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Observation, analysis and modelling in complex fluid media

Sedimentation of small particles: how can such a simple problem be so difficult?

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Abstract

Although sedimentation can be considered as one of the simplest examples of suspension flow, much remains unknown about the fundamental properties of sedimenting suspensions. The problem that one encounters lies in the long range nature of the multibody hydrodynamic interactions between particles. This will be illustrated for sedimenting suspensions of spheres, of non-spherical particles such as fibers, and for sedimenting clouds of particles. ***To cite this article: É. Guazzelli, C. R. Mecanique 334 (2006).***
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Résumé

Sédimentation de petites particules : comment un problème si simple peut-il être si compliqué ? La sédimentation de particules à bas nombre de Reynolds peut être considérée comme un des exemples les plus simples d'écoulement de suspension. Et pourtant ce problème est compliqué à cause de la dominance des interactions hydrodynamiques multicorps à longues portées. Trois situations illustreront cette difficulté : la sédimentation d'une suspension de sphères, de particules anisotropes (des fibres) et d'un nuage sphérique de particules. ***Pour citer cet article : É. Guazzelli, C. R. Mecanique 334 (2006).***
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Mots-clés : Mécanique des fluides ; Sédimentation ; Interactions hydrodynamiques multicorps ; Bas nombre de Reynolds

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1. Introduction

The sedimentation of particles is found in a variety of natural or biological phenomena such as the flow of sediment in rivers, the sedimentation of rain drops and dust in the atmosphere, or the flow of red blood cells. It is also one of the oldest engineering processes used to clarify liquid as well as to separate particles of different densities or sizes. Even though it is a long-standing problem, the sedimentation of particles at low Reynolds number remains not completely understood.

In this article, we shall confine our attention to suspensions composed of non-Brownian rigid particles of equal size and density sedimenting in a Newtonian fluid under creeping flow condition. Three situations will be reviewed. In Section 2, we will present some results on the mean velocity and velocity fluctuations for sedimenting suspensions of monodisperse non-Brownian spheres. In Section 3, we will examine the sedimentation of non-spherical particles

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such as fibers which, unlike a suspension of spheres, is unstable to perturbations in concentration. In Section 4, we will considered the sedimentation of clouds of particles which, surprisingly, do not maintain a spherical shape.

2. Sedimentation of a suspension of spheres

One of the classical results for a sedimenting sphere comes from Stokes in the mid-19th century and concerns the drag force $-6\pi\mu a\mathbf{U}$ on a sphere of radius a moving at velocity $\mathbf{U}$ in a fluid of viscosity μ at rest at infinity. Balancing the Stokes drag force with the weight minus buoyancy, one obtains the sedimentation velocity for an isolated sphere, also called Stokes velocity:

$$\mathbf{U_S} = \frac{2}{9}\frac{a^2}{\mu}(\rho_s - \rho)\mathbf{g} \tag{1}$$

where ρ_s is the sphere density and ρ that of the fluid. The settling sphere creates a velocity perturbation in the fluid which has also been computed by Stokes. The dominant disturbance has a very slow decrease as $\mathrm{O}(1/r)$ around the sphere.

This long-range nature of the hydrodynamic interaction created by the settling of the sphere is the cause of all the trouble. If one wants to compute the mean velocity of a suspension of uniformly dispersed spheres by simply summing the hydrodynamics interactions induced on a test sphere by all the other spheres, one ends up with a divergent integral! Of course, this is not observed experimentally as the mean velocity is found to be strongly hindered, i.e., there is a decrease in settling velocity relative to the Stokes velocity of an isolated sphere as the concentration is increased. The theoretical problem was solved by Batchelor [1] (about 120 years after Stokes' result for an isolated sphere) who found the first correction in concentration c to the Stokes velocity:

$$\frac{\langle U\rangle}{U_S} = 1 - 6.55c + \mathrm{O}\big(c^2\big) \tag{2}$$

The main hindrance effect is coming from the fluid back-flow induced by the particles settling towards the impenetrable bottom of the settling vessel. The result also depends upon the structure of the suspension which itself is in turn determined by the hydrodynamics. Batchelor' calculation assumes that particle position are sufficiently random and his prediction seems to agree well with experimental data at low particle volume fraction. At higher volume fractions, only sophisticated computer-simulation can capture the phenomenon.

The mean velocity does not completely characterize the sedimentation process; individual particle motions fluctuate about the mean as can be seen in Fig. 1. The fluctuations in the velocity of the particles are due to the constantly changing configuration of the suspension microstructure [2,3]. These fluctuations have been found experimentally to be very large and to be anisotropic as the vertical fluctuations which are of order of the mean are twice as large as the horizontal fluctuations [3].

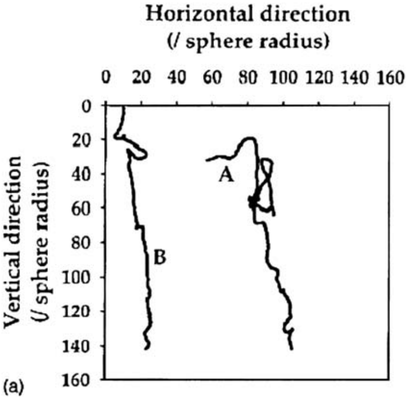

Fig. 1. Trajectories of spheres obtained by particle-tracking in an index-matched sedimenting suspension of spheres from [3].

Fig. 1. Trajectoires de sphères obtenues par suivi particulaire dans une suspension de sphères en sédimentation [3].

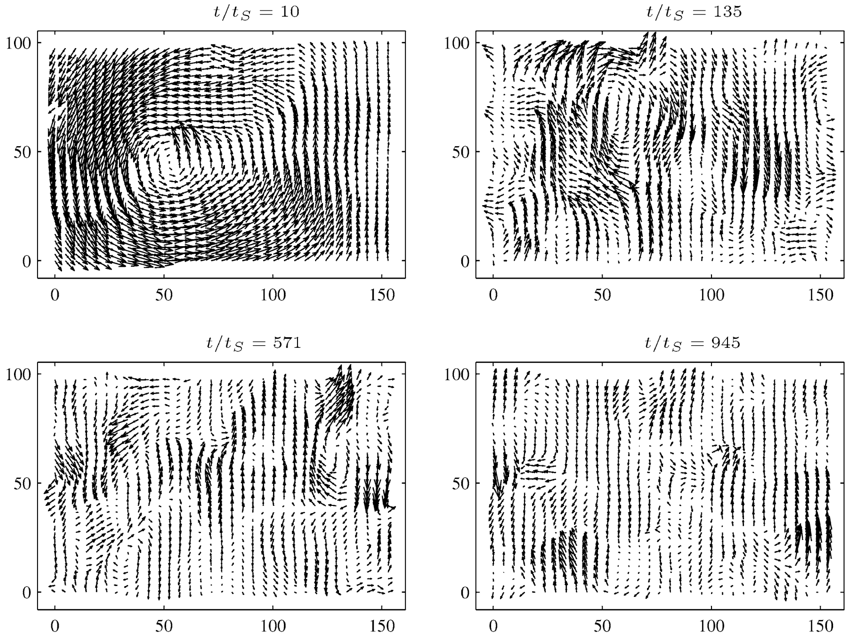

Fig. 2. Fluctuation evolution from large-scale of the order of the size of the container down to small-scale of the order of 20 mean interparticle spacings obtained by particle image velocimetry measurements in a dilute sedimenting suspension of spheres from [20]. The time has been made dimensionless by the Stokes' time $t_S = a/U_S$, i.e., the time for an isolated sphere to fall its radius. The width of the imaging window is approximately that of the cell.

Fig. 2. Evolution des fluctuations depuis les grandes échelles de l'ordre de la taille du récipient jusqu'au petites échelles de l'ordre de 20 fois la distance interparticules moyenne obtenue par mesures PIV dans une suspension en sédimentation diluée de sphères de [20]. Le temps a été normalisé par le temps de Stokes $t_S = a/U_S$, soit le temps pour une sphère de tomber de son rayon. La taille de la fenêtre de l'image est approximativement celle de la cellule.

However, there is again a divergence problem for the amplitude of the fluctuations. Analytical calculations for dilute random sedimenting suspensions predict that the size of the fluctuations diverges with the size of the container [4,5]. However, no such divergence is seen in experiments [6–8]. There have been a number of theoretical models and simulations attempting to explain this screening of the fluctuations [9–19]. Schematically, some studies consider additional physical mechanisms. For instance, a vertical gradient in the mean concentration may suppress the diverging fluctuations [13]. Some other studies consider that these additional effects are not needed as the bottom of the container and the upper top of the suspension act as sinks of the large-scale disturbances [5]. Our recent experiments support this last point of view [20,21]. The initial mixing of the suspension creates density fluctuations also called 'blobs' on all length-scales (from container size down to the mean interparticle spacing) which give rise to convection currents also called 'swirls' on all length-scales. Initially, the large-scale fluctuations dominate the dynamics in agreement with analytical calculations [4,5] as can be seen in Fig. 2. However, they are transient, as the heavy blobs settle to the bottom and light blobs rise to the top. Then, smaller-scale fluctuations or swirls of typically 20 mean interparticle spacings are remaining and are dominant until the arrival of the upper sedimentation front. The origin of this particular length-scale is still an open question.

3. Sedimentation of a suspension of fibers

The sedimentation of non spherical particles such as fibers is qualitatively different than the sedimentation of spheres. Theoretical calculations predict that the coupling between the fiber orientation and flow field generated by the sedimenting fiber leads to a clustering of the particles and to a subsequent enhancement of the sedimentation velocity [22]. Experiments confirmed the existence of such structure instability in dilute sedimenting fibre suspension. The mean sedimentation velocity was found to exceed the Stokes' velocity of an isolated vertical fiber, i.e., the maximum sedimentation speed [23,24]. The fibers were observed to orient in the direction of gravity and to form clusters. These

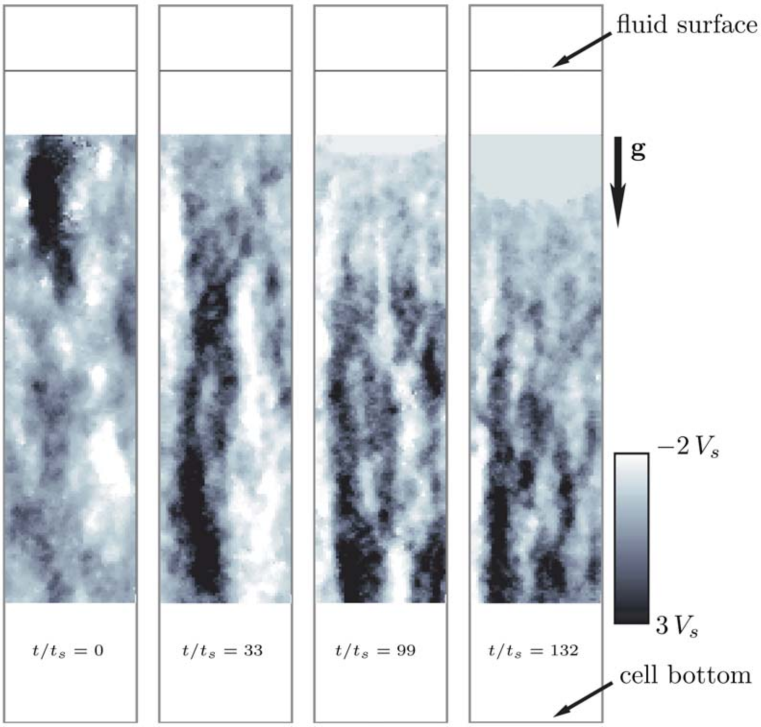

Fig. 3. Large-scale streamers breaking into smaller-scale streamers obtained by particle image velocimetry measurements in a dilute sedimenting suspension of fibers from [25]. Here, the Stokes' velocity U_S is the sedimentation velocity of an isolated vertical fiber and the Stokes' time t_S is the time for an isolated vertical fiber to fall half its length.

Fig. 3. Structures de grande échelle se brisant en structures de plus petite échelle obtenues par mesures PIV dans une suspension en sédimentation diluée de fibres de [25]. Ici, la vitesse de Stokes U_S est la vitesse de sédimentation d'une fibre verticale isolée et le temps de Stokes t_S est temps pourune fibre verticale isolée de tomber de sa demi-longueur.

clusters organized into high density downwards streamers balanced by low density backflow regions [25]. The flow structure evolved from long-wavelength to shorter wavelength organisation as can be seen in Fig. 3. The instability has been also predicted by numerical simulations of varying degrees of sophistication. When using periodic boundary conditions, the fibers were observed to orient in the direction of gravity and to cluster into an unique streamer which spanned the whole height of the periodic cell [26–28]. With a bottom bounding wall, the single large-scale streamer disrupted into smaller streamers as time progressed, in qualitative agreement with the experimental observations [29]. While the mean velocity and fibre orientation reach a steady value, the flow structure evolves in time and does not seem to reach a steady state.

4. Sedimentation of a cloud of particles

A cloud composed of spherical particles can be regarded as an effective medium of excess mass and the problem has been related to that of the sedimentation of a spherical drop of heavy fluid in an otherwise lighter fluid solved by Hadamard and Rybczyński, see, e.g., [30,31]. It has been often considered that a single sedimenting spherical cloud would not deform substantially and would maintain its spherical shape [30,32]. In fact, the spherical cloud is unstable. It has been observed first to remain roughly spherical with a leakage of particles in a vertical tail and then to evolve into a torus which breaks up into two droplets in a repeating cascade, as can be seen in Fig. 4. Simple simulations using point-particles at zero-Reynolds number reproduce very well these successive instabilities, but it remains to understand the physical instability mechanism.

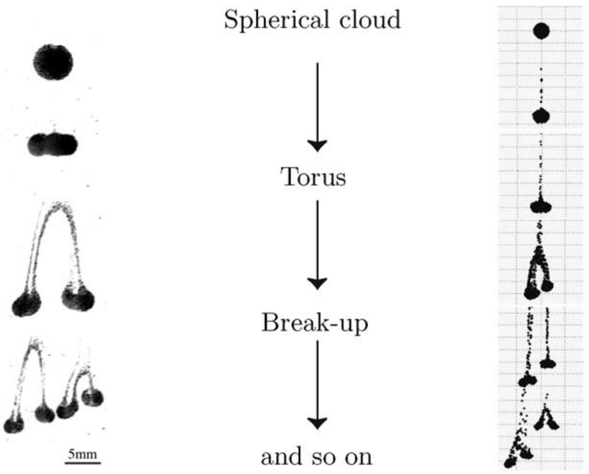

Fig. 4. Successive instabilities and break-up of a spherical cloud of spherical particles. Left: experiments and right: numerical simulations with point-particles at zero-Reynolds number.

Fig. 4. Instabilités successives et désintégration d'un nuage sphèrique de particules sphèriques. A Gauche : expériences et à droite : simulations numériques de particules point à nombre de Reynolds nul.

5. Conclusion

Although sedimentation can be considered as one of the simplest examples of suspension flow, much remains unknown about the fundamental properties of sedimenting suspensions. The problem that one encounters lies in the long range nature of the multibody hydrodynamic interactions between particles. The hydrodynamics are then determined by the suspension microstructure, i.e., the orientation and relative position of the particles, which is itself determined by the hydrodynamics. This coupling results in collective dynamics: swirls of order 20 mean interparticle spacings, i.e., about 8000 particles for sedimenting spheres, large-scale streamers composed of many sedimenting fibers, and a collective motion leading to instabilities of a cloud of spherical particles.

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