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On the formation of crack networks in high cycle fatigue

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Abstract

A probabilistic model based on an initial distribution of sites is proposed to describe different aspects of the formation, propagation and coalescence of crack networks in thermomechanical fatigue. The interaction between cracks is modeled by considering shielding effects. ***To cite this article: N. Malésys et al., C. R. Mecanique 334 (2006).***

Résumé

De la formation de réseaux de fissures en fatigue à grand nombre de cycles. La création de fissures, leur propagation et coalescence ultérieure dans un réseau sont traitées à l'aide d'un modèle probabiliste. Les interactions entre fissures sont prises en compte par la description des effets d'écran. ***Pour citer cet article : N. Malésys et al., C. R. Mecanique 334 (2006).***

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Mots-clés : Fatigue ; Effet d'écran ; Processus ponctuel de Poisson ; Modèle de Weibull

Version française abrégée

Le modèle proposé dans cette Note traite de manière unifiée de trois échelles de fissuration en fatigue [2]. L'échelle microscopique est liée au stade I de propagation. L'échelle mésoscopique correspond à la propagation en stade II des fissures. Enfin, la coalescence des fissures se produit à l'échelle macroscopique. Le stade II sera le plus important dans cette étude. La microstructure est modélisée par des sites sur lesquels des fissures peuvent s'amorcer par l'intermédiaire d'un processus ponctuel de Poisson.

En fatigue à grand nombre de cycles, de toutes les microfissures, seule une fraction peut former des mésofissures. Un processus de germination continue est introduit. Une microfissure peut conduire à l'amorçage d'une mésofissure

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à condition qu'elle ne soit pas écrantée par une autre mésofissure. Un domaine d'obscurcissement autour de mésofissures (i.e., une zone dans laquelle les contraintes sont inférieures au niveau appliqué, ce qui empêche tout nouvel amorçage) est défini. Ainsi, de nouvelles mésofissures sont amorcées s'il existe un site potentiel dans la zone considérée et, lorsque la condition d'amorçage est satisfaite, si cette fissure n'est pas écrantée par les mésofissures existantes (Fig. 1(a)). L'élément fondamental pour analyser l'amorçage est l'horizon (Fig. 1(b)). Il s'agit de regarder dans le passé de l'histoire de chargement. Une microfissure en S peut amorcer une mésofissure si aucune mésofissure ne se trouve dans son horizon. L'incrément de densité de mésofissures λ_m est calculé à l'aide de la probabilité d'obscurcissement P_{obs}. À partir de cette information, un modèle d'endommagement est écrit à l'échelle mésoscopique. Enfin, la coalescence peu alors être décrite à l'aide d'une condition de localisation de l'endommagement.

1. Introduction

Various components in nuclear power plants are subjected to thermomechanical loadings during their lifetime. Thermal striping was observed in the mixing zones of the auxiliary cooling system of nuclear power plants [1]. This is not troublesome as long as a fracture analysis indicates that the cracks will not grow notably during the remaining service life. Consequently, the evaluation of crack initiation, their subsequent propagation and coalescence in structures subjected to thermomechanical loadings is very important to determine investigation periods and maintenance programs. It is proposed to analyze the three stages of cracking by using a unified probabilistic framework.

2. Initiation, propagation and coalescence in a crack network

The following model aims at bridging three scales [2]. First the microscale, which is related to stage I of the fatigue process, depends upon the details of the microstructure. The cracks are considered as microstructurally short (they will be referred to as microcracks). This scale ends when mesocracks are initiated. The second scale corresponds to the propagation of mesocracks (i.e., stage II fatigue) that form the network. The cracks are considered as physically small. Last, the third scale is concerned with coalesced mesocracks that form a macrocrack (i.e., a long crack). Since stage II fatigue is predominant, the cracking directions are assumed to be aligned along the principal stress directions $\mathbf{d}_i$ (here constant during the whole load history). Each direction will be considered independently. The stress σ will denote any of the in-plane local principal stresses σ_1, σ_2 since initiation is assumed to occur on the surface of samples or structures.

The microstructure is modeled in terms of sites where cracks may initiate. The sites are approximated by points of *density* λ_t (i.e., their average number per unit surface or length). For example, these points have a random yield stress σ_y accounting for microplasticity in their vicinity. In the present setting, they correspond to the distribution of local fatigue limits $\Delta\sigma_\infty$ $(=\sigma_y)$. A Poisson point process of intensity λ_t is considered herein. A power law function is assumed and leads to a Poisson–Weibull model [3]

$$\lambda_t(\Delta\sigma) = \lambda_0\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^m \tag{1}$$

where m is the Weibull modulus (i.e., it characterizes the scatter in local fatigue limits or similarly in yield stress level), $\Delta\sigma$ the stress amplitude, and $\Delta\sigma_0$ the scale parameter relative to a reference density λ_0. The probability P of finding $N_\mu = \nu$ microcracks within a uniformly loaded domain Ω is expressed in terms of a Poisson distribution

$$P(N_\mu = \nu, \Omega) = \frac{[\lambda_t(\Delta\sigma)Z]^\nu}{\nu!}\exp\bigl[-\lambda_t(\Delta\sigma)Z\bigr] \tag{2}$$

The product $\lambda_t(\Delta\sigma)Z$ corresponds to the *average* number of microcracks in a domain Ω of measure Z. With a weakest link hypothesis, a two-parameter Weibull law [4] is retrieved (i.e., the failure probability P_F is given by $P_F = 1 - P(N_\mu = 0, \Omega) = 1 - \exp\{-\lambda_t(\Delta\sigma)Z\}$). The Weibull parameters may therefore be determined by analyzing *endurance* data for which the majority of the number of cycles is used in the formation of a mesocrack.

In high cycle fatigue, among all these microcracks, there is only a fraction for which the mesoscopic initiation condition is satisfied. Let λ_{tI} denote the corresponding density that depends upon the stress amplitude $\Delta\sigma$ and the number of cycles N. For instance, a threshold $\Delta\sigma_u(N)$ accounting for *continuous* mesoscopic initiation is considered

$$\lambda_{tI}(N; \Delta\sigma) = \lambda_t\bigl[\bigl\langle\Delta\sigma - \Delta\sigma_u(N)\bigr\rangle\bigr] \tag{3}$$

where $\langle\cdot\rangle$ are the Macauley brackets. Equation (3) shows that the initiation process needs a *minimum* number of cycles $N_{\min}$ (i.e., such that $\Delta\sigma - \Delta\sigma_u(N_{\min}) = 0$) to initiate the first mesocrack. In Eq. (3), the principal variable is N and $\Delta\sigma$ appears as a parameter. Similarly, with a weakest link hypothesis, a three-parameter Weibull law is retrieved as long as $\Delta\sigma_u > 0$. To be consistent with the previous analysis, it is assumed that $\Delta\sigma_u(N \to +\infty) \to 0$ so that high cycle fatigue (i.e., $N < +\infty$) and endurance (i.e., $N \to +\infty$) are described in the same framework. Therefore, $\Delta\sigma_\infty + \Delta\sigma_u(N)$ corresponds to the equation of a constant failure probability in a Woehler diagram.

To understand why a microcrack may initiate a mesocrack, one has to model its interaction with other mesocracks. An obscuration domain of measure Z_{obs} around mesocracks (i.e., a zone in which the stresses are less than the applied stresses, thus do not allow for new initiations) has to be defined. The obscuration domain of measure Z_{obs} is assumed to be proportional to the current size a of propagating mesocracks

$$Z_{\text{obs}}(N - N_I; \Delta\sigma) = S\big[a(N - N_I; \Delta\sigma)\big]^n \tag{4}$$

where N_I the number of cycles to mesoscopic initiation, $n = 1$ or 2 the space dimension, S a shape parameter independent of the Weibull modulus m but dependent on the space dimension n [5]. It may be noted that the initial mesocrack size $a(0; \Delta\sigma)$ is different from zero and depends upon microstructural parameters [2]. By using this set of hypotheses, microcracks do not obscure each other and mesocracks obscure microcracks, thereby partly inhibiting mesocrack initiation, and some mesocrack propagations.

New mesocracks will be initiated only if a microcrack exists in the considered zone, if the initiation condition is met and if the crack does not belong to *any* relaxed zone depicted in gray in Fig. 1(a). The spatial position of the microcracks is represented as a simple abscissa (instead of a two- or one-dimensional representation) of an x–y graph where the y-axis denotes the number of cycles. The microscopic growth is depicted by the black zones. A first mesocrack initiation occurs at point 1 for a number of cycles equal to N_1. The initiated mesocrack creates a stress relaxation zone or an 'obscured zone'. For a number of cycles N_3, the second mesocrack will be initiated at point 3, which is outside the obscuration zone of mesocrack 1. The second initiated mesocrack creates its own obscured zone. The sites 2 and 4 do not create mesocracks because they are obscured by the first and second mesocracks. The space-time (i.e., number of cycles) diagram is composed of the union of the obscured zones where no crack initiates and their complementary zones where any active site initiates a mesocrack. The key element to analyze the interaction between existing mesocracks and microcracks is the horizon of a given site S (Fig. 1(b)). It consists in looking at the past of S. A microcrack S will initiate a mesocrack if it is not obscured by other mesocracks. The horizon is a space-time zone where S is at least obscured by another mesocrack. Consequently, for a mesocrack to be formed, its horizon should not contain any mesocrack. In the present setting, obscuration occurs at the mesoscopic level. Therefore, the density λ_{tI} is split into two parts, namely, λ_m, the density of mesocracks and the obscured density. The increment of λ_m is related to that of λ_{tI} by

$$\frac{\mathrm{d}\lambda_m}{\mathrm{d}N}(N; \Delta\sigma) = \frac{\mathrm{d}\lambda_{tI}}{\mathrm{d}N}(N; \Delta\sigma) \times \big[1 - P_{\text{obs}}(N; \Delta\sigma)\big] \tag{5}$$

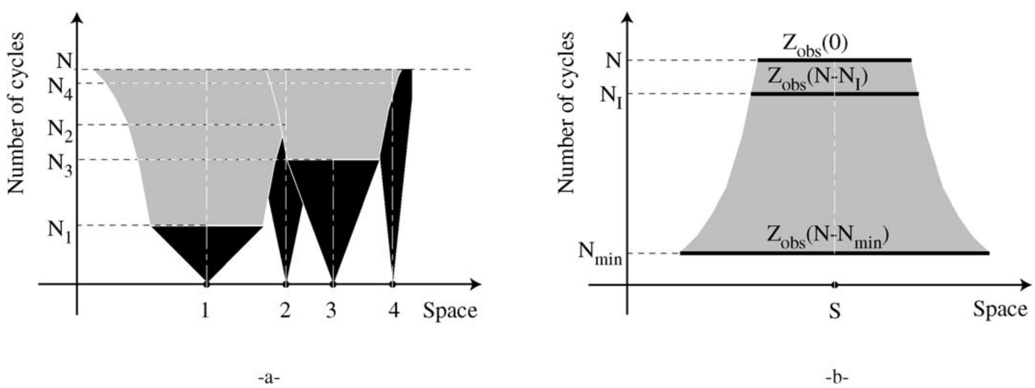

Fig. 1. (a) Depiction of two mesocrack propagations (1 and 3) and obscuration for two microcracks (2 and 4). (b) Horizon for a given location S.

Fig. 1. (a) Schématisation de la propagation de deux mésofissures (1 et 3) et de l'écrantage de deux microfissures (2 et 4). (b) Horizon en un lieu donné S.

with $\lambda_m(0; \Delta\sigma) = 0$ and P_{obs} the probability of obscuration

$$P_{\text{obs}}(N; \Delta\sigma) = 1 - \exp\left[-\widehat{Z}_{\text{obs}}(N; \Delta\sigma)\lambda_{tI}(N; \Delta\sigma)\right] \tag{6}$$

where $\widehat{Z}_{\text{obs}}$ is the measure of the *mean* obscuration zone [6]

$$\widehat{Z}_{\text{obs}}(N; \Delta\sigma)\lambda_{tI}(N; \Delta\sigma) = \int_{N_{\min}}^{N} Z_{\text{obs}}(N - N_I; \Delta\sigma)\frac{\mathrm{d}\lambda_{tI}}{\mathrm{d}N_I}(N_I; \Delta\sigma)\,\mathrm{d}N_I \tag{7}$$

It is worth noting that Eq. (6) accounts for overlappings of obscuration zones. Furthermore, in the context of mathematical morphology, the above-described approach corresponds to a Boolean islands model [7]. By using Eq. (3), Eq. (7) is rewritten in terms of the underlying distribution of endurance limits $\Delta\sigma_\infty$

$$\widehat{Z}_{\text{obs}}(N; \Delta\sigma)\lambda_t\left[\langle\Delta\sigma - \Delta\sigma_u(N)\rangle\right] = \int_{0}^{\langle\Delta\sigma - \Delta\sigma_u(N)\rangle} Z_{\text{obs}}\left[N - N_I(\Delta\sigma_\infty); \Delta\sigma\right]\frac{\mathrm{d}\lambda_t}{\mathrm{d}\Delta\sigma_\infty}(\Delta\sigma_\infty)\,\mathrm{d}\Delta\sigma_\infty \tag{8}$$

where $N_I(\Delta\sigma_\infty)$ corresponds to the number of cycles to initiation associated with $\Delta\sigma_\infty$ (e.g., $N_I(0) = N_{\min}$). When the crack propagation law is known, the change of Z_{obs} is written (see Eq. (4)) and the current density of mesocracks is derived.

From this information, a damage model at the mesoscale is written. To each principal direction $\mathbf{d}_i$ is associated a damage variable $D_i \equiv P_{\text{obs}}(N; \Delta\sigma_i)$ that characterizes the overall mesocracking state. The 'effective' stress $\Delta\sigma_i$ is here taken equal to $\sqrt{2E\Delta Y_i}$, where ΔY_i is the amplitude of the thermodynamic force associated with the damage variable D_i, E the Young's modulus of the virgin material. The mesoscopic principal stress amplitudes $\Delta\Sigma_i$ are related to the corresponding strain amplitudes $\Delta\varepsilon_i$ by the reduced stiffness tensor $\mathcal{K}$ that depends upon the two damage variables. Its inverse reads

$$\mathcal{K}^{-1}(D_1, D_2) = \frac{1}{E}\begin{bmatrix} \frac{1}{1-D_1} & -\nu \\ -\nu & \frac{1}{1-D_2} \end{bmatrix}_{(\mathbf{d}_1, \mathbf{d}_2)} \tag{9}$$

With this damage description, $\sqrt{2E\Delta Y_i} = \Delta\Sigma_i/(1 - D_i)$. The onset of coalescence is written as a damage localization condition. The number of cycles to coalescence N_{coal} is obtained from the condition

$$\mathrm{d}(\Delta\Sigma_i) = 0 \quad \text{or equivalently} \quad \Delta\sigma_i\frac{\partial\ln(1 - D_i)}{\partial\Delta\sigma_i} = -1 \tag{10}$$

that corresponds to the onset of strain and damage localization *perpendicular* to the considered eigen direction $\mathbf{d}_i$ [8].

3. Example

The mesoscopic initiation condition is described by using a damage model [9] written at a microscopic scale

$$\frac{\delta D_\mu}{\delta N} = \left(\frac{\langle\Delta\sigma - \Delta\sigma_\infty\rangle}{\Delta S_D}\right)^\eta \tag{11}$$

where ΔS_D and η are material-dependent constants, and D_μ the microscopic damage variable. Since the microscopic yield stress σ_y is equal to the local fatigue limit amplitude $\Delta\sigma_\infty$, the latter appears in Eq. (11). Consequently, the number of cycles to initiation (i.e., when $D_\mu = 1$) and the threshold stress become

$$N_I(\Delta\sigma_\infty) = \left(\frac{\Delta S_D}{\langle\Delta\sigma - \Delta\sigma_\infty\rangle}\right)^\eta \quad \text{and} \quad \Delta\sigma_u(N) = \Delta S_D N^{-1/\eta} \tag{12}$$

When initiation occurs, the mesocrack size is equal to $a(0; \Delta\sigma) = \Phi(\Delta\sigma_\infty)$, where Φ is a characteristic size of the microstructure (e.g., grain size). If the Hall–Petch relationship applies, $\Delta\sigma_\infty$ $(= \sigma_y)$ and Φ are related by

$$\Phi(\Delta\sigma_\infty) = \left(\frac{K}{\Delta\sigma_\infty}\right)^2 \tag{13}$$

where K is a material-dependent parameter. To get closed-form results, it is assumed that the mesocrack propagation is such that

$$a(N - N_I; \Delta\sigma) = \Phi(\Delta\sigma_\infty)\Psi(N - N_I; \Delta\sigma) \tag{14}$$

where the function Ψ describes the propagation stage in a simple way. By definition, $\Psi(0; \Delta\sigma) = 1$, and $\Psi(0; \Delta\sigma) \leqslant \Psi(N - N_I; \Delta\sigma) \leqslant \Psi(N - N_{\min}; \Delta\sigma)$, where the latter describes the propagation of the largest crack. Consequently, bounds to the obscuration probability and the density of mesocracks are derived. Bounds to the obscuration probability become $(m > 2n)$

$$\begin{aligned} P_{\text{obs}}(N; \Delta\sigma) &\geqslant 1 - \exp\left[-\frac{m}{m-2n}\left\{\frac{\langle \Delta\sigma - \Delta\sigma_u(N)\rangle}{\Delta\sigma_c^*}\right\}^{m-2n}\right] \\ P_{\text{obs}}(N; \Delta\sigma) &\leqslant 1 - \exp\left[-\frac{m}{m-2n}\left\{\frac{\langle \Delta\sigma - \Delta\sigma_u(N)\rangle}{\Delta\sigma_c(N - N_{\min}; \Delta\sigma)}\right\}^{m-2n}\right] \end{aligned} \tag{15}$$

where $\Delta\sigma_c^*$ is a characteristic (initiation) stress depending upon the Weibull and Hall–Petch parameters and $\Delta\sigma_c$ is a characteristic (propagation) stress depending in addition on the crack propagation law

$$\Delta\sigma_c^* = \Delta\sigma_0\left[\Delta\sigma_0\sqrt{a_0}/K\right]^{\frac{2n}{m-2n}} \quad \text{and} \quad \Delta\sigma_c(N - N_{\min}; \Delta\sigma) = \Delta\sigma_c^*\left[\Psi(N - N_{\min}; \Delta\sigma)\right]^{-\frac{n}{m-2n}} \tag{16}$$

with $a_0 = (S\lambda_0)^{-1/n}$ a reference crack size. Bounds to the density of mesocracks read $(m > 2n)$

$$\begin{aligned} \frac{\lambda_m(N; \Delta\sigma)}{\lambda_c(N - N_{\min}; \Delta\sigma)} &\geqslant \left(\frac{m-2n}{m}\right)^{\frac{2n}{m-2n}} \gamma\left[\frac{m}{m-2n}; \frac{m}{m-2n}\left\{\frac{\langle \Delta\sigma - \Delta\sigma_u(N)\rangle}{\Delta\sigma_c(N - N_{\min}; \Delta\sigma)}\right\}^{m-2n}\right] \\ \frac{\lambda_m(N; \Delta\sigma)}{\lambda_c^*} &\leqslant \left(\frac{m-2n}{m}\right)^{\frac{2n}{m-2n}} \gamma\left[\frac{m}{m-2n}; \frac{m}{m-2n}\left\{\frac{\langle \Delta\sigma - \Delta\sigma_u(N)\rangle}{\Delta\sigma_c^*}\right\}^{m-2n}\right] \end{aligned} \tag{17}$$

where γ is the incomplete gamma function $\gamma[p, x] = \int_0^x t^{p-1}\exp(-t)\,\mathrm{d}t$, λ_c^* and λ_c characteristic densities

$$\lambda_c^* = \lambda_0\left[\Delta\sigma_0\sqrt{a_0}/K\right]^{\frac{2mn}{m-2n}} \quad \text{and} \quad \lambda_c(N - N_{\min}; \Delta\sigma) = \lambda_c^*\left[\Psi(N - N_{\min}; \Delta\sigma)\right]^{-\frac{mn}{m-2n}} \tag{18}$$

The characteristic quantities are related with one another by the following condition

$$Z_{\text{obs}}(0; \Delta\sigma)\lambda_t[\Delta\sigma_c^*] = 1 \quad \text{and} \quad Z_{\text{obs}}(N - N_{\min}; \Delta\sigma)\lambda_t\left[\Delta\sigma_c(N - N_{\min}; \Delta\sigma)\right] = 1 \tag{19}$$

i.e., the average number of site in a domain of measure $1/\lambda_c^*$ (resp., $1/\lambda_c(N - N_{\min}; \Delta\sigma)$) is equal to 1.

An upper bound to the number of cycles to coalescence N_{coal} is obtained from the condition

$$D_i = 1 - \exp\left(-\frac{1}{m-2n}\right) \quad \text{or} \quad \left\langle\sqrt{2E\Delta Y_i} - \Delta\sigma_c^* m^{-1/(m-2n)}\right\rangle = \Delta\sigma_u(N) \tag{20}$$

leading to

$$N_{\text{coal}} < \left[\frac{\Delta S_D}{\langle \mathrm{e}^{1/(m-2n)}\Delta\Sigma - m^{-1/(m-2n)}\Delta\sigma_c^*\rangle}\right]^{\eta} \tag{21}$$

This type of analysis is valid as long as the horizon remains included in any examination zone. Otherwise, a weakest link hypothesis applies at the considered level.

4. Perspectives

The probabilistic model was described in the simple case of the initiation of cracks on a surface along two perpendicular directions. A more detailed analysis on the interaction between cracks aligned along any direction is needed. In thermomechanical fatigue, crack propagation is also driven by the stress profile induced by temperature variations through the thickness of a structure. This effect has also to be accounted for.

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