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Comparison between methods of generation of waveriders derived from conical flows

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Abstract

Waveriders are supersonic or hypersonic lifting configurations which are generated from a known flowfield, preventing leakage from the high-pressure undersurface to the uppersurface, and allowing high lift-to-drag ratios. A method based on an Euler code is used to study waveriders generated from the flow field around axisymmetric bodies. In the case of a cone, the results are compared with those given by the Taylor–Maccoll system and inviscid hypersonic small-disturbance theory. In the present study conditions at Mach 5, the last method paradoxically works at high cone angles (greater than 10°) while the best lift-to-drag ratio are obtained with small angles. ***To cite this article: B. Mangin et al., C. R. Mecanique 334 (2006).***
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Résumé

Comparaison de méthodes de génération de waveriders derivés d'écoulements coniques. Les waveriders sont des configurations portantes supersoniques ou hypersoniques qui sont générées à partir d'un écoulement connu empêchant tout déversement latéral de la zone haute pression de l'intrados vers l'extrados assurant ainsi de hautes finesses. Une méthode utilisant un code Euler est utilisée pour étudier des waveriders créés à partir d'écoulements axisymétriques. Dans le cas du cône, les résultats sont comparés avec ceux du système de Taylor–Maccoll et de la théorie des petites perturbations hypersoniques. Dans les conditions de la présente étude à Mach 5 la dernière méthode fournit paradoxalement des résultats comparables à ceux de Taylor–Maccoll aux grands angles de cône (supérieurs à 10°) tandis que la meilleure finesse est obtenue pour de petits angles. ***Pour citer cet article : B. Mangin et al., C. R. Mecanique 334 (2006).***
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Keywords: Fluid mechanics; Hypersonic; Waveriders; Cone; Small-disturbances; Lift-to-drag ratio

Mots-clés : Mécanique des fluides ; Hypersonique ; Waveriders ; Cône ; Petites perturbations ; Finesse

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1. Introduction

In the 1950s, confronted with the challenge of avoiding leakage due to shock stand-off (for bodies with blunt noses), Nonweiler [1] found a method for defining hypersonic vehicle shapes which simultaneously solved this problem and allowed him to take into account the 3D geometry in a simple way. Waveriders are generated by an inverse method: the undersurface is a streamsurface of a known inviscid flow around a reference body after its refraction by the front shock (see Fig. 1). The leading edge is the intersection of a freestream surface (which provides the uppersurface) and the shock created by the reference body. Leading edge sharpness prevents leakage and so provides high lift-to-drag ratios (the compression zone is isolated by the shock). The undersurface being a streamsurface, the waverider cruising at supersonic or hypersonic speeds creates the same shock wave as that due to the reference body and seems to be riding on top of this shock (thus its name). If the reference body is a wedge or an axisymmetric body, the flowfield will be 2D.

High lift-to-drag ratios are very difficult to obtain at hypersonic speeds due to high wave drag and massive viscous effects. Kuchemann [2] exhibits an empirical correlation for $(L/D)_{\max}$ based on experiments: $(L/D)_{\max} = 4(M_\infty + 3)/M_\infty$. Studies lead by Corda [3] and Bowcutt [4] show that waveriders break this barrier.

Rasmussen [7] lead to a waverider renaissance by using hypersonic small disturbance theory but the accuracy of this depends on the hypothesis that the reference body is slender enough and the Mach number high enough. In the present study, this method has been used in the cone case and its results have been compared to the results of the Taylor–Maccoll system at Mach 5.

2. Creation of a waverider

Waveriders are considered as the forebody of an entire configuration and the base design is not considered. For the determination of the flow around the reference body, the hypersonic flowfield around an axisymmetric body is obtained using a code which solves Euler equations (a version of the ONERA code FLU3M). In the case of a cone, results have also been obtained by solving the Taylor–Maccoll system and with hypersonic small disturbances assumptions as well.

X defines the flow direction. The cylindrical upper surface is defined by its cross section in a (Z, Y) plane, with Y as an even function of Z.

The upper surface being parallel to the freestream, its pressure is at freestream conditions so that there is no drag or lift created. The base drag is not considered.

Some geometrical characteristics are given. In particular, under and upper surface areas, planform area (on '$Y = 0$' plane) and volume are calculated (see [5]).

2.1. Flowfields

Aerodynamic flowfields on cones have been calculated using the Euler code and two other methods: the first solves the Taylor–Maccoll system derived from the Euler equations with conical assumption, and the second is based on the hypersonic small-disturbance theory.

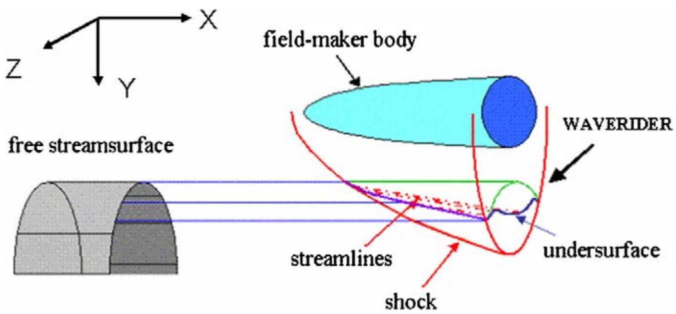

Fig. 1. Inverse method to create a waverider.

Fig. 1. Méthode inverse de création d'un waverider.

The conical assumption is that polar angle θ is the only variable (axisymmetric field and no length-scale). So Euler equations can take the form of the Taylor–Maccoll system [6]. Flow conditions downstream of the shock are obtained, thanks to attached-shock relations for a perfect gas. The Taylor–Maccoll system is solved using a fourth-order Runge–Kutta algorithm, the shock angle being obtained by iterations (shock-fitting).

Hypersonic small-disturbance theory assumption [7] enables to simplify the Taylor–Maccoll system and to solve it analytically. It requires the assumption that the cone is slender enough and the freestream Mach number high enough to apply small angle hypothesis. The ratio between the shock angle and the cone angle δ depends also on the infinite Mach number M_0, and is given as a function of the self-similarity parameter $K = Mo.\delta$. A streamline is created from the mass flow conservation between it and the cone and from the hypothesis of a constant density downstream of the shock.

The Euler version of the ONERA FLU3M code uses a Roe scheme without Harten correction on structured mono-zone grids. The two-dimensional grid is designed to obtain high point density around the body.

Grid convergence is guaranteed by the comparison of the wall pressure and the shock location obtained with various mesh definitions. The final grid definition has 480 points in I and 110 in J coordinates (spatial convergence of the calculation is obtained with less points, especially along I coordinates but this high definition is needed to obtain smooth streamlines and a thin shock).

2.2. *Viscous effects*

Viscous stresses are evaluated using two integral methods based on empirical formulations of the friction coefficient, Eckert's Reference Temperature Method [8] and Cousteix–Michel Method [9] respectively, considering uniform and non-uniform external flowfields.

Transition location is evaluated thanks to experimental results on flat plates [10]. If the waverider length is lower than the transition location, the boundary layer is considered as entirely laminar.

In the shock downstream area, the pressure increases along a streamline in the case of a conical flow while it decreases in the case of the power law blunt body flow. The two methods have been used to evaluate these variations effects on wall shear stresses. The relative difference between friction coefficients evaluated thanks to these methods is never greater than five per cent. The resulting friction drag difference is lower than two per cent (in both cones and blunt body cases).

2.3. *Forces*

Each generating streamline is defined by the same number of points (100 points for the 100 streamlines to ensure forces convergence). These 100×100 points define the grid structuring the undersurface (defined by two integers I and J). Wave drag (X-component) and lift (Y-component) of each cell are the components of the average pressure strain multiplied by the cell area. For each cell, the average component of the force due to the skin friction coefficient Cf, parallel to the local inviscid speed, gives the local contribution to the friction drag.

The flow conditions are given in Table 1. The Reynolds number based on the waverider length, Re_{Lw}, is 10e6. Experiments led on slender cones furnished a correlation between unitary Reynolds number and transition Reynolds number (see [10] and Table 1). Hence it appears that the waverider would have a laminar boundary layer on its first half and a turbulent one downstream. Meanwhile, the two extreme cases 'entirely laminar' and 'entirely turbulent' are considered here, rather than considering a non precise transition location.

Table 1
Flow conditions

Tableau 1
Caractéristiques de l'écoulement

Mach number	5
Unitary Reynolds number (m-1)	$40^{E}6$
Transition Rey. number	$4.78^{E}6$

3. Results

The results of the above subjects are presented in the following.

3.1. Comparison Taylor–Maccoll/small-disturbances

It is known that Hypersonic Small-Disturbance Theory works for small cone angles (less than 5°) and for high Mach numbers (greater than 10). Also, results for a Mach number of 3 can be seen in literature [7]. This method will be called the Rasmussen Method. To appreciate how it is performing at Mach 5 with the considered cone angles, its results are compared to those given by Taylor–Maccoll calculations in the particular case of cone-derived waveriders with flat upper surfaces.

Each method allows its own determination of the under surface geometry. In each case, forces are evaluated by the previously defined method. *Ro* is the distance from the upper surface to the symmetry axis in the (Z, X) plane and *Lw* is the waverider length. In Fig. 2 are plotted lift-to-drag ratio evolutions given by the two methods versus the cone angle for a fixed Ro/Lw equal to 0.2. The shift paradoxically decreases when the cone angle increases. All the curves are the same if the cone angle is greater than 13° because friction forces are negligible compared to pressure forces.

'Inviscid' lift-to-drag ratios (L/Dw) are close to each other. In the viscous laminar and turbulent cases, lift-to-drag ratios given by the two methods are different for lower cone angles. This is due to the fact that waverider geometries do not coincide. Indeed, for $\delta = 5°$, there is a non-negligible difference between the streamsurfaces given by the two methods while these are very close for $\delta = 12.9°$. (See in Fig. 3 the under surfaces represented by their projection in various transverse cutting planes). This is due to the fact that K, the hypersonic small-disturbance similarity parameter ($K = Mo.\delta$ where *Mo* is the freestream Mach number) must be high enough: according to Anderson [11] K must be higher than 0.5 and 1.5 for δ equal to 5° and 20°, respectively, corresponding to a Mach number higher than 5.73 and 4.30, respectively. At Mach 5, a linear interpolation implies a minimum K about 0.71 corresponding to a minimum cone angle about 8.1° which is higher than the cone angles where the lift-to-drag ratio is the highest. Moreover the similarity relation provides shock angles whose relative shifts compared to Taylor–Maccoll results reach 4% for $\delta = 5°$. This explains why the Hypersonic Small-Disturbance Theory has not been used to study Mach 5 blunt body-derived waveriders [13]. Those last ones have been created from the flowfields calculated by an Euler code. The validation of this last method is presented in the following part.

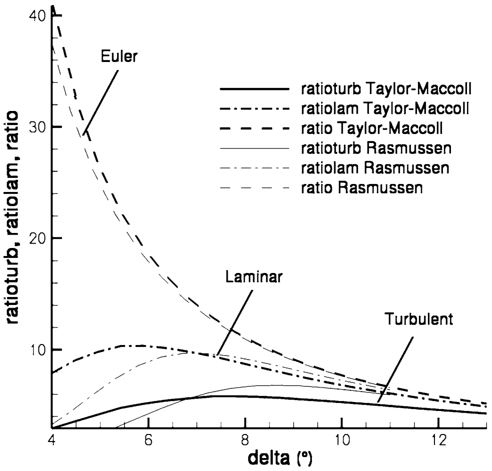

Fig. 2. Lift-to-drag ratios versus cone angle (Taylor–Maccoll and Rasmussen Methods for $Ro/Lw = 0.2$, $Re_{Lw} = 10e6$).

Fig. 2. Finesse en fonction de l'angle du cône (méthodes de Taylor–Maccoll et de Rasmussen pour $Ro/Lw = 0{,}2$, $Re_{Lw} = 10e6$).

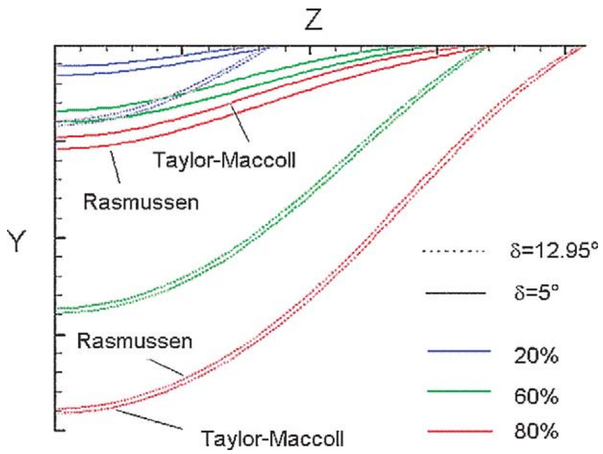

Fig. 3. Profiles of waveriders undersurfaces from Taylor–Maccoll and Rasmussen methods in transverse cutting planes $Y-Z$ ($Ro/Lw = 0.2$, $\delta = 5°$ and $\delta = 12.9°$).

Fig. 3. Profils des intrados des waveriders des méthodes de Taylor–Maccoll et de Rasmussen dans des sections transverses $Y-Z$ ($Ro/Lw = 0{,}2$, $\delta = 5°$ et $\delta = 12{,}9°$).

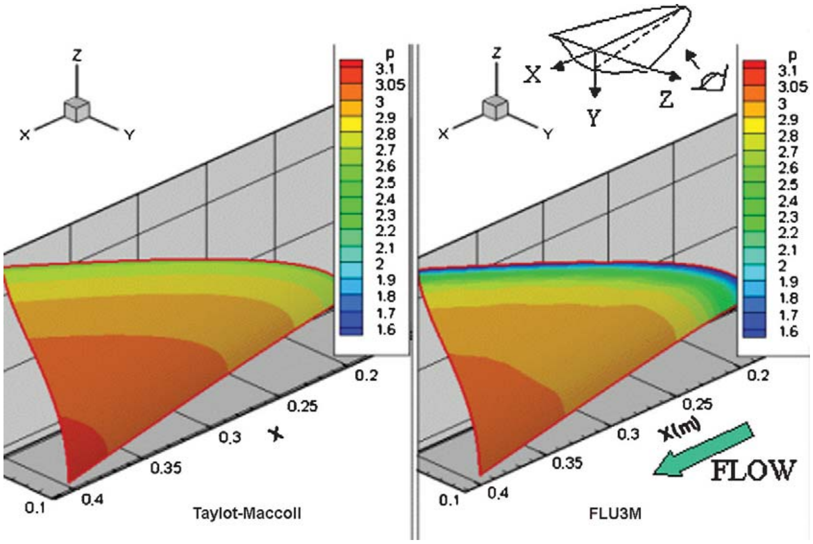

Fig. 4. Pressure to freestream pressure ratio at the undersurface (Taylor–Maccoll and Euler code).

Fig. 4. Pression à l'intrados adimensionnée par la pression à l'infini (Taylor–Maccoll et Euler code).

Table 2
Waveriders characteristics (Taylor–Maccoll/FLU3M)

Tableau 2
Caractéristiques des waveriders (Taylor–Maccoll/FLU3M)

$Ro/Lw = 0.2$, $Re_{Lw} = 10e6$, $\delta = 12.95°$	Taylor–Maccoll	FLU3M	ε (%)
CDw (wave drag coeff.)	1.031^{E}–02	0.991^{E}–02	3.88
CL (lift coeff.)	5.334^{E}–02	5.176^{E}–02	2.96
Wetted surface area (m2)	7.634^{E}–02	7.625^{E}–02	0.12
Inviscid lift-to-drag ratio	5.175	5.231	1.18

3.2. Comparison of Taylor–Maccoll versus Euler calculations

The ONERA CFD code FLU3M has been used to obtain the flowfields around cones. The results of the Euler code based method are compared to those given by the use of Taylor–Maccoll equations.

A difference between these two methods appears when looking at the shock localization. Indeed, the Taylor–Maccoll method fits the position of an infinitely thin shock while the Euler code based method creates a shock which smears over several grid points. Lobbia [12] encountered this problem with resulting drag and lift shifts of 13% and 22% respectively between a 3D Euler code based method and Taylor–Maccoll for a 10° cone. The use of a 3D code prevented them from excessively increasing the number of points to thin the shock. Shock smearing effect is the low pressure zone at the leading edge in Fig. 4.

For a cone angle equal to 12.95° and Ro/Lw equal to 0.2, waveriders with a flat uppersurface are obtained by using either method. Strains are very close, relative differences being lower than 4% (see Table 2, aerodynamic coefficients are relative to the wetted surface area). Geometries are very close too (see Fig. 4).

These results validate the Euler code based method to create waveriders.

4. Conclusion

Small disturbance theory showed its limits at Mach 5, what explains the use of an Euler code to create the flowfields around axisymmetric bodies. This last method has been well validated in the cone case by a comparison with Taylor–Maccoll results. Non-uniformity of the external flowfield has negligible effect on the friction stresses.

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