



Observation, analysis and modelling in complex fluid media

Turbulent channel flow concentration profile and wall deposition of a large Schmidt number passive scalar

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Available online 7 September 2006

Abstract

The transport of a passive scalar within a turbulent plane channel flow has been theoretically analyzed by assuming that the Schmidt number Sc , associated to the molecular diffusivity of the passive scalar, is a large parameter. Throughout most of the channel cross-section the mean passive scalar density is constant, but adjacent to the walls a thin boundary layer develops embedded in the viscous sublayer, with a relative thickness of order $Sc^{-1/3}$. In this narrow region a passive scalar profile arises due to the non-vanishing flux normal to the wall. This profile is parameter independent (universal) and leads to a constant flux of passive scalar that results from the addition of both the molecular diffusion flux and the turbulent transport one. The Sc -asymptotic matching of this profile with the constant core value provides an analytical expression for the wall-normal flux that depends on the fluid dynamics of the carrier flow. By using a DNS code to solve the external turbulent flow, the analytical expression has been quantified and compared with empirical expressions based on experimental data, showing excellent agreement. **To cite this article: P.L. Garcia-Ybarra, A. Pinelli, C. R. Mecanique 334 (2006).**

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Résumé

Profil de concentration et dépôt à la paroi pour un scalaire passif à grand nombre de Schmidt dans un écoulement turbulent de canal. Le transport d'un scalaire passif au sein d'un écoulement turbulent de canal plan est analysé de façon théorique en supposant que le nombre de Schmidt Sc , associé à la diffusion moléculaire de ce scalaire, est un grand paramètre. Dans la plus grande partie de la section transverse, la densité moyenne du scalaire est constante, mais, près des parois, une fine couche limite située à l'intérieur de la sous-couche visqueuse se développe, avec une épaisseur qui varie comme $Sc^{-1/3}$. Dans cette région étroite, le profil de scalaire passif qui apparaît résulte du fait que le flux de scalaire normal à la paroi ne s'annule pas sur celle-ci. Ce profil est universel et conduit à un flux de scalaire constant qui est la somme du flux de diffusion laminaire et du flux de diffusion turbulent. Pour la valeur asymptotique de Sc , le raccordement de ce profil avec la valeur constante correspondant au cœur de l'écoulement produit une expression du flux à la paroi qui dépend des propriétés de l'écoulement porteur. En utilisant un calcul numérique basé sur des DNS pour simuler l'écoulement turbulent, cette expression analytique a été étudiée et validée, et la comparaison avec des données expérimentales montre un excellent accord. **Pour citer cet article : P.L. Garcia-Ybarra, A. Pinelli, C. R. Mecanique 334 (2006).**

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Keywords: Turbulence; Passive scalar deposition; Turbulent concentration profile; Turbulent channel flow; High Schmidt number asymptotics

Mots-clés : Turbulence ; Dépôt d'un scalaire passif ; Profil turbulent de concentration ; Écoulement turbulent de canal ; Lois asymptotiques à très grand nombre de Schmidt

1. Introduction

In the case of an admixture with a high Schmidt number (e.g., a dye or a macromolecule) conveyed by a turbulent stream along a channel, the admixture follows closely the flow streamlines behaving almost as a passive scalar: forced by the turbulence fluctuations, the admixture undergoes a turbulent dispersion which would lead to its uniform average distribution within the channel. However, the admixture may deposit on the walls because of Fickian diffusion. According to the assumption of high values of the Schmidt number, this is a very weak effect which only affects a very thin layer adjacent to the solid wall. In this layer, Fick diffusion generates a diffusional flux toward the walls driven by the local values of the admixture concentration gradient [1, p. 327]. This classical problem still encloses difficulties associated, in particular, with the near-wall turbulence modeling [2] and with its numerical computation due to the strong sensitivity to the near-wall grid size [3]. It is interesting to recall the long standing controversy about the near-wall behavior of the turbulent passive scalar diffusion coefficient [4], [5, pp. 148–151]. From heuristic arguments, Landau and Levich proposed a coefficient growing with the fourth power of the distance to the wall whereas a third power dependence was supported by other authors. Finally, correlations of several experimental data allowed to settle the controversy by giving credit to the third power dependence [1] which, on the other hand, is a theoretically well-founded result in turbulent channel flows (cf., [6, p. 284]). Experimental results on mass deposition can be correlated in terms of the mass Stanton number by expressions like [7], [8, p. 345]:

$$St_m = \alpha \left(\frac{c_f}{2} \right)^{1/2} Sc^{-\beta} \quad (1)$$

where c_f is the skin-friction coefficient. Slightly different values can be found in the literature for the numeric prefactor α , also the exponent β of the Schmidt number Sc is taken equal to $-2/3$ by most authors. Relations of this form were obtained with semiquantitative two-layer models by patching the solutions, valid in the different layers, at reasonably (although arbitrarily) selected borders [9], [1, p. 343] but no rigorous derivation has been published so far.

The present work provides a general framework to address the problems pointed out above that allows one to describe the effects of the different phenomena and their interplay. The article is organized as follows: in the next section the governing equations for a passive scalar field and a carrier fluid are written in dimensionless form. Then, the classical Reynolds-average method is applied to the equations of the passive scalar and the mean passive scalar density equation is analytically integrated in the near wall region. Asymptotic matching of this solution with the constant core value in the limit of a relatively weak molecular diffusion (high Schmidt number) provides the mean passive scalar density distribution across the channel and, therefore, allows one to obtain an expression for the mean passive scalar flux towards the walls. In the last section, the main results are summarized and discussed. Finally some conclusions are drawn.

2. Formulation of the problem: governing equations

Let us consider the flow of a fluid consisting of a carrier gas and an advected passive scalar. The governing equations are

$$\nabla \cdot \mathbf{V} = 0 \quad \text{and} \quad \rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{V} \quad (2)$$

i.e., the incompressible continuity equation and the Navier–Stokes equation respectively, with \mathbf{V} being the fluid velocity, p the fluid pressure, ρ the fluid density, μ the dynamic viscosity and D/Dt denoting the material derivative. In addition, the continuity equation for the passive scalar is

$$\frac{\partial c}{\partial t} + \mathbf{V} \cdot \nabla c = \mathcal{D} \nabla^2 c \quad (3)$$

where c is the passive scalar density (i.e.: the mass fraction in the case of a dilute admixture) and \mathcal{D} is the molecular (Brownian) diffusion coefficient.

We will restrict our study to the case of a turbulent flow in a channel of infinite extension in the streamwise and spanwise directions. This choice is made to simplify the analysis, even though the results are valid for any straight duct with a small curvature along the spanwise direction relatively to the inverse of the viscous sublayer thickness, a condition that is not very restrictive in practice. Turbulence will be assumed to be statistically steady and homogeneous along the streamwise and spanwise directions. Time averaged quantities will be denoted by an over-bar. The length h , characterizing half of the channel height, will be taken as the reference length to define dimensionless orthogonal coordinates (x, y, z) , with y being the wall-normal coordinate (with origin on the wall), x the streamwise coordinate and z the spanwise coordinate. For the mean values, the spanwise homogeneity hypothesis reduces the problem to a two-dimensional analysis. Thus, the only non-vanishing (time) average velocity component is the streamwise velocity, denoted by $\bar{U}(y)$, which depends on the distance to the wall. On the other hand, the mean velocity across the channel, U_0 , will be used as the reference velocity to define the channel Reynolds number, that will be assumed to take on very large values,

$$Re \equiv \frac{U_0 h}{\nu} \gg 1 \tag{4}$$

where ν is the fluid kinematic viscosity. Also, ρU_0^2 will be taken as the pressure reference value.

As usual, in the near-wall viscous sublayer, the shear stress,

$$\tau_{\text{wall}} \equiv \left. \frac{\rho \nu d\bar{U}}{h dy} \right|_{\text{wall}} \tag{5}$$

characterizes the friction velocity and the viscous length, given by

$$u_\tau \equiv \sqrt{\tau_{\text{wall}}/\rho} \quad \text{and} \quad \delta_\nu \equiv \nu/u_\tau \tag{6}$$

respectively. These are used as the reference wall units to define the friction Reynolds number,

$$Re_\tau \equiv \frac{u_\tau h}{\nu} = \frac{h}{\delta_\nu} < Re \tag{7}$$

and the wall coordinate,

$$y^+ \equiv \frac{h}{\nu/u_\tau} y = Re_\tau y \tag{8}$$

In the following, superscript $+$ will denote the magnitudes in wall units (i.e., viscous units).

With these definitions, the dimensionless governing equations, corresponding to (2), (3), can be expressed:

$$\nabla \cdot \mathbf{v} = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \frac{1}{Re} \nabla^2 \mathbf{v} \tag{9}$$

and

$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c = -\nabla \cdot \mathbf{J} \tag{10}$$

where $\mathbf{v} = (u, v, w)$ and P are the dimensionless fluid velocity and pressure, respectively and, according to (3), the molecular diffusive flux of the passive scalar is given by

$$\mathbf{J} \equiv -\frac{1}{Re Sc} \nabla c \tag{11}$$

In this equation, the Schmidt number will be considered to be a large parameter,

$$Sc \equiv \frac{\nu}{\mathcal{D}} \gg 1 \tag{12}$$

and, according also to the high Reynolds number assumption, the Péclet number, $Pe = Re Sc$, will be also a very large parameter,

$$Pe \equiv \frac{U_0 h}{\mathcal{D}} = Re Sc \gg 1 \tag{13}$$

In fact, the large values taken by the Schmidt number make the Péclet number very large as compared to the Reynolds number and, correspondingly, boundary layers of the passive scalar field will be thinner than viscous boundary layers (e.g., the viscous sublayer)

$$\delta_s \equiv \frac{D}{u_\tau} \ll \delta_v \quad (14)$$

3. Reynolds averaging and spatial scales

Let us introduce the following decomposition for the velocity field,

$$\mathbf{v} = \bar{u}(y)\mathbf{i} + \mathbf{v}'(x, y, z, t) \quad (15)$$

where \mathbf{i} is the unit vector in the (streamwise) x direction and the prime denotes the fluctuations. In spite of the homogeneity of the turbulence along the streamwise direction, the outwards or inwards wall-normal passive scalar flux along the channel walls produces a continuous decrease or increase, respectively, of the mean passive scalar density in the streamwise direction. Thus, the statistics of the passive scalar density cannot be homogeneous along the streamwise coordinate but varies according to the passive scalar wall deposition or injection and, when the passive scalar density is Reynolds-averaged, still a longitudinal dependence remains in the average term accounting for the scalar density decay or increase, respectively. On the other hand, using the decomposition (15), the advective flux of passive scalar becomes after averaging,

$$\overline{c\mathbf{v}} = \bar{c}\bar{u}\mathbf{i} + \overline{c'\mathbf{v}'} \quad (16)$$

which, when used in (10), gives the Reynolds averaged passive scalar transport equation,

$$\left(\bar{u} \frac{\partial \bar{c}}{\partial x} + \frac{\partial \overline{c'u'}}{\partial x} \right) + \frac{\partial \overline{c'v'}}{\partial y} - Pe^{-1} \frac{\partial^2 \bar{c}}{\partial y^2} = o(Pe^{-1}) \quad (17)$$

In this equation, the first two terms in parenthesis are the passive scalar streamwise transport and account for the longitudinal scalar density variation, the third term is the passive scalar turbulent transport in the transverse direction and the last term is the higher order contribution from molecular diffusion. By integrating (17) across the channel mid-height and accounting for the no-slip condition on the wall, and the mirror symmetry condition (vanishing fluxes) through the channel centerline (at $y = 1$), the following integral relation for the scalar density conservation is obtained:

$$\frac{\partial}{\partial x} \int_0^1 (\bar{u}\bar{c} + \overline{c'u'}) dy + Pe^{-1} \left. \frac{\partial \bar{c}}{\partial y} \right|_{y=0} = 0 \quad (18)$$

The integral term in (18) is just the total streamwise flux of scalar density through the channel half cross-section. Therefore, its longitudinal variation is shown to equal the flux of passive scalar by diffusion at the wall. This relation imposes that the scale of the streamwise variation is Pe^{-1} , the same as the molecular diffusion magnitude. According to (13), this allows one to argue that passive scalar diffusion is a *slow* transport mechanism compared to passive scalar advective transport and is effective over a longitudinal scale much larger than the scale of the *fast* turbulent fluctuations. To deal with this two-scale process, the *slow* streamwise coordinate

$$X = Pe^{-1}x \quad (19)$$

is introduced to separate the *long* scale of passive scalar streamwise variation from the *short* scales of turbulence and the passive scalar density is decomposed like

$$c = \bar{c}(X, y) + c'(x, X, y, z, t) \quad (20)$$

Moreover, Eq. (17) can be equivalently written in terms of the wall coordinate (8), more appropriate to analyze the near wall region where deposition/injection effects take place. Thus,

$$Pe^{-1} Re_\tau^{-1} \left(\bar{u}^+ \frac{\partial \bar{c}}{\partial X} + \frac{\partial \overline{c'u'^+}}{\partial X} \right) + \frac{\partial \overline{c'v'^+}}{\partial y^+} - Sc^{-1} \frac{\partial^2 \bar{c}}{\partial y^{+2}} = o(Pe^{-1}) \quad (21)$$

Here the derivative rule of the multiscale formalism [10] has been used to evaluate the longitudinal derivatives. Eq. (21) shows that, at each X location and to leading order in the Péclet number expansion, turbulent dispersion and molecular diffusion control the passive scalar distribution through the channel cross-section. Moreover, due to the high Schmidt number value, whose inverse affects the derivative of larger order, the equation shows a singular character when solved by perturbations with Sc^{-1} playing the role of a small expanding parameter.

4. Average passive scalar distribution across the channel. High-Schmidt-number asymptotics

In the core region of the channel, molecular diffusion is negligible to leading order and turbulent stirring leads to a homogeneous distribution of the passive scalar. This constant profile plays the role of the *outer solution* in a high- Sc asymptotic analysis whereas molecular diffusion only matters in a very thin layer next to the wall, embedded in the viscous sublayer, whose relative thickness is dictated by (21). This two-layer analysis is performed next by working out both the *outer* and *inner solutions*.

4.1. Outer region

To leading order, streamwise advection and molecular diffusion are negligible and Eq. (21) reduces to the turbulent flux contribution,

$$\frac{\partial \overline{c'v'}}{\partial y} = 0 \tag{22}$$

Integration of (22) across the channel mid-section, with the condition of vanishing average scalar flux through the middle point, leads simply to

$$\overline{c'v'} = 0 \tag{23}$$

which expresses the vanishing of the average turbulent flux of passive scalar in the wall-normal direction through all the channel cross-section, i.e., the average passive scalar density is constant across the channel core,

$$\bar{c} = c_M(X) \tag{24}$$

Here the slow dependence of this mean passive scalar density c_M on X , due to the wall-normal scalar flux along the channel wall, has been emphasized.

Solution (24) corresponds to a vanishing average flux of passive scalar across the channel and cannot describe the scalar flux through the wall. In other words, solution (24) cannot verify the boundary condition on the wall which is the typical footprint of a singular perturbation problem [11]. Actually, the scalar flux through the wall develops in a thin boundary layer adjacent to the wall, whose thickness is controlled by the Schmidt number and where molecular transport plays a leading role. Solution in this *inner* region is chosen to verify the boundary condition on the wall and the corresponding analysis is performed in the next section.

4.2. Inner diffusional layer

The structure of the diffusional layer is obtained from the passive scalar density conservation equation in the viscous sublayer (21). The gradient-diffusion hypothesis is used to model the wall-normal scalar transport, with a turbulent diffusivity coefficient defined such that [6, p. 94]

$$\overline{c'v'} = -\frac{\nu_T}{Sc_T} \frac{\partial \bar{c}}{\partial y} \tag{25}$$

where ν_T is the dimensionless fluid eddy viscosity and the turbulent Schmidt number Sc_T takes on values close to the unity (several studies point towards the value 0.7 but there is a large uncertainty about this value). Furthermore, the momentum turbulent diffusivity coefficient is defined by a relation analogous to (25),

$$\nu_T = -\overline{u'v'} \left(\frac{d\bar{u}}{dy} \right)^{-1} \tag{26}$$

In the vicinity of the wall, the limit form of this relation can be shown to be [6, p. 284]

$$v_T = \frac{\sigma}{Re} y^{+3} + O(y^{+4}) \quad (27)$$

where the positive constant σ is related to the coefficient of the fourth order term in the expansion of the average streamwise velocity in powers of the distance to the wall (in fact, σ is minus four times this coefficient). Thus, the turbulent transport (25) can be modeled as

$$\overline{c'v'^+} \cong -y^{+3} \frac{\sigma}{Sc_T} \frac{\partial \bar{c}}{\partial y^+} \quad (28)$$

By inspection of the passive scalar transport equation (21), it is found that the strained coordinate that brings the molecular diffusion to play a leading role is

$$Y = \left(\frac{\sigma Sc}{Sc_T} \right)^{1/3} y^+ \quad (29)$$

at the same time it is found that streamwise advection is a secondary transport effect in the diffusive layer and does not matter at the leading order that is controlled only by molecular and turbulent diffusion. In this scenario, the governing equation accounting for these two effects becomes

$$\frac{\partial^2 \bar{c}}{\partial Y^2} + \frac{\partial}{\partial Y} \left(Y^3 \frac{\partial \bar{c}}{\partial Y} \right) = 0 \quad (30)$$

Thus, in this region both diffusive mechanisms cooperate to build up a constant net flux of passive scalar through this diffusive sublayer to comply with the boundary condition at the wall. Due to the zero-velocity condition on the wall, the closer to the wall the smaller becomes the turbulent flux contribution and the molecular flux is gaining in importance until, at the wall surface, molecular diffusion remains ultimately as the only mechanism responsible for passive scalar deposition on the wall.

The two boundary conditions to impose on Eq. (30) are: the matching with the outer solution far from the wall and a given value at the wall (for instance, the saturation concentration in the case of an admixture),

$$\bar{c}|_{Y \rightarrow \infty} = c_M \quad (31)$$

$$\bar{c}|_{Y=0} = c_S \quad (32)$$

The solution of (30) accounting for the wall condition (32) can be written as

$$\bar{c} = \frac{2\pi}{3\sqrt{3}} \left(\frac{\partial \bar{c}}{\partial Y} \right)_{Y=0} F(Y) + c_S \quad (33)$$

with

$$F(Y) \equiv \frac{\sqrt{3}}{2\pi} \ln \frac{1+Y}{\sqrt{Y^2-Y+1}} + \frac{3}{2\pi} \arctan \frac{2Y-1}{\sqrt{3}} + \frac{1}{4} \quad (34)$$

On the other hand, the limit of (33) far from the wall ($Y \rightarrow \infty$) is

$$\bar{c}|_{Y \rightarrow \infty} = \left(\frac{\partial \bar{c}}{\partial Y} \right)_{Y=0} \frac{2\pi}{3\sqrt{3}} + c_S \quad (35)$$

which together with the matching condition (31) allows to determine the slope at the wall ($Y = 0$) and to get the following universal profile for the inner passive scalar density distribution from Eq. (33):

$$\frac{\bar{c} - c_S}{c_M - c_S} = F(Y) \quad (36)$$

with $F(Y)$ given by (34).

The shape of the profile (36) is depicted in Fig. 1. In the particular case of an admixture, its deposition flux is ruled by the slope of this profile at the wall and will be analyzed in the next section.

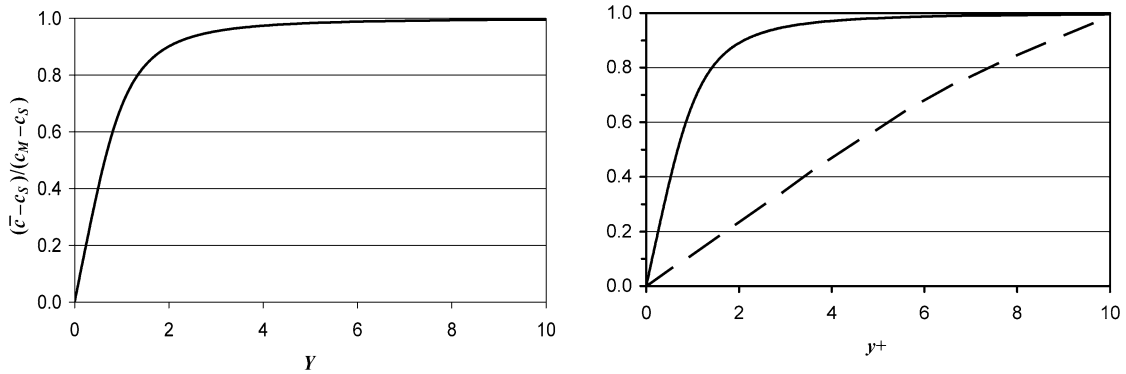


Fig. 1. Universal profile of the passive scalar density in the diffusional sublayer (left). Comparison of the scalar profile (solid line) with the normalized velocity profile, $\bar{u}^+ / \bar{u}^+ (10)$, for $Sc = 700, Sc_T = 1$, in wall units (dashed line).

Fig. 1. Profil universel de la densité du scalaire passif dans la sous-couche diffusive (gauche). Comparaison du profil de scalaire (ligne continue) avec le profil de vitesse normalisé, $\bar{u}^+ / \bar{u}^+ (10)$, pour $Sc = 700, Sc_T = 1$, en unités de paroi (ligne pointillée).

5. Passive scalar flux at the wall

According to (11), the dimensionless flux of passive scalar at the wall, in absolute value, is

$$J_{\text{wall}} \equiv Pe^{-1} \left. \frac{\partial \bar{c}}{\partial y} \right|_{y=0} \tag{37}$$

The slope of the inner profile (36) at the wall is determined by (31) and (35). Thus, the passive scalar flux at the wall (37) written in terms of the original outer variables, becomes

$$J_{\text{wall}} = \frac{3^{3/2}}{2\pi} \left(\frac{\sigma}{Sc_T} \right)^{1/3} \frac{Re_\tau}{Re} Sc^{-2/3} (c_M - c_S) \tag{38}$$

The numeric prefactor depends on the phenomenological coefficient σ that has to be determined from experiments or from numerical simulations. By using a DNS code [12,13], the following approximated value has been obtained (with $Re = 3250, Re_\tau = 180$):

$$\sigma \cong 1.2 \times 10^{-3} \tag{39}$$

It is customary to write the passive scalar flux at the wall in terms of the Stanton number, defined by $St \equiv J_{\text{wall}} / (c_M - c_S)$. Thus (38) leads to

$$St \cong \frac{3^{3/2}}{2\pi} \left(\frac{\sigma}{Sc_T} \right)^{1/3} \left(\frac{c_f}{2} \right)^{1/2} Sc^{-2/3} \tag{40}$$

where $c_f = 2(Re_\tau / Re)^2$ is the skin-friction coefficient. This expression is to be compared with (1), which is compatible with a large number of experimental data compiled in [7]. Expression (40) provides $\beta = -2/3$ and values of α between 0.0990 and 0.0879 for Sc_T between 0.7 and 1, respectively, whereas Shaw and Hanratty compilation [7] gave $\beta = -0.704$ and $\alpha = 0.0889$. Thus, an excellent quantitative agreement is found between experimental and theoretical expressions, which becomes optimum for $Sc_T = 0.966$.

With respect to the passive scalar deposition/injection along the channel wall, it was stressed in Section 3 that it produces a *slow* streamwise decrease/increase of the mean passive scalar density, which occurs over *long* distances of the order of the Péclet number (times the channel mid-height) according to Eq. (18).

6. Discussion of results and conclusions

In the case of a turbulent channel flow carrying a passive scalar with large Schmidt number, it is shown that the streamwise variation of the average scalar density, due to the flux at the walls, is a second order effect because the Péclet number is a very large parameter. Then, at each position along the channel, turbulent dispersion controls the

scalar density distribution through most of the channel cross-section, leading to a constant distribution except in the vicinity of the walls. This solution implies a vanishing flux of passive scalar and can not comply with the prescribed value on the walls. Near to the wall a boundary layer develops, of relative thickness $\delta_s/\delta_v \propto Sc^{-1/3}$, where molecular diffusion becomes a leading order effect together with turbulent dispersion. A solution in this thin zone is found that verifies the boundary condition at the wall. The whole problem is solved by asymptotic matching of the solutions in both zones and the predicted passive scalar flux at the wall has been shown to agree satisfactorily with experimental results.

Additionally, it is also found that the typical streamwise length of the passive scalar variation along the flow is of the order of hPe . According to (13), for a turbulent flow carrying a weakly diffusive admixture, this length is very large compared to the channel height which shows the long persistence of the admixture in air streams if no other kind of fluxes were present to increase the deposition rate. Among these ones, the phenomenon of thermal-diffusion (Ludwig–Soret effect) provides a mechanism to easily raise the deposition rate by orders of magnitude [14] being then of crucial practical importance the study of thermal-diffusion in non-isothermal turbulent flows. However, in laminar flows, it is known that thermal-diffusion changes completely the structure of the molecular deposition layer [15] and the extension of the present work to account for thermal-diffusion would need of deep modifications that will be the subject of future studies.

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