the basin drain (bathymetry in grey).

Fig. 3. Comparaison entre notre modèle (trait plein), et un schéma ADI (pointillés) testés par Balzano : niveaux d'eau au stade final du drainage (bathymetrie en trait gris).

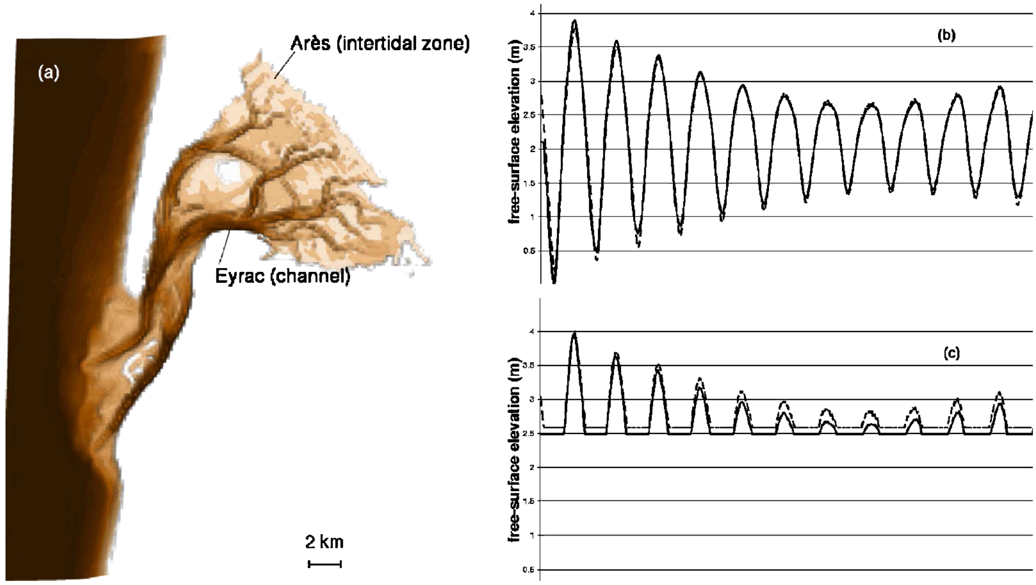

Fig. 4. (a) Bathymetry of the Arcachon lagoon. Numerical water elevation at (b) Eyrac and (c) Arès: our model (solid line), Ifremer model (dotted line).

Fig. 4. (a) Bathymétrie du Bassin d'Arcachon. Elévation de la surface libre à (b) Eyrac, et à (c) Arès : notre modèle (trait plein), le modèle d'Ifremer (pointillés).

alternating wetting and drying due to the tidal oscillation of the free surface. In such areas, experiencing strong water elevation fluctuations and covering 52% of the total surface of the lagoon [22], sediment transport processes are strongly dependent on the tidal flow [23]. Consequently, a good treatment of dry zones as well as of varying bathymetry and the use of large time steps are particularly essential for this 2DH-SW model when studying hundreds of tidal periods interacting with the Arcachon Lagoon hydrodynamics.

5.2. First results

After validating the numerical method on academic test cases, the lagoon hydrodynamics has been modelized. At the western open boundary of the domain, an offshore measured sequence of 11 real tides is imposed. Dimensions of the domain are 31 km × 36 km, discretized into a regular Cartesian meshgrid of 200 m × 200 m and the time step of

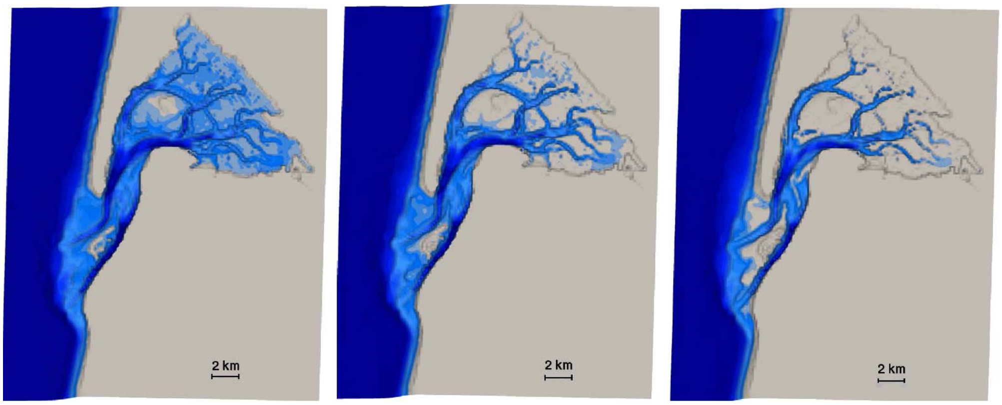

Fig. 5. Tide in the Arcachon lagoon: water elevations at (a) high tide; (b) mid-ebb tide; (c) low tide.

Fig. 5. Marée dans le bassin d'Arcachon : hauteur d'eau à (a) marée haute, (b) mi-marée, (c) marée basse.

the simulations is 20 s. The first results enabled to control the good management of strongly irregular bathymetry and the intertidal zones. These results are compared to simulations of the numerical 2DH model MARS developed by the French Research Institute for Exploitation of the Sea (Ifremer), validated beforehand thanks to field measurements [24,25]. The conditions imposed at the western liquid border of both models are identical. As Figs. 4(b) and 4(c) show, the water depth at Eyrac (tidal chanel) or Arès (intertidal mudflat) match the Ifremer results. Fig. 5. represents different stages of the water depth when the lagoon is draining during the ebb.

5.3. *Further developments*

Further developments required to improve our model are twofold. On one hand, in order to pursue the validation of the method accuracy and stability, highly unsteady test cases, such as dam-break or bore run-up, are being studied. On the other hand, the application of our model to a wider range of real cases requires to take into account more physical processes such as wind stresses, complex friction laws, salinity and temperature distributions or turbulent diffusion. The behavior of turbulence in shallow flows has been recently studied by numerous experimental and numerical studies [26,27]. In particular, the implementation of Horizontal Large Eddy Simulation [28,29] method in our model should allow to represent more accurately the essential turbulent processes: diffusion, mixing, transport and formation of vortex structures.

6. Conclusion and perspectives

Simulation of hydro- and morpho-dynamics of coastal areas requires robust numerical methods, able to deal with the particular features of shallow water flows. Numerical models have to handle large time steps required by sediment transport phenomena, to accurately represent the processes of wetting and drying in intertidal flats and to cope with strongly irregular bathymetry. In the present Note, a novel numerical approach is presented, based on the introduction of a Darcy term into the classical 3D Navier–Stokes equations. An original Shallow Water Equation system is solved thanks to an implicit resolution procedure provided by an adaptation of the Augmented Lagrangian algorithm and a penalty method. The model accuracy and robustness have been tested on several academic test cases. A real application is finally carried out to show the model accuracy in representing the tidal propagation in a complex coastal embayment: the Arcachon lagoon. Our results are compared with Ifremer simulations, which have been previously validated with fields measurements. Comparison of water level shows a very good agreement between both models.

The first results do encourage us to carry out more detailed studies on the basin. Studying sediment processes in its inner zone would locally require a finer meshgrid in order to better describe this area. An adaptative mesh refinement technique [23] will be used to increase the local mesh accuracy according to bathymetry, velocity or water level gradients. The new treatment of the dry zone will then provide more accurate velocity fields on the intertidal flats and, consequently, a better transport computation.

Acknowledgements

We would like to thank Martin Plus, from IFREMER, for fruitful discussions and for having accepted to provide us with water level and velocity data in different stations of the Arcachon lagoon.

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