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A micromechanical analysis of damage propagation in fluid-saturated cracked media

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Abstract

We first revisit the well known framework of Linear Elastic Fracture Mechanics (LEFM) in the case of a fluid-saturated crack. We next consider a r.e.v. of cracked medium comprising a family of cracks characterized by the corresponding crack density parameter ε. Generalizing the classical energy approach of LEFM, the proposed damage criterion is written on the thermodynamic force associated with ε, which is estimated by means of standard homogenization schemes. This criterion proves to involve a macroscopic effective strain tensor, or alternatively the Terzaghi effective stress tensor. The stability of damage propagation is discussed for various homogenization schemes. A comparison with experimental results is presented in the case of a uniaxial tensile test on concrete. ***To cite this article: L. Dormieux et al., C. R. Mecanique 334 (2006).***

Résumé

Une analyse micromécanique de la propagation de l'endommagement d'une fissure sous pression de fluide. On reprend tout d'abord le cadre classique de la Mécanique Linéaire de la Rupture (MLR) en considérant le cas d'une fissure sous pression de fluide. Puis l'on s'intéresse à un v.e.r. de milieu fissuré saturé comprenant une famille de fissures caractérisée par le paramètre de densité ε correspondant. Généralisant l'approche énergétique usuelle de la MLR, le critère d'endommagement est formulé sur la force thermodynamique associée à ε, celle-ci pouvant être estimée à l'aide de divers schémas d'homogénéisation. On montre que ce critère s'exprime en fonction d'une déformation effective macroscopique, ou alternativement, en fonction de la contrainte effective de Terzaghi. La stabilité de la propagation de l'endommagement est discutée en fonction du schéma utilisé. On présente une comparaison avec les résultats d'un essai de traction uniaxiale sur une éprouvette de béton. ***Pour citer cet article : L. Dormieux et al., C. R. Mecanique 334 (2006).***

Keywords: Damage; Homogenization; Micromechanics; Cracking; Anisotropy; Eshelby tensor

Mots-clés : Endommagement ; Homogénéisation ; Micromécanique ; Fissure ; Anisotropie ; Tenseur d'Eshelby

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1. Introduction

The classical thermodynamics approach to damage is based on the identification of the internal variables characterizing the damage amount and of the associated thermodynamics forces. The damage criterion then takes the form of a mathematical condition written on these forces [1]. This approach is phenomenological in nature. The present Note revisits it from a micromechanical point of view.

Usually, the contribution of micromechanics to damage theory focuses on the determination of the homogenized properties of cracked media [2–5]. The crack density parameter [6] provides a micromechanical interpretation to the macroscopic damage variable. Nevertheless, some attempts have been devoted to the other essential issue, namely the propagation of damage [7–10]. Indeed, based on an appropriate choice of a homogenization scheme, micromechanics is able to provide estimates of the thermodynamics force associated with damage, interpreted as an energy release rate.

Considering fluid-saturated cracks, it is shown that this energy release rate depends on effective strain or stress tensors (Section 3). The dependence of the critical energy release rate as a function of the damage state is investigated. The approach is based on an estimate of the dissipated energy during the damage propagation. We then assess the stability of the damage propagation which turns out to depend on the crack density parameter (Section 4). Finally, we discuss the role of the homogenization scheme (Section 5). Comparisons with experimental data are presented at Section 6.

2. Elements of linear fracture mechanics in fluid-saturated conditions

Consider a 3-dimensional structure Ω made up of a linear elastic material and of a single plane crack. $|\Omega_0|$ denotes the volume of the considered structure in its reference configuration. The crack of initial (resp. current) volume V_0 (resp. V) is saturated by a fluid at pressure P. The displacement $\underline{\xi}$ is prescribed on the external boundary $\partial\Omega$. In order to scale the magnitude of the prescribed displacement, a scalar loading parameter $\mathcal{E}(t)$ and a time-independent function $\underline{\xi}^o(\underline{z})$ defined on $\partial\Omega$ are introduced:

$$(\forall \underline{z} \in \partial\Omega) \quad \underline{\xi}(\underline{z}) = \mathcal{E}(t)\underline{\xi}^o(\underline{z}) \tag{1}$$

Extending the classical reasoning [11,12] to the case of a saturated crack, we introduce the densities of elastic energy Ψ and potential energy Ψ^* of the solid, defined as:

$$\Psi = \frac{1}{2|\Omega_0|}\int_{\Omega} \boldsymbol{\varepsilon} : \mathbb{C}^s : \boldsymbol{\varepsilon}\, \mathrm{d}V; \qquad \Psi^* = \Psi - P(V - V_0)/|\Omega_0| \tag{2}$$

in which $\boldsymbol{\varepsilon}$ is the elastic strain field in the solid. Let $\mathrm{d}W$ denote the increment of mechanical work provided to Ω. The dissipation associated with the crack propagation can be expressed:

$$\mathrm{d}\mathcal{D} = \mathrm{d}W - |\Omega_0|\,\mathrm{d}\Psi = \mathrm{d}W - |\Omega_0|\,\mathrm{d}\Psi^* - \mathrm{d}\big(P(V - V_0)\big) \geqslant 0 \tag{3}$$

The work $\mathrm{d}W$ is the sum of the contributions of (i) the surface forces $\underline{T} = \boldsymbol{\sigma} \cdot \underline{n}$ acting on $\partial\Omega$ in the incremental displacement $\mathrm{d}\underline{\xi}$, and (ii) the fluid pressure P acting on the crack lips. Considering first reversible evolutions of the system provides its state equations in the form:

$$\frac{1}{|\Omega_0|}\int_{\partial\Omega} \underline{T} \cdot \underline{\xi}^o\, \mathrm{d}S = \frac{\partial \Psi^*}{\partial \mathcal{E}}; \qquad \frac{V - V_0}{|\Omega_0|} = -\frac{\partial \Psi^*}{\partial P} \tag{4}$$

Introducing (4) into (3), it is readily recognized that the driving force of the creation of additional crack surface $\mathrm{d}\ell > 0$ is the energy release rate $\mathcal{G}$:

$$\mathrm{d}\mathcal{D} = \mathcal{G}\,\mathrm{d}\ell; \qquad \mathcal{G} = -|\Omega_0|\frac{\partial \Psi^*}{\partial \ell} \geqslant 0 \tag{5}$$

A fracture propagates when the energy release rate $\mathcal{G}$ reaches the critical threshold G_f, which is an intrinsic material property:

$$\mathcal{G}(\ell) - G_f \leqslant 0; \qquad \mathrm{d}\ell \geqslant 0; \qquad (\mathcal{G} - G_f)\,\mathrm{d}\ell = 0 \tag{6}$$

and

$$\mathrm{d}\ell > 0 \quad \Rightarrow \quad \mathrm{d}\mathcal{D} = G_f \,\mathrm{d}\ell \tag{7}$$

The uniform strain boundary conditions

$$(\forall \underline{z} \in \partial\Omega) \quad \underline{\xi}(\underline{z}) = \mathbf{E} \cdot \underline{z} \tag{8}$$

can be seen as a particular case of the boundary condition (1). In a given Cartesian orthonormal frame $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$, it corresponds to $\mathcal{E} \to E_{ij}$ and $\underline{\xi}^o \to z_j \underline{e}_i$. Introducing the normalized crack volume $\phi = V/|\Omega_0|$ and the average stress $\overline{\boldsymbol{\sigma}}$, (4) determining the global response of the structure, can be put in the form:

$$\overline{\boldsymbol{\sigma}} = \frac{1}{|\Omega_0|} \int_{\Omega} \boldsymbol{\sigma} \,\mathrm{d}\Omega = \frac{\partial \Psi^*}{\partial \mathbf{E}}; \qquad \phi - \phi_0 = -\frac{\partial \Psi^*}{\partial P} \tag{9}$$

Note the formal analogy between (9) and the state equations of poroelasticity.

3. LEFM-damage analogy

Our purpose is now to investigate the response of a representative elementary volume (r.e.v.) of a saturated cracked material in non reversible conditions. By definition, the latter does not comprise only one single crack but a disordered distribution of cracks. Consider one single family of identical penny-shaped parallel cracks[1] (surface $\ell = \pi a^2$, density $\mathcal{N}$) in an r.e.v. The loading applied to the latter is defined by (8), in which $\mathbf{E}$ is now interpreted as the macroscopic strain tensor, and by a uniform fluid pressure P in the crack network. The damage parameter controlling the damaging effect on the poroelastic coefficients is the crack density parameter $\varepsilon = \mathcal{N} a^3$ [6], which is also equal to $\mathcal{N}(\ell/\pi)^{3/2}$. Hence, assuming that the crack density $\mathcal{N}$ is constant, ℓ and ε are equivalent state variables:

$$\ell = \pi \left(\frac{\varepsilon}{\mathcal{N}}\right)^{2/3}; \quad \mathrm{d}\ell = \frac{2\pi}{3\mathcal{N}^{2/3}} \varepsilon^{-1/3} \,\mathrm{d}\varepsilon \tag{10}$$

Owing to the linearity of the solid behavior, the potential energy of the solid phase in the r.e.v. is expected to be a quadratic function of $\mathbf{E}$ and P [13]:

$$\Psi^*(\mathbf{E}, P, \varepsilon) = \frac{1}{2}\mathbf{E} : \mathbb{C}^{\mathrm{hom}}(\varepsilon) : \mathbf{E} - \frac{P^2}{2N(\varepsilon)} - P\mathbf{B}(\varepsilon) : \mathbf{E} \tag{11}$$

Introduction of the above equation into (9) allows us to identify the state equations of the cracked porous medium. They relate the macroscopic stress $\boldsymbol{\Sigma} = \overline{\boldsymbol{\sigma}}$ and the normalized crack volume ϕ to the macroscopic strain $\mathbf{E}$ and to the fluid pressure P:

$$\boldsymbol{\Sigma} = \frac{\partial \Psi^*}{\partial \mathbf{E}} = \mathbb{C}^{\mathrm{hom}}(\varepsilon) : \mathbf{E} - \mathbf{B}(\varepsilon) P \tag{12}$$

$$\phi - \phi_0 = -\frac{\partial \Psi^*}{\partial P} = \frac{P}{N(\varepsilon)} + \mathbf{B}(\varepsilon) : \mathbf{E} \tag{13}$$

Note that ϕ in (13) represents the ratio $V/|\Omega_0|$, in which V is the sum of the crack volumes.

Relation (5) now takes the form:

$$\frac{\mathrm{d}\mathcal{D}}{|\Omega_0|} = -\frac{\partial \Psi^*}{\partial \ell} \,\mathrm{d}\ell = -\frac{\partial \Psi^*}{\partial \varepsilon} \,\mathrm{d}\varepsilon \geqslant 0 \tag{14}$$

The driving force of damage propagation therefore reads:

$$\mathcal{G}_\varepsilon = -\frac{\partial \Psi^*}{\partial \varepsilon} = -\frac{1}{2}\mathbf{E} : \frac{\partial \mathbb{C}^{\mathrm{hom}}}{\partial \varepsilon} : \mathbf{E} + \frac{P^2}{2} \frac{\partial}{\partial \varepsilon}\left(\frac{1}{N}\right) + P \frac{\partial \mathbf{B}}{\partial \varepsilon} : \mathbf{E} \tag{15}$$

[1] The reasoning is also relevant for a system of identical cracks with orientations that are isotropically distributed. However, in this case, an isotropic loading must be considered.

Following the reasoning introduced in [7] in the case of spherical or cylindrical cavities, it is appealing to adopt a criterion of the form (6) to describe damage propagation:

$$\mathcal{G}_\varepsilon - G_c \leqslant 0; \qquad \mathrm{d}\varepsilon \geqslant 0; \qquad (\mathcal{G}_\varepsilon - G_c)\,\mathrm{d}\varepsilon = 0 \tag{16}$$

where G_c is the analogous of the fracture energy G_f. Still, as opposed to G_f, G_c is not an intrinsic property of the solid and is expected to depend on the damage parameter ε.

Clearly enough, the proposed criterion is thermodynamic in nature, and is not derived from a micromechanics reasoning. This criterion addresses the propagation of a family of cracks in which all elements are assumed to behave identically and in a self-similar manner. It is this very assumption which enables to characterize the state of damage (microcracking) by a single scalar parameter, namely ε, together with the orientation of the cracks. However, the specific details of the geometrical configuration in the family are disregarded. We now take advantage of the Biot relations (see [10]):

$$\mathbf{B} = \mathbf{1} - \mathbb{C}^{\text{hom}} : \mathbb{S}^s : \mathbf{1}; \qquad \frac{1}{N} = \big((1-\phi_0)\mathbf{1} - \mathbb{C}^{\text{hom}} : \mathbb{S}^s : \mathbf{1}\big) : \mathbb{S}^s : \mathbf{1} \tag{17}$$

Inserting (17) into (15), the damage criterion now becomes:

$$\mathcal{G}_\varepsilon = -\frac{1}{2}(\mathbf{E} + P\mathbb{S}^s : \mathbf{1}) : \frac{\partial}{\partial \varepsilon}(\mathbb{C}^{\text{hom}}) : (\mathbf{E} + P\mathbb{S}^s : \mathbf{1}) \leqslant G_c(\varepsilon) \tag{18}$$

$\mathbf{E} + P\mathbb{S}^s : \mathbf{1}$ appears as an 'effective strain' controlling the damage propagation. Alternatively, a stress formulation is obtained from a combination of (18) with the state equation (12) together with the expression (17) of the Biot tensor $\mathbf{B}$:

$$\mathcal{G}_\varepsilon = -\frac{1}{2}(\mathbf{\Sigma} + P\mathbf{1}) : \mathbb{S}^{\text{hom}} : \frac{\partial}{\partial \varepsilon}(\mathbb{C}^{\text{hom}}) : \mathbb{S}^{\text{hom}} : (\mathbf{\Sigma} + P\mathbf{1}) \leqslant G_c(\varepsilon) \tag{19}$$

It is interesting to note that the damage criterion (19) is controlled by Terzaghi's effective stress $\mathbf{\Sigma} + P\mathbf{1}$.

4. Stability analysis

We now seek a criterion for stable damage propagation. The forthcoming analysis is strongly related to the characterization of damage by the crack density parameter ε, that is, to the assumption that all cracks grow in a self-similar and identical way. A possible mechanism in which a single crack propagates is not addressed. Let us assume that the condition (16) for damage propagation is reached for the loading level $(\mathbf{E}, P)$ and the crack density parameter ε:

$$\mathcal{G}_\varepsilon(\mathbf{E}, P, \varepsilon) = G_c(\varepsilon) \tag{20}$$

The crack propagation is stable if $\mathcal{G}_\varepsilon(\mathbf{E}, P, \varepsilon + \mathrm{d}\varepsilon) < G_c(\varepsilon + \mathrm{d}\varepsilon)$, that is, if:

$$\frac{\partial \mathcal{G}_\varepsilon}{\partial \varepsilon}(\mathbf{E}, P, \varepsilon) < G_c'(\varepsilon) \tag{21}$$

which can also be written:

$$\frac{1}{2}(\mathbf{E} + P\mathbb{S}^s : \mathbf{1}) : \frac{\partial^2}{\partial \varepsilon^2}(\mathbb{C}^{\text{hom}}) : (\mathbf{E} + P\mathbb{S}^s : \mathbf{1}) + G_c'(\varepsilon) > 0 \tag{22}$$

In order to implement this stability condition, we need to characterize the dependence of G_c on ε which can be investigated through an energy reasoning. More precisely, the total dissipation in the r.e.v. is the sum of the contributions of all cracks in the r.e.v. which are assumed to propagate by the same amount $\mathrm{d}\ell$. The elementary contribution of a single crack is given by (7) so that the total dissipation reads:

$$\frac{\mathrm{d}\mathcal{D}}{|\Omega_0|} = \mathcal{N} G_f\,\mathrm{d}\ell = G_c\,\mathrm{d}\varepsilon \tag{23}$$

Recalling (10) then yields

$$G_c = \frac{2\pi}{3} G_f \left(\frac{\mathcal{N}}{\varepsilon}\right)^{1/3} = \frac{2\pi}{3}\frac{G_f}{a} \tag{24}$$

Hence, as already mentioned, G_c is not a independent material property, but depends on the crack size, and hence on the loading history.

The previous LEFM-damage analogy holds as well for parallel or randomly oriented open cracks of same crack radius a. A straightforward extension to multiple cracks of different radii and densities requires a statistical analysis [14] which lies beyond the purpose of this Note.

We now consider the simpler situation where the crack network is represented by several crack families, each one being associated with a crack density parameter ε^i. The radius a^i is assumed to be uniform in each family, but can be different from one family to the other. An immediate extension of the methodology presented in Section 3 consists in introducing a damage criterion (16) for each crack family (note that G_c^i is a priori different for each crack family because of its dependence on the crack size (see (24)):

$$(\forall i) \quad \mathcal{G}_\varepsilon^i(\mathbf{E}, P, \varepsilon^1, \ldots, \varepsilon^n) - G_c^i \leqslant 0 \quad \Leftrightarrow \quad -\frac{\partial \Psi^*}{\partial \varepsilon^i} \leqslant \frac{2\pi}{3}\frac{G_f}{a^i} \tag{25}$$

The discussion concerning the stability in the case of a finite number of crack families can be achieved in a way similar to the one presented in [11] in the LEFM framework.

5. The role of the homogenization scheme on the damage criterion

As indicated before, the homogenized poroelastic properties of a cracked medium depend on the choice of the homogenization scheme. This choice therefore should as well impact the expression of the damage criterion. In this section, the discussion is presented in the isotropic case: the distribution of cracks is isotropic and the macroscopic strain is of the form $\mathbf{E} = 1/3\,\mathrm{tr}\,\mathbf{E}\mathbf{1}$. The loading defined by $\mathrm{tr}\,\mathbf{E}$ and P is expected to induce an isotropic stress state $\boldsymbol{\Sigma} = \Sigma_m \mathbf{1}$. The damage growth is therefore controlled by a single crack density parameter ε. The implementation of the damage criterion (18) as well as of the stability condition only requires an estimate of the homogenized bulk modulus k^{hom}. The values provided by the dilute scheme, the Mori–Tanaka (MT) scheme and the Ponte–Castaneda–Willis one (PCW) [15] are recalled below:

$$k_{\mathrm{dil}}^{\mathrm{hom}} = k^s(1 - \varepsilon Q_1); \qquad k_{\mathrm{mt}}^{\mathrm{hom}} = \frac{k^s}{1 + \varepsilon Q_1}; \qquad k_{\mathrm{pcw}}^{\mathrm{hom}} = k^s\left(1 - \frac{Q_1\varepsilon}{1 + \varepsilon Q_1'}\right) \tag{26}$$

where

$$Q_1 = \frac{16}{9}\frac{1 - \nu^{s2}}{1 - 2\nu^s}; \qquad Q_1' = Q_1\frac{1 + \nu^s}{3(1 - \nu^s)} \tag{27}$$

in which ν^s is the Poisson ratio of the solid. Following (18) and (19), the onset of damage is characterized by the following effective strain (resp. stress) criterion:

$$\mathcal{G}_\varepsilon(\mathbf{E}, P) = -\frac{1}{2}\frac{\partial}{\partial \varepsilon}(k^{\mathrm{hom}})\left(\mathrm{tr}\,\mathbf{E} + \frac{P}{k^s}\right)^2 \leqslant G_c; \qquad \mathcal{G}_\varepsilon(\Sigma, P) = \frac{1}{2}\frac{\partial}{\partial \varepsilon}(1/k^{\mathrm{hom}})(\Sigma_m + P)^2 \leqslant G_c \tag{28}$$

In turn, the stability condition (22) now reads:

$$\frac{1}{2}\frac{\partial^2}{\partial \varepsilon^2}(k^{\mathrm{hom}})\left(\mathrm{tr}\,\mathbf{E} + \frac{P}{k^s}\right)^2 + G_c'(\varepsilon) > 0 \tag{29}$$

According to the dilute scheme, $k_{\mathrm{dil}}^{\mathrm{hom}}$ is a linear function of ε. Hence, the stability condition (29) would be fulfilled only if $G_c(\varepsilon)$ were a strictly increasing function, which is not the case (see (24)).

In the case of the MT scheme, it can be shown from (29) that the conclusion of the stability analysis depends on the value of the initial crack density parameter ε_o which should be compared to the critical value $\varepsilon_{\mathrm{cr}}^{\mathrm{mt}} = 1/(5Q_1)$ (see Fig. 1). It appears that the damage propagation is only stable if $\varepsilon_o > \varepsilon_{\mathrm{cr}}^{\mathrm{mt}}$. Furthermore, introduction of (24) into (28) provides the link between ε and the 'effective' loading parameter $E' = \mathrm{tr}\,\mathbf{E} + P/k^s$:

$$E' = E_o'\frac{1 + \varepsilon Q_1}{1 + \varepsilon_o Q_1}\left(\frac{\varepsilon_o}{\varepsilon}\right)^{1/6} \tag{30}$$

where ε_o denotes the crack density parameter at the onset of propagation and E_o' the corresponding value of the effective loading parameter.

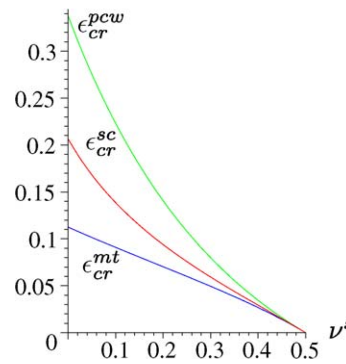

Fig. 1. Effect of the Poisson ratio ν^s on $\varepsilon_{\rm cr}^{\rm mt}$, $\varepsilon_{\rm cr}^{\rm pcw}$ and $\varepsilon_{\rm cr}^{\rm sc}$.

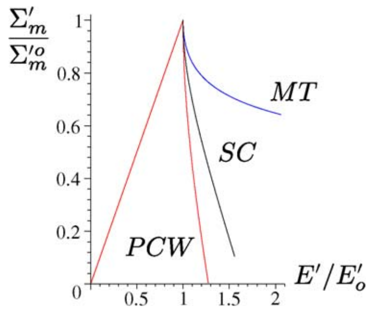

Fig. 2. Effective stress response of a damaging cracked porous material when G_c is given by (24). For each scheme, ε_o is set equal to the critical density $\varepsilon_{\rm cr}$ ($\nu^s = 0.2$).

When the stability condition $\varepsilon > \varepsilon_{\rm cr}^{\rm mt}$ is satisfied, it is readily seen from (30) that ε is an increasing function of E'. In turn, (28) reveals that the mean effective stress $\Sigma'_m = \Sigma_m + P$ decreases as the propagation goes on, that is, as ε increases:

$$\Sigma'_m = \Sigma'^o_m \left(\frac{\varepsilon_o}{\varepsilon}\right)^{1/6} \tag{31}$$

where Σ'^o_m denotes the value of Σ'_m at the onset of propagation. The typical shape of the stress-strain curve is displayed in Fig. 2.

The PCW scheme aims at separating the effects of inclusion shape and spatial distribution. In the case of an isotropic distribution of crack orientation and of an isotropic spatial distribution of cracks, this line of reasoning leads to the expression of the effective bulk modulus given in (26). The conclusion of the stability analysis again depends on the value of the initial crack density parameter ε_o which should now be compared to the critical value $\varepsilon_{\rm cr}^{\rm pcw} = 1/(5Q'_1)$ (see Fig. 1). The damage propagation is only stable if $\varepsilon_o > \varepsilon_{\rm cr}^{\rm pcw}$. As in the MT case, when the stability condition $\varepsilon_o > \varepsilon_{\rm cr}^{\rm pcw}$ is satisfied, the crack density parameter proves to be an increasing function of the loading parameter $E' = {\rm tr}\,\mathbf{E} + P/k^s$. We just have to replace Q_1 by Q'_1 in (30), while the effective mean stress Σ'_m is a decreasing function of ε as well as of E':

$$E' = E'_o \frac{1+\varepsilon Q'_1}{1+\varepsilon_o Q'_1}\left(\frac{\varepsilon_o}{\varepsilon}\right)^{1/6}; \qquad \Sigma'_m = \Sigma'^o_m \frac{27-32(1+\nu^s)\varepsilon}{27-32(1+\nu^s)\varepsilon_o}\left(\frac{\varepsilon_o}{\varepsilon}\right)^{1/6} \tag{32}$$

In contrast to (31), (32) predicts that the macroscopic effective stress Σ'_m vanishes when the damage parameter ε reaches the value $27/(32(1+\nu^s))$ (see Fig. 2). However, the latter lies out the domain of validity of the PCW estimate, which is bounded by $\varepsilon = 3/(4\pi)$.

As regards the self-consistent scheme (SC), no analytical expression of the effective properties of the microcracked material could be derived. Nevertheless, numerical estimates of the critical density $\varepsilon_{\rm cr}^{\rm sc}$ and of the normalized stress-strain behavior are plotted at Figs. 1 and 2. The softening predicted by the SC scheme and the critical density lie in between the predictions of the MT and PCW schemes.

6. Comparisons with experimental data

The comparison between the theoretical predictions of the macroscopic softening behavior and experimental data can be better achieved in the context of a strain driven uniaxial tensile test [16]. The macroscopic stress state therefore takes the form $\Sigma_{33}\underline{e}_3 \otimes \underline{e}_3$. The microcracking of the sample is described by a single family of identical parallel cracks propagating in the plane normal to the direction $\underline{e}_3$ of traction. This crack family is characterized by the parameter ε with initial value ε_o. The normalized stress $\Sigma_{33}/\Sigma_{33}^o$ and strain E_{33}/E_{33}^o respectively read:

$$\frac{\Sigma_{33}}{\Sigma_{33}^o} = \left(\frac{G_c(\varepsilon)}{G_c(\varepsilon_o)}\frac{\partial S_{3333}^{\rm hom}/\partial\varepsilon(\varepsilon_o)}{\partial S_{3333}^{\rm hom}/\partial\varepsilon(\varepsilon)}\right)^{1/2}; \qquad \frac{E_{33}}{E_{33}^o} = \frac{S_{3333}^{\rm hom}(\varepsilon)}{S_{3333}^{\rm hom}(\varepsilon_o)}\frac{\Sigma_{33}}{\Sigma_{33}^o} \tag{33}$$

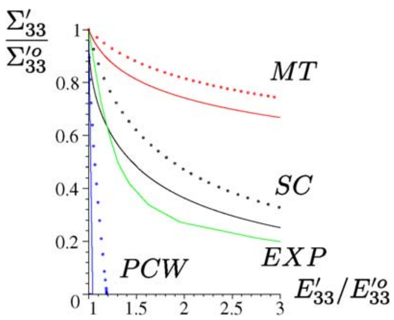

Fig. 3. Effective stress-strain response under simple traction: comparison between theory and experimental data (EXP) for $\varepsilon_o = 0.1$ (line) and $\varepsilon_o = 0.3$ (∘), $\nu^s = 0.2$.

It was not possible to find any value of ε_o providing a reasonable agreement between experimental data and the theoretical predictions of the MT and PCW schemes. They respectively significantly under- and overestimate the softening behavior associated with crack propagation. In contrast, as shown in Fig. 3, much better results are obtained with the self-consistent scheme.

7. Conclusions

Based on a energy reasoning, the present Note provides an estimate of the critical energy G_c required in a damage criterion as function of the fracture energy G_f and of the cracks density parameter ε. It appears that G_c is not an independent material property but seems to depend on the loading history. For a given homogenization scheme, it turns out that the discussion of the stability of the damage propagation is strongly sensitive to the way G_c depends on the cracks density parameter. The relevance of the proposed analysis for the description of the post peak non-linear softening regime has been assessed in the context of the tensile test.

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