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ScienceDirect
C. R. Mecanique 334 (2006) 555–559
COMPTES RENDUS MECANIQUE
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Observation, analysis and modelling in complex fluid media

On the modelling of piping erosion

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Abstract

A phenomenon called 'piping' often occurs in hydraulics works, involving the formation and evolution of a continuous tunnel between the upstream and the downstream sides. The hole erosion test is commonly used to quantify the rate of piping erosion. However, few attempts have been made to model these tests. From the equations for diphasic flow with diffusion, and the equations of jump with erosion, a piping model is developed. A characteristic time of internal erosion process is proposed. Comparison with experimental data validates our results. ***To cite this article: S. Bonelli et al., C. R. Mecanique 334 (2006).***

Résumé

Sur la modélisation de l'érosion interne. Un phénomène dénommé « renard » survient souvent sur les ouvrages hydrauliques. Il est lié à la formation et au développement d'un tunnel continu entre l'amont et l'aval. L'essai d'érosion au trou est très utilisé pour quantifier la cinétique d'érosion par renard. Toutefois, peu de travaux ont porté sur la modélisation de cette expérimentation. A partir des équations d'écoulement diphasique avec diffusion, et des équations de saut avec érosion, nous développons un modèle pour l'érosion par renard. Nous proposons un temps caractéristique de ce mécanisme d'érosion. Nous validons notre approche par comparaison avec des résultats expérimentaux. ***Pour citer cet article : S. Bonelli et al., C. R. Mecanique 334 (2006).***

Keywords: Soils; Two-phase flow; Internal erosion; Piping

Mots-clés : Sols ; Écoulements diphasiques ; Erosion interne ; Renard

1. Introduction

Internal erosion of soil induced by seepage flow is the main cause of major hydraulic works failures (dykes, dams). The issue is defined by the risk of flooding of areas located downstream. When internal erosion is suspected to occur or is already detected in situ, the amount of warning time before failure is difficult to predict. The development of effective emergency action plans which will lead to preventing heavy loss of life and property damage is strongly linked to the knowledge of a characteristic time.

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doi:10.1016/j.crme.2006.07.003

During the last decades, several investigations were carried out to study the internal erosion on the laboratory. Four types of this process were, particularly, identified: (1) evolution of defect (cracks or microfissures) in the soil matrix; (2) regressive erosion; (3) internal suffusion which modifies the soil structure; and (4) external suffusion between two soils. This study concerns the first mechanism: the enlargement of a crack which leads to an internal erosion called 'piping' in soil mechanics.

Numerous experimental methods have been performed in order to reproduce the internal erosion process in the laboratory and different types of equipment were developed with particular attention focussed on the hole erosion tests [1–3]. However, few attempts have been made to model these tests. The purpose of this article is to propose a useful model for the interpretation of the hole erosion test.

On the first part, equations of diphasic flow and equations of jump with erosion are presented. In the second part, a model is developed by spatial integration of simplified equations obtained from asymptotic developments in the case of a circular hole. Some comparison of this modelling with experiments is finally shown on the third part.

2. Two-phase flow equation with interface erosion

We study the surface erosion phenomenon of a fluid/soil interface under a flow parallel to the interface. The soil, considered here as saturated, is eroded by the flow which then carries out the eroded particles. As far as the particles are smaller enough compared to the characteristic length scale of the flow, this two-phase flow can be considered as a continuum. We note Ω the two-phase mixture volume and Γ the fluid/soil interface. For simplification, sedimentation and deposition processes are neglected. The mass conservation equations for the water-particles mixture and for the mass of particles as well as the balance equation of momentum of the mixture within Ω can be written as follows in an Eulerian framework [4]:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0, \qquad \frac{\partial \rho Y}{\partial t} + \vec{\nabla} \cdot (\rho Y \vec{u}) = -\vec{\nabla} \cdot \vec{J}, \qquad \frac{\partial \rho \vec{u}}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u} \otimes \vec{u}) = \vec{\nabla} \cdot \sigma \tag{1}$$

In these equations, ρ is the density mixture, depending on the particles mass fraction Y, $\vec{u}$ is the mass-weighted average velocity, $\vec{J}$ is the mass diffusion flux of particles, and σ is the Cauchy stress tensor in the mixture.

The two media, i.e., the soil and the two-phase fluid, are separated by the interface Γ. The water particles mixture is assumed to flow as a fluid above Γ, while a solid-like behaviour is considered underneath. As erosion occurs, a mass flux crosses this interface and so undergoes a transition from solid-like to fluid-like behaviour. As a consequence, Γ is not a material interface: at different moments, Γ is not defined by the same particles. We assume that Γ is a purely geometric separation and has no thickness. Let us denote by $\vec{n}$ the normal unit vector of Γ oriented outwards the soil, and $\vec{v}_\Gamma$ the normal velocity of Γ. The jump equations over Γ are:

$$[\![\rho(\vec{v}_\Gamma - \vec{u}) \cdot \vec{n}]\!] = 0, \qquad [\![\rho Y(\vec{v}_\Gamma - \vec{u}) \cdot \vec{n}]\!] = [\![\vec{J} \cdot \vec{n}]\!], \qquad [\![\rho \vec{u}(\vec{v}_\Gamma - \vec{u}) \cdot \vec{n}]\!] = -[\![\sigma \cdot \vec{n}]\!] \tag{2}$$

where $[\![a]\!] = a_g - a_b$ is the jump of any physical variable a across the interface, and a_g and a_b stands for the limiting value of a from the solid and fluid sides of the interface, respectively. The soil is supposed homogeneous, rigid and without seepage. The co-ordinate system is linked to the soil. In this case, the total flux of eroded material (both particles and water) crossing the interface is $\dot{m} = -\rho_g \vec{v}_\Gamma \cdot \vec{n}$ where ρ_g is the density of the soil.

Erosion laws, dealing with soil surface erosion by a tangential flow are often written as threshold laws such as:

$$\dot{m} = \begin{cases} k_{\mathrm{er}}\big(|\tau_b| - \tau_c\big) & \text{if } |\tau_b| > \tau_c \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $|\tau_b| = \sqrt{(\sigma \cdot \vec{n})^2 - (\vec{n} \cdot \sigma \cdot \vec{n})^2}|_b$ is the tangential shear stress at the interface, τ_c is the critical (threshold) shear stress for erosion and k_{er} is the coefficient of soil erosion.

This complete set of Eqs. (1)–(3) were already used to study different situations of permanent flow (boundary layer and free surface flow) over an erodable soil [5]. Here, these equations can be extended to the study of internal erosion by mean of a spatial integration over Ω.

3. Application to piping erosion

We consider a cylinder Ω of length L and radius R (with initial value R_0) (Fig. 1). The reference velocity is $V_{\mathrm{fl}} = Q_{\mathrm{fl}}/\pi R_0^2$ where Q_{fl} is the initial entrance flow, and flow time is $t_{\mathrm{fl}} = R_0/V_{\mathrm{fl}}$. By assuming an axisymmetrical flow, we

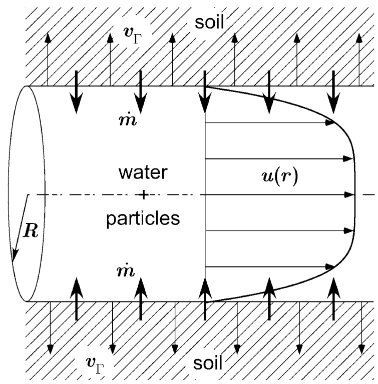


Fig. 1. Sketch of the axisymmetrical flow with erosion of the soil and transport of the eroded particles.

Fig. 1. Schéma de l'écoulement axisymétrique avec érosion du sol et transport des particules érodées.

eliminate one momentum equation. We introduce the small parameter Re^{-1} to simplify the dimensionless equations in a boundary layer theory spirit [6], with Reynolds number $Re = V_{fl} R_0/\nu^f$ assumed to be large, where ν^f is the kinematic viscosity. Navier–Stokes equations are written with the time scaled by t_{fl}, the axial coordinate by $R_0 Re$, the radial coordinate by R_0, the axial velocity by V_{fl}, the radial velocity by V_{fl}/Re, and stresses by $\rho^f V_{fl}^2$. Supposing turbulent stress viscosity and diffusivity, we make regular asymptotic expansion of unknown and we neglect the terms of order $O(Re^{-1})$ in (1) and (2) (as in [5]). As a result, we have now only one momentum equation, and the pressure is uniform across a section.

Therefore, we integrate the obtained system first on a cross-section, and secondly along the axis. We denote the mean value of a in a cross-section as $\langle a\rangle_R$, and the mean value in Ω as $\langle a\rangle_\Omega$. The mean longitudinal velocity is $V = \langle u\rangle_R$. The mean density of the fluid is $\bar{\rho} = \langle\rho\rangle_\Omega$. Some assumptions are made:

(A1) the tangential velocities are supposed continuous across Γ (no-slip condition on the interface);
(A2) the radial profile of the velocity field is given by the Nikuradze approximation;
(A3) the concentration is uniform in a section;
(A4) we introduce a phenomenological friction coefficient f_b by $\tau_b = f_b\bar{\rho}V^2$; and
(A5) the radius R is axially uniform (as a consequence, $V = \langle u\rangle_\Omega$).

This leads to an ordinary differential system with unknowns $(R, \bar{\rho}, V)$ which can be solved numerically.

If the erosion time scale is chosen, dimensional analysis reveals that the four basic parameters of the system are:

$$\tilde{a}_\phi = \frac{\alpha R_0 \rho_g}{2 f_b L \rho^f}, \qquad \tilde{b}_\phi = \left(\frac{\langle u^2\rangle_\Omega}{\langle u\rangle_\Omega^2} - 1\right)\left(1 - \frac{\rho^f}{\rho_g}\right), \qquad \tilde{k}_{ref} = \frac{k_{er} V_{fl}}{1 + k_{er} V_{fl}}, \qquad \tilde{\tau}_c = \frac{\tau_c}{P_{fl}}$$

The stress $P_{fl} = R_0(p_{in} - p_{out})/2L$ represents the hydraulic gradient and depends on the input and output pressures, respectively p_{in} and p_{out}. The positive dimensionless number $\alpha = (\rho_{out} - \rho_f)/(\bar{\rho} - \rho_f)$ represents the fact that the input fluid is pure water, while the output fluid is a two-phase mixture ($\rho_f < \bar{\rho} < \rho_{out}$).

The erosion velocity appears to be $V_{er} = k_{er} P_{fl}/(1 + k_{er} V_{fl})\rho_g$. The eroded flow is thus $Q_{er} = 2\pi R_0 L V_{er}$ and the erosional time is $t_{er} = R_0/V_{er}$. As a consequence, the erosional flow scale ratio is $Q_{er}/Q_{fl} = \alpha\tilde{k}_{ref}\tilde{a}_\phi^{-1}$, the erosional time scale ratio is $t_{fl}/t_{er} = \tilde{k}_{ref} f_b \rho^f/\rho_g$ and the maximum volumic concentration is $c_{ref} = (1-n)/(1 + \tilde{a}_\phi \tilde{k}_{ref}^{-1})$ where n is the porosity of the soil.

Now we assume that $\tilde{a}_\phi \geqslant O(1)$ and $\tilde{b}_\phi \approx O(1)$, which is the case in the experiments described below. We call $\tilde{k}_{ref}$ the kinetics of erosion (dimensionless) number. If $\tilde{k}_{ref} \ll 1$ is a small parameter, then asymptotic analysis leads to important conclusions: (1) the concentration is low and becomes a secondary unknown as it does not influence the density, the inertia, the velocity or the stress; (2) the flow is quasisteady; and (3) the interface velocity is low and does not contribute to inertia. We call this case, which arises when $k_{er} \ll V_{fl}^{-1}$, the situation of low kinetics of erosion.

Table 1
Range of characteristic values of data and results

Tableau 1
Domaine des valeurs caractéristiques des données et des résultats

	Soil	Flow				Erosion		
	n	P_{fl} (Pa)	t_{fl} (s)	V_{fl} (m/s)	c_{ref}	τ_c (Pa)	t_{er} (s)	V_{er} (m/s)
min	0.20	6.91	1.02×10^{-3}	0.63	7.74×10^{-7}	6.41	57.43	1.37×10^{-6}
max	0.54	129	4.73×10^{-3}	2.92	9.36×10^{-5}	128.22	2182.56	5.22×10^{-5}

Starting from initial condition ($R(0) = R_0$, $V(0) = 0$), and under constant hydraulic gradient $P_{fl} > \tau_c$, the solution of the system can be written as follows:

$$\frac{R(t)}{R_0} = 1 + \left(1 - \frac{\tau_c}{P_{fl}}\right)\left[\exp\left(\frac{t}{t_{er}}\right) - 1\right], \qquad \frac{V(t)}{V_{fl}} = \sqrt{\frac{R(t)}{R_0}} \tag{4}$$

with $t_{er} = 2L\rho_g / k_{er}(p_{in} - p_{out})$. The shear stress at the interface is $\tau_b(t) = P_{fl} R(t)/R_0$, and the flow is $Q(t) = Q_{fl}(R(t)/R_0)^{5/2}$.

It is important to note that the limiting case $\tilde{k}_{ref} \to 1$ (corresponding to $k_{er} \to \infty$) may be of interest. In this case, the concentration can be high, and even more close to the compacity of the soil $c_{ref} = (1 - n)/(1 + \tilde{a}_\phi)$. The erosion law (3) leads to $\tau_b = \tau_c$, but the rheological law depends strongly upon the concentration [7] (assumption (A4) has to be modified) so the velocity remains unknown. Moreover, the concentration influences most probably the velocity profile: assumptions (A2) and (A4) are not relevant anymore. To our knowledge, radial profiles of concentrations in pipe flow with erosion remains to be investigated.

4. Comparisons with experimental data

According to the logic of the derivation given above, the obtained scaling laws (4) should hold in all past and future experiments performed in erosional pipe flow with constant pressure drop, in the situation of low kinetics of erosion. The ultimate justification is a comparison with experiment.

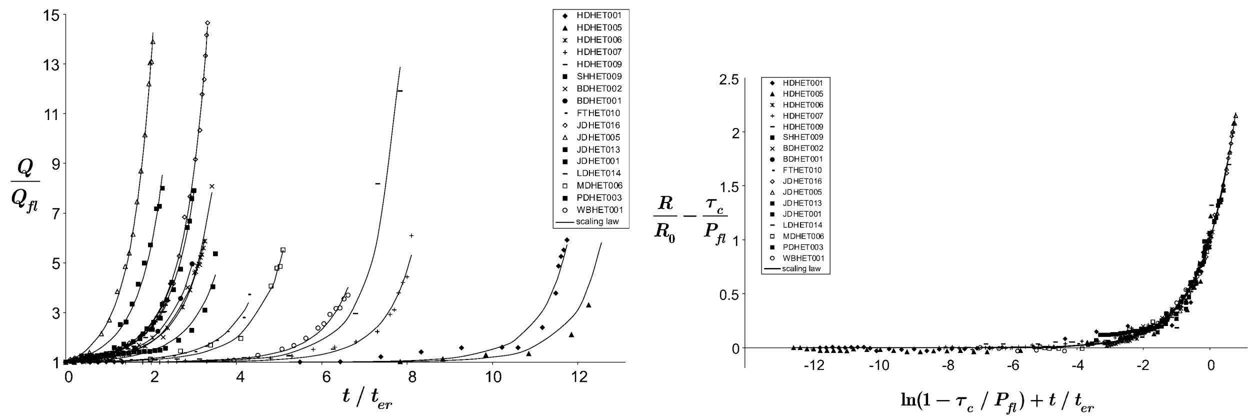
The Hole Erosion Test has been designed to reproduce piping flow erosion in a hole [1]. The soil specimen is compacted inside a standard mould used for the Standard Compaction Test. A hole is drilled along the longitudinal axis of the soil sample. An eroding fluid is driven through the soil sample to initiate erosion of the soil along the pre-formed hole. The test result is described by the flow rate versus time curve under constant pressure drop. For further details on this test, see [1–3].

The predicted scaling law (4) is now compared to available data produced by [1]. Simulations were performed on 17 tests, concerning 10 different soils (clay, sandy clay, clayey sand or silty sand). The range of soil density ρ_g/ρ^f were 1.78 to 2.35. The range of initial water content were 8 to 38.7%. The initial radius and the length of the pipe were $R_0 = 3$ mm and $L = 117$ mm. The range of Reynolds numbers were 2000 to 8800. The comparison between characteristic values is given in Table 1. The range of $\tilde{a}_\phi$ numbers is 2.37 to 4.82, and the range of $\tilde{k}_{er}$ numbers is 7.54×10^{-5} to 1.19×10^{-2}, so all cases correspond to the situation of low kinetics of erosion. The range of coefficient of erosion k_{er} is 3.50×10^{-5} to 1.47×10^{-2} s/m and the range of critical stress is 6.41 to 128.22 Pa.

Fig. 2(a) shows the increase of the flow in $Q \propto R^{5/2}$ and shows that the use of t_{er} leads to an efficient dimensionless scaling. In Fig. 2(b), we plot the experimental data of [1] in the $(R(t)/R_0 - \tau_c/P_{fl})$, $(t/t_{er} + \ln(1 - \tau_c/P_{fl}))$ plane. We observe that all the data except for few fall on a single curve. Taking into account the many simplifying assumptions, the agreement with the scaling law (4) speaks for itself: in spite of the large range of k_{er} (three orders of magnitude), no further manipulation is needed to bring its consequences into line with the experimental data.

5. Conclusion

Many laboratory tests are commonly used to study internal erosion in a soil. One of them, the hole erosion test appears to be efficient and simple means of quantifying the rate of piping erosion, but few attempts have been made to model this. We started from the field equations for diphasic flow with diffusion, and the equations of jump with



(a) (b)

Fig. 2. Erosion with constant pressure drop tests (symbols) / model (continuous lines) comparison, flow (a) and radius (b) as a function of time. The experimental data are taken from [1].

Fig. 2. Erosion à pression constante, comparaison tests (symboles) / modèle (lignes continues) pour le débit (a) et le rayon (b) en fonction du temps. Les données expérimentales sont celles publiées dans [1].

erosion. After many simplifying assumptions, using asymptotic developments and dimensional analysis, we proposed some characteristic numbers, among which the two most significant are the kinetics of erosion dimensionless number and the erosion time. We defined a particular case: the situation of low kinetics of erosion. This situation arises when the erosion kinetics are much smaller. In this case, the influence of both the concentration and inertial effects can be neglected. We obtained an analytical scaling law for the interpretation of the hole erosion test with a constant pressure drop. We made comparison with available experimental data on seventeen tests concerning nine different soils. This comparison has confirmed the validity of our scaling law, which can be used for the interpretation of the hole erosion test. More research works are needed to investigate if this characteristic time could be used in practical situations to predict the development of internal erosion in hydraulic works.

Acknowledgements

This project was sponsored by the Région Provence Alpes Côte d'Azur. This research effort is continuing under the sponsorship of the French National Research Agency under grant 0594C0115 (ERINOH). The authors wish to thank Pr. Robin Fell and Dr. Chi Fai Wan for their valuable experimental data.

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