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Experimental determination of the concentration Probability Density Function for a saltating particle layer

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Abstract

A horizontal saltation layer of glass particles in air was investigated experimentally in a wind tunnel. Particle concentrations are measured by light scattering diffusion and image processing and all the statistical characteristics were evaluated and thus the probability density function. Our experimental results confirm that the mean concentration decreases exponentially with height, the mean Eulerian dispersion height H being a characteristic lengthscale and that the instantaneous concentration distribution $\tilde{c}(\vec{x}, t)$ is a random variable following quite well a lognormal distribution. ***To cite this article: X. Zhang et al., C. R. Mecanique 334 (2006).***

Résumé

Détermination expérimentale de la densité de probabilité de concentration d'une couche de particules en saltation. La dispersion de particules de verre en mouvement de saltation à l'intérieur d'une couche limite turbulente est étudiée expérimentalement. Les concentrations ont été déterminées par diffusion de la lumière et analyse d'images. Certaines caractéristiques statistiques ont été mesurées telles les concentrations moyennes, les différents moments statistiques ainsi que les densités de probabilité. Les résultats expérimentaux confirment que la concentration moyenne décroît exponentiellement avec la hauteur, la hauteur moyenne de dispersion eulérienne H étant une échelle de longueur caractéristique. On montre en plus que la distribution de concentration instantanée $\tilde{c}(\vec{x}, t)$ est une variable aléatoire qui suit une loi de distribution Lognormale. ***Pour citer cet article : X. Zhang et al., C. R. Mecanique 334 (2006).***

Keywords: Granular media; Turbulence; Saltation; Dispersion; Solid particles

Mots-clés : Milieux granulaires ; Turbulence ; Saltation ; Dispersion ; Particules solides

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Parmi les trois différents modes de transport de particules dans l'atmosphére, la saltation est le plus important dans le processus physique d'érosion [1]. Initialement soulevées par le vent et accélérées, elles retombent sur le sol, rebondissant et/ou éjectant d'autres particules dont de la poussiére. Bien que les trajectoires de particules en saltation soient essentiellement déterministes, la saltation doit être considérée comme un processus stochastique. La nature stochastique provient du processus d'ejection. Dans ce cadre, la concentration de particules est naturellement une variable aléatoire $\tilde{c}(\vec{x},t)$ fonction de la position et du temps. La couche de saltation considérée comme un continuum, le développement de modèles de prévision statistiques nécessite la connaissance des propriétés statistiques et notamment de la densité de probabilité.

Les expériences ont été réalisées dans la soufflerie du 'Key Laboratory of Desert and desertification' de l'Académie des Sciences de Chine, Lanzhou, Chine. Une description complète de cet équipement peut-être trouvée dans Dong et al. [3]. Les particules utilisées sont des particules de verre, de densité $\rho_\mathrm{p} = 2650\ \mathrm{kg\,m^{-3}}$ et de diamètre moyen 208 µm. Le plancher de la soufflerie fut recouvert de particules sur une épaisseur de 0,02 m, à une distance de 8 m en aval de l'entrée et sur une longueur de 3 m. Les mesures par visualisation et analyse d'images [4] furent effectuées dans une section où la couche de saltation est stationnaire et établie. Un descriptif complet des expériences et des résultats expérimentaux peut être trouvé dans Zhang et al. [5] et Wang et al. [6].

Les résultats expérimentaux concernant la concentration moyenne reportés Fig. 1 indiquent une forme de décroissance exponentielle en fonction de la hauteur. Déterminons la forme générale. L'équation de la concentration moyenne peut-être établie à partir de l'équation de Boltzmann (Zhu et al. [7]) mais aussi plus simplement à partir de l'équation de continuité des particules (1). L'équation d'évolution eulérienne de la concentration moyenne $\langle C\rangle = \langle nm\rangle$ s'écrit selon (2). La couche de saltation pouvant être considérée comme une couche bidimensionnelle, incompressible et stationnaire, en utilisant le modèle de fermeture de gradient (3) avec le coefficient de diffusivité K_t, on retrouve l'équation de la concentration moyenne (4) sous la forme proposée par Pasquill [8]. Si K_t est supposé constant et après une double intégration, on obtient la loi d'évolution de la concentration moyenne (5) qui décroît exponentiellement en fonction de la hauteur. La hauteur moyenne de dispersion eulérienne H apparaît comme une échelle de longueur caractéristique.

Les courbes théoriques reportées sur la Fig. 1 sont en bon accord avec les résultats expérimentaux.

Chatwin et al. [2] et Mole et Clarke [11] ont montré dans le cas de la dispersion d'un scalaire au sein d'un écoulement turbulent que la courbe du coefficient d'aplatissement F en fonction du coefficient de dissymétrie S était parabolique. De plus, Mole et Clarke [11] ont montré que les coefficients S et F devaient vérifier la relation $F \geqslant S^2 + 1$. Dans le cas d'une distribution lognormale de paramètres b et σ et dont la densité de probabilité s'écrit selon (6), l'intensité de concentration I s'écrit $I^2 = \mathrm{e}^{\sigma^2} - 1$. Dans le cas limite où $\sigma \ll 1$ et $I \ll 1$, la relation devient $F \approx \frac{16}{9}S^2 + 3$.

Les résultats expérimentaux présentés Figs. 2, 3 et 4 permettent de penser que nos expériences entrent bien dans le cadre de ces analyses théoriques. La forme parabolique (Fig. 2) ainsi que la relation $F \geqslant S^2 + 1$ (Fig. 4) sont bien vérifiées. On observe de plus que l'approximation limite de distribution lognormale $F \approx \frac{16}{9}S^2 + 3$ coïncide bien avec les résultats expérimentaux (Fig. 4). La Fig. 5 représente la densité de probabilité obtenue expérimentalement pour trois hauteurs différentes au sein de la couche de saltation, $z = H/2, H, 2H$, et pour les trois vitesses étudiées. La comparaison avec la distribution lognormale théorique est en bon accord.

En conclusion, on peut considérer que la concentration instantanée $\tilde{c}(\vec{x},t)$ de particules solides en mouvement de saltation au sein d'une couche limite turbulente est une variable aléatoire qui suit une loi lognormale.

1. Introduction: the need for the probability density function

The wind-induced movement of small particles has been classified into three transport processes: suspension, saltation and creep. From the three different solid particle motions, saltation is the primary wind erosion mechanism. The main reason is that the entrainment, the transport and the impact of saltating sand particles have a severe impact on the natural environment and human activity as for example soil erosion and dust entrainment (see for example Shao et al. [1]). Initially aerodynamically lifted off by the wind in short hops and although accelerated by the wind, particles will return to the bed and will impact the ground, rebounding and/or ejecting other particles as dust. Most mass transportation occurring near the ground surface is due to saltation. So, the elaboration of statistical models needs the knowledge of statistical properties of the flow, particularly the probability density function.

Although the trajectories of saltating particles are mainly deterministic and weakly influenced by the turbulent flow, saltation must be considered as a stochastic process. The stochastic nature is mainly coming at the time of the interaction between particles and bed. The ejection process, i.e. particles rebounding or not, the angle of ejection and the velocity of ejection, are all stochastic effects. In this framework, the instantaneous particle concentration $\tilde{c}$ (number of particles situated inside an elementary volume) is naturally a random variable, function of time and space, $\tilde{c}(\vec{x}, t)$. As noted by Chatwin et al. [2] for the scalar dispersion in turbulent flows, "*it has too often been forgotten that the concentrations are random variables and that the mean concentration is but one of its statistical properties*". The same sentence for saltating particles and particle concentration could be asserted.

2. Experimental set-up

The experiments were run in the blowing sand wind tunnel in the Key Laboratory of Desert and desertification, the Chinese Academy of Science, Lanzhou, China. A complete description of the facility could be found in Dong et al. [3]. The boundary layer developed on the horizontal flat floor of the 1 m × 0.6 m test section of the wind tunnel. For the $U_\mathrm{e} = 6, 8, 10\ \mathrm{m\,s^{-1}}$ speed of the free stream flow considered, the boundary layer thickness was about 0.12 m and the friction velocity $u_* = 0.29, 0.32, 0.38\ \mathrm{m\,s^{-1}}$, respectively. The particles used are glass particles with a density $\rho_\mathrm{p} = 2650\ \mathrm{kg\,m^{-3}}$ and a mean diameter of 208 µm. The floor of the wind tunnel was covered with particles to a depth of 0.02 m, installed 8 m downstream of the entrance of the test section and over a length of more than 3 m. The measure area was set at 3 m from the upwind edge of the sand bed where the saltation layer is stationary and well established.

3. The physical problem

We are interested by the transportation of solid particles inside an air boundary layer. This physical problem is a typical two phase flow problem. However, when the optical magnification of the imaging system is small enough, the saltation layer can be considered as a continuum, a one phase flow and we could tackle the problem as the dispersion of some scalar in air. Concentrations were measured using Light Scattering Diffusion and image processing (Ayrault and Simoëns [4]). The light intensity of each respective pixel from the ensemble of 400 concentration images was averaged, the concentration fluctuations about the mean values were obtained and so, all the ensemble averaged statistics determined. A detailed report of the experimental results can be found in Zhang et al. [5] and Wang et al. [6].

Experimental mean concentration profiles are shown in Fig. 1. Mean concentration and height are normalized respectively by the maximum of concentration $\langle C\rangle_{\max}$ and by the mean Eulerian height H defined by

$$H = \frac{\int_0^\infty z\langle C(x,z)\rangle\,\mathrm{d}z}{\int_0^\infty \langle C(x,z)\rangle\,\mathrm{d}z}$$

$H = 14.3, 21.1, 24.4$ mm for $U_\mathrm{e} = 6, 8, 10\ \mathrm{m\,s^{-1}}$. The different curves exhibit the same exponential shape and are similar.

Mean concentration could be estimated from the mean Eulerian dispersion equation. This equation could be established from the Boltzmann equation (Zhu et al. [7]) but also more simply from the continuity equation for particles. Consider the distribution function of particle velocity $p(\vec{V}, \vec{x}, t)$. The instantaneous density number of particles per unit volume is expressed as $n(\vec{x}, t) = \iiint p(\vec{V}, \vec{x}, t)\,\mathrm{d}\vec{V}$. By averaging the instantaneous continuity equation for particles

$$\frac{\partial \tilde{c}}{\partial t} + \frac{\partial(\tilde{c}V_i)}{\partial x_i} = 0 \tag{1}$$

and introducing the Reynolds decomposition for the instantaneous mass concentration $\tilde{c} = \langle C\rangle + c$, with $\tilde{c} = nm, m$ being the mass of a particle, $\langle C\rangle = \langle nm\rangle$ the mean concentration and c the fluctuation, we obtain the equation for the mean mass concentration

$$\frac{\partial\langle C\rangle}{\partial t} + \frac{\partial(\langle C\rangle\langle V_i\rangle)}{\partial x_i} + \frac{\partial\langle cv_i\rangle}{\partial x_i} = 0 \tag{2}$$

The saltation layer can be considered as a mean two-dimensional, incompressible and stationary flow. By introducing the simple mean gradient model for $\langle c v_z \rangle$ formulated as

$$\langle c v_z \rangle = -K_t \frac{\partial \langle C \rangle}{\partial z} \tag{3}$$

where K_t represents the eddy diffusivity coefficient and if we consider $\langle V_z \rangle \approx -V_{\rm lim}$ where $V_{\rm lim}$ is the terminal velocity of the particle, we get the simplified mean concentration equation as postulated by Pasquill [8]

$$\frac{\partial}{\partial z}\left[K_t \frac{\partial \langle C \rangle}{\partial z}\right] + V_{\rm lim} \frac{\partial \langle C \rangle}{\partial z} = 0 \tag{4}$$

The knowledge of the eddy diffusivity coefficient will allow us to solve this equation. In the atmospheric surface layer, the eddy diffusion coefficient is given by $K_t = u^* k z$ where k is von Karman's constant and u^* the friction velocity. With this value, the mean concentration of particles is expressed by a power law (Gillette and Goodwin [9]), $\langle C(z) \rangle = \langle C(z_0) \rangle (z/z_0)^{\gamma}$ with $\gamma = V_{\rm lim}/k u^*$. For wind tunnel experiments, the eddy diffusivity K_t should have a different value. Suppose that for our experiment we can consider the eddy diffusivity coefficient as a constant. After integrating twice the mean concentration equation, we obtain the exponential solution

$$\frac{\langle C(z) \rangle}{\langle C(z=0) \rangle} = \exp\left\{-\frac{z}{H}\right\} \tag{5}$$

4. Results

The exponential curves together with the corresponding experimental results are plotted in Fig. 1. The different values are in quite good agreement. We verify what is generally admitted (Dong et al. [10]) that mean concentration in a laboratory saltation layer decreases exponentially with height, provided that the eddy diffusivity coefficient be a constant.

Consider now the third and fourth statistical moments of concentration namely the skewness and the flatness coefficients. The experimental flatness coefficient F plotted against the skewness coefficient S is represented in Fig. 2. The collapse of the experimental data for the three fluid velocities is remarkable and the profiles have a parabolic form as clearly seen with the best fit quadratic curve. This fact was previously reported by Chatwin et al. [2] and Mole and Clarke [11] for scalar dispersion in turbulent shear flows.

Mole and Clarke [11] have pointed out some other characteristics of these statistical moments. First of all they reported that for all probability distributions, the skewness coefficient S and the flatness coefficient F should verify the relation $F \geqslant S^2 + 1$, the equality being verified only in the case when the sample space consists of exactly 2 discrete values i.e. without any molecular diffusion. Second, for the lognormal distribution with parameters b and σ and whose pdf expression is

$$p(C) = \frac{1}{\sigma C \sqrt{2\pi}} \exp\left\{-\frac{(\ln C - b)^2}{2\sigma^2}\right\} \tag{6}$$

they showed that the intensity of concentration $I = c'/\langle C \rangle$ is expressed by $I^2 = e^{\sigma^2} - 1$. c' represents the rms concentration value and $\langle C \rangle$ the mean. Then, for the limiting cases $\sigma \ll 1$ and $I \ll 1$, the relation between the flatness and the skewness coefficients was written as $F \approx \frac{16}{9} S^2 + 3$.

The experimental results for the intensity of concentration I are reported in Fig. 3. Since I is small, it may be assumed that our experiments take place in the framework of this limiting case.

The experimental results of the flatness coefficient plotted against the skewness coefficient, the theoretical curve$F = \frac{16}{9} S^2 + 3$, and the relation $F = S^2 + 1$ are plotted for the $U_{\rm e} = 8\ {\rm m\,s^{-1}}$ fluid velocity in Fig. 4. We verify first the relation $F \geqslant S^2 + 1$. We can observe that the lognormal approximation for the limiting case fit quite well the experimental data. This supports the view that the particle concentration variable $\tilde{c}(x, z, t)$ is lognormal distributed. This will be verified experimentally.

Fig. 5 represents the experimental probability density functions at three different heights $z = H/2, H, 2H$ inside the saltation layer and for the three different velocities. The agreement of the experimental values with the theoretical lognormal distribution is quite reasonable.

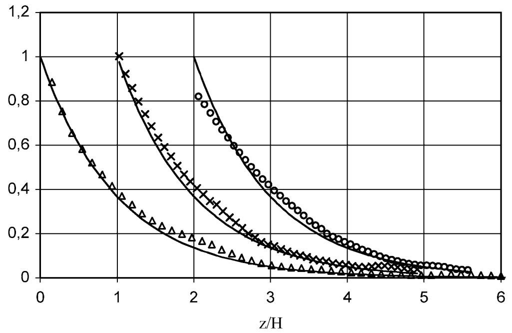

Fig. 1. Non-dimensionalized mean concentration profiles $\langle C(z)\rangle/\langle C\rangle_{\max}$ against non-dimensionalized height z/H for the three $U_e = 6, 8, 10\ \mathrm{m\,s^{-1}}$. Triangles: $U_e = 6\ \mathrm{m\,s^{-1}}$; crosses: $U_e = 8\ \mathrm{m\,s^{-1}}$; circles: $U_e = 10\ \mathrm{m\,s^{-1}}$; Lines: Theoretical exponential curves $\langle C(z)\rangle/\langle C(z=0)\rangle = \exp\{-\frac{z}{H}\}$. Note: The deviation between measurements and exponential curve for $U_e = 10\ \mathrm{m\,s^{-1}}$ is due to light absorption near the bed.

Fig. 1. Profils de concentration adimensionalisés $\langle C(z)\rangle/\langle C\rangle_{\max}$ en fonction de la hauteur adimensionalisée z/H pour les trois vitesses $U_e = 6, 8, 10\ \mathrm{m\,s^{-1}}$. Triangles : $U_e = 6\ \mathrm{m\,s^{-1}}$; croix : $U_e = 8\ \mathrm{m\,s^{-1}}$; cercles : $U_e = 10\ \mathrm{m\,s^{-1}}$; Lignes : courbes théoriques exponentielles $\langle C(z)\rangle/\langle C(z=0)\rangle = \exp\{-\frac{z}{H}\}$. Remarque : L'écart entre les résultats expérimentaux et la courbe théorique pour la vitesse $U_e = 10\ \mathrm{m\ s^{-1}}$ est dû à l'absorption de la lumière par les particules près du sol.

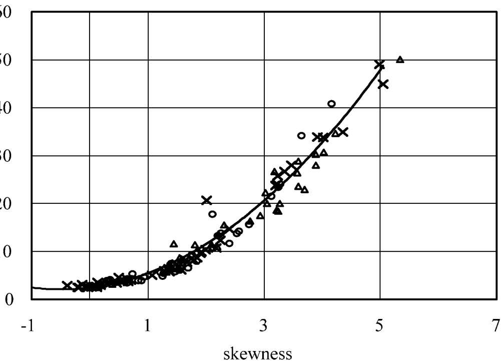

Fig. 2. Profiles of the flatness coefficient of concentration F versus the skewness coefficient S. Line is the parabolic best fit for $U_e = 8\ \mathrm{m\,s^{-1}}$.

Fig. 2. Profils du coefficient d'aplatissement F en fonction du coefficient de dissymétrie S. La courbe représente le meilleur lissage parabolique pour la vitesse $U_e = 8\ \mathrm{m\,s^{-1}}$.

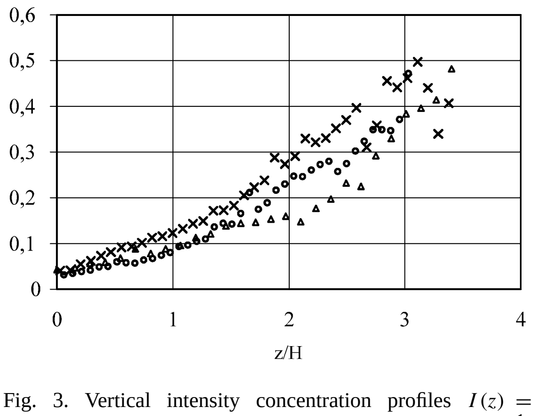

Fig. 3. Vertical intensity concentration profiles $I(z) = c'(z)/\langle C(z)\rangle$ against $\frac{z}{H}$ for velocities $U_e = 6, 8, 10\ \mathrm{m\,s^{-1}}$. Triangles: $U_e = 6\ \mathrm{m\,s^{-1}}$; crosses: $U_e = 8\ \mathrm{m\,s^{-1}}$; circles: $U_e = 10\ \mathrm{m\,s^{-1}}$.

Fig. 3. Profils verticaux de l'intensité de concentration $I(z) = c'(z)/\langle C(z)\rangle$ en fonction de la hauteur adimensionalisée $\frac{z}{H}$ pour les vitesses $U_e = 6, 8, 10\ \mathrm{m\,s^{-1}}$. Triangles : $U_e = 6\ \mathrm{m\,s^{-1}}$; croix : $U_e = 8\ \mathrm{m\,s^{-1}}$; cercles : $U_e = 10\ \mathrm{m\,s^{-1}}$.

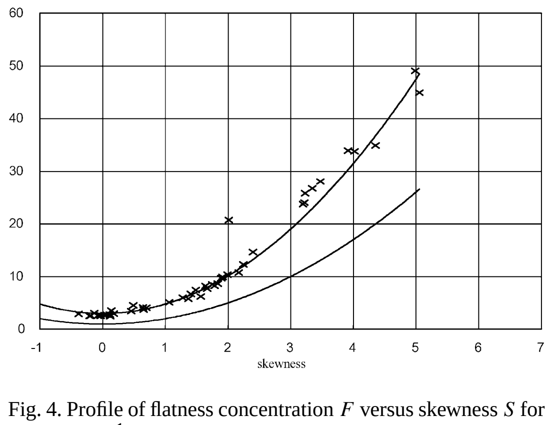

Fig. 4. Profile of flatness concentration F versus skewness S for $U_e = 8\ \mathrm{m\,s^{-1}}$ (crosses). Line represents the lognormal distribution $F = \frac{16}{9}S^2 + 3$ and the dashed line, the relation $F = S^2 + 1$.

Fig. 4. Profil du coefficient d'aplatissement F en fonction du coefficient de dissymétrie S pour $U_e = 8\ \mathrm{m\,s^{-1}}$ (croix). Ligne : limite de la distribution normale $F = \frac{16}{9}S^2 + 3$; pointillés : relation $F = S^2 + 1$.

In conclusion, we can consider that inside the saltation layer of solid particles in wind tunnel and for the three fluid flow speeds $U_e = 6, 8, 10\ \mathrm{m\,s^{-1}}$ studied, the mean concentration decreases exponentially with height, the mean Eulerian dispersion H being a characteristic lengthscale, the instantaneous concentration $\tilde{c}(x, z, t)$ of saltating glass particles is a lognormal random variable.

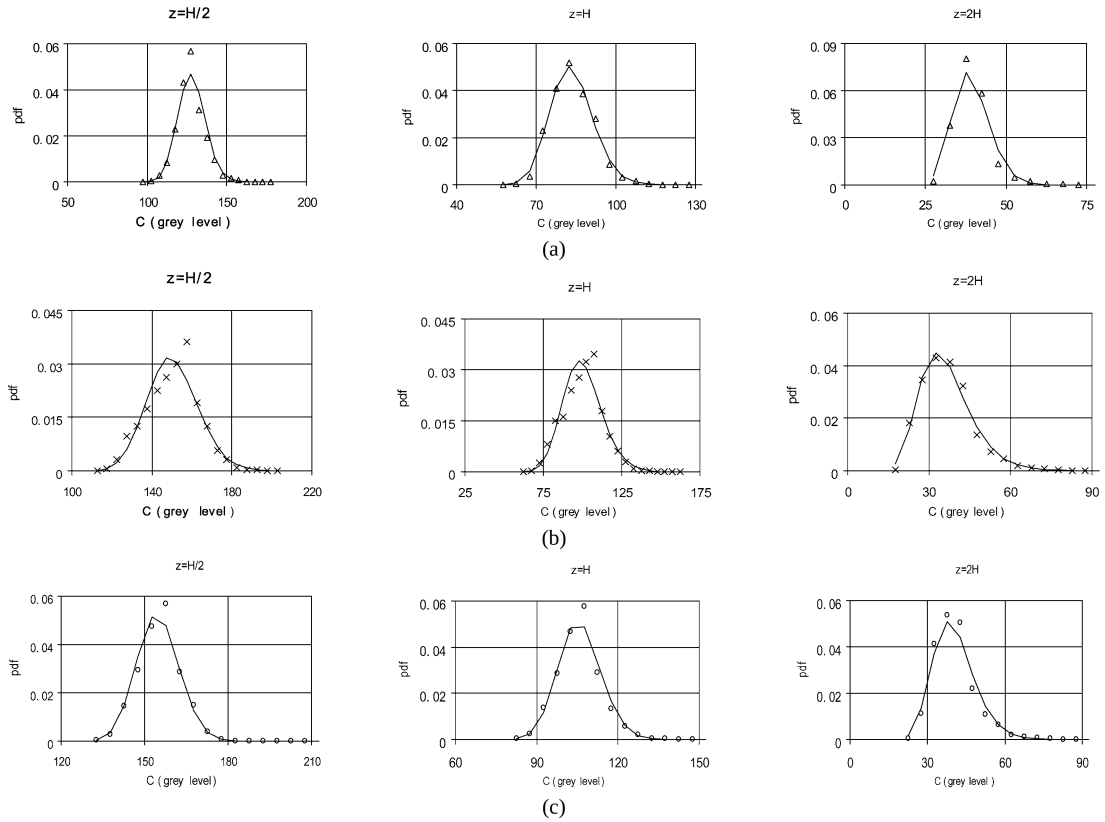

Fig. 5. Pdf of concentration at heights $z = H/2, H, 2H$ (a) $U_e = 6\ \mathrm{m\,s^{-1}}$; (b) $U_e = 8\ \mathrm{m\,s^{-1}}$; (c) $U_e = 10\ \mathrm{m\,s^{-1}}$. Experimental results: symbols; Lognormal theoretical curve: line.

Fig. 5. Densité de probabilité de concentration pour les hauteurs $z = H/2, H, 2H$. (a) $U_e = 6\ \mathrm{m\,s^{-1}}$; (b) $U_e = 8\ \mathrm{m\,s^{-1}}$; (c) $U_e = 10\ \mathrm{m\,s^{-1}}$. Symboles : ddp de concentration expérimentale ; Courbe : courbe lognormale théorique.

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