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Experimental evidence of energy pumping in acoustics

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Abstract

This Note presents an experimental vibro-acoustic set-up that aims to reproduce the energy pumping phenomenon between an acoustic medium and an essentially nonlinear oscillator. It shows a one-way irreversible transfer of energy between the first acoustic mode in a tube and a thin visco-elastic membrane. ***To cite this article: B. Cochelin et al., C. R. Mecanique 334 (2006).***

Résumé

Mise en évidence expérimentale de pompage énergétique en acoustique. Cette Note décrit un montage expérimental destiné à reproduire le phénomène de pompage énergétique entre un milieu acoustique et un oscillateur purement non linéaire. Il montre un transfert irréversible d'énergie entre le premier mode acoustique d'un tube et une membrane visco-élastique mince. ***Pour citer cet article : B. Cochelin et al., C. R. Mecanique 334 (2006).***

Keywords: Acoustics; Vibro-acoustic; Nonlinear; Pumping; Irreversible energy transfer; Dynamic absorber

Mots-clés : Acoustique ; Vibro-acoustique ; Non linéaire ; Pompage ; Transfert d'énergie irréversible ; Absorbeur dynamique

1. Introduction

This Note deals with the energy pumping phenomenon for acoustic applications. The energy pumping concept has been introduced by Vakakis and Gendelman [1–3] in 2001: the idea is to passively reduce the vibrations in a linear system (discrete or continuous) by attaching to it an essentially nonlinear damped oscillator. Under suitable conditions, a one-way irreversible transfer of energy occurs from the linear system to the nonlinear one, leading to an efficient cancellation of the oscillations in the linear system. Many recent papers have been published around this interesting new concept,[1] and to its application to various fields. For instance, the idea of using several nonlinear

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[1] Because the attached nonlinear oscillator has no linear stiffness, the concept differs significantly from the classical tuned linear dynamic absorber.

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oscillators to improve the pumping has been investigated in [4], and a first experimental validation of pumping has been demonstrated in [5] on a two d.o.f. mechanical system.

In this Note, we describe an experimental set-up that permits us to obtain an irreversible transfer of energy between an acoustic medium and a thin visco-elastic membrane. The membrane has been designed to perform large amplitude oscillations in order to exhibit an essentially nonlinear stiffening behavior. This first academic verification of pumping in acoustics is a first step toward the design of new generation of passive acoustic absorbers that would be efficient in the low frequency range.

2. The theory of pumping: a brief overview

We consider a two degrees of freedom system consisting of two oscillators connected by mean of a small stiffness coupling spring (Fig. 1). The first oscillator is a classical linear spring-mass system. The second one is composed by a mass, a nonlinear cubic spring (without linear stiffness), and a damper. Rescaling time with the natural frequency of the linear oscillator, the nondimensional equations for the positions $u_1(t)$ and $u_2(t)$ of the masses are

$$\begin{aligned} &\ddot{u}_1 + u_1 + \beta(u_1 - u_2) = 0 \\ &\gamma\ddot{u}_2 + \mu\dot{u}_2 + \alpha_3 u_2^3 + \beta(u_2 - u_1) = 0 \end{aligned} \tag{1}$$

with γ the mass ratio of the two masses, β the small coupling coefficient, μ the damping factor in the nonlinear oscillator and α_3 the cubic stiffness coefficient.

The curves in Fig. 2 show the free oscillations of such a system when an initial velocity is given to the linear mass, i.e. the mass of the linear oscillator. For small initial velocity, the motion looks like that obtained for $\dot{u}_1(0) = 3$: after a transient phase, the two masses perform (almost) out of phase damped oscillations with a very slow (exponential) decrease of the amplitude. For high initial velocity, the motion looks like the one for $\dot{u}_1(0) = 4$. We can see a first transient zone, a pumping zone where a fast decrease of the linear mass amplitude occurs, another transient zone, and again a slow decrease of both amplitudes. In the pumping zone, the decrease of the linear mass amplitude is

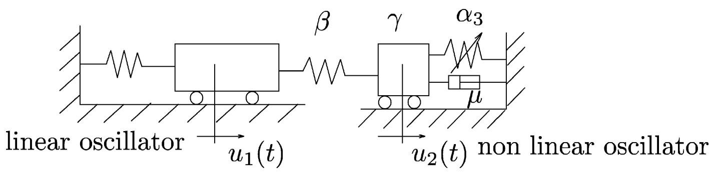

Fig. 1. A simple two d.o.f. vibrating system for illustrating the pumping phenomenon.

Fig. 1. Un système vibrant simple à deux degrés de liberté pour illustrer le phénomène de pompage.

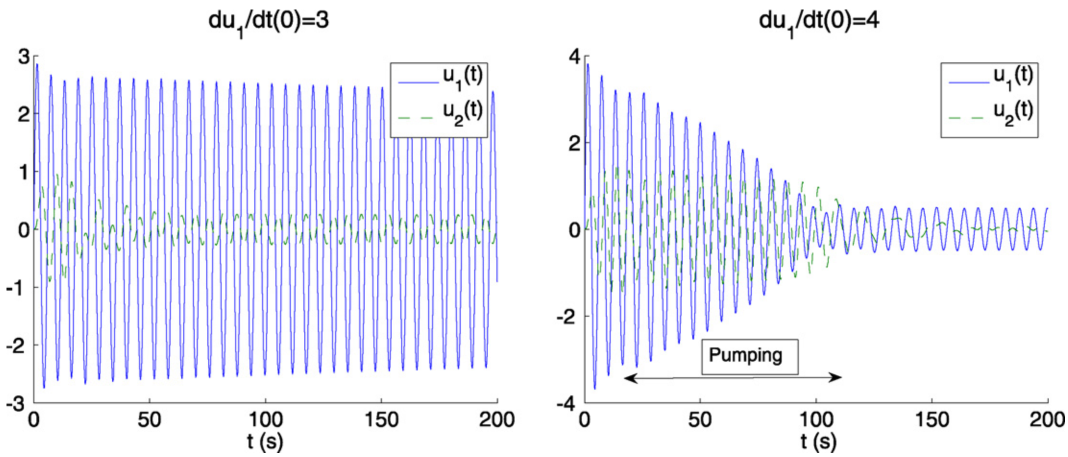

Fig. 2. Free oscillations for the initial condition $\dot{u}_1(0) = 3$ (left) and $\dot{u}_1(0) = 4$ (right). The parameters are $\gamma = 1$, $\beta = 0.1$, $\mu = 0.1$, $\alpha_3 = 1$.

Fig. 2. Oscillations libre à partir de la condition initiale $\dot{u}_1(0) = 3$ (à gauche) et $\dot{u}_1(0) = 4$ (à droite).

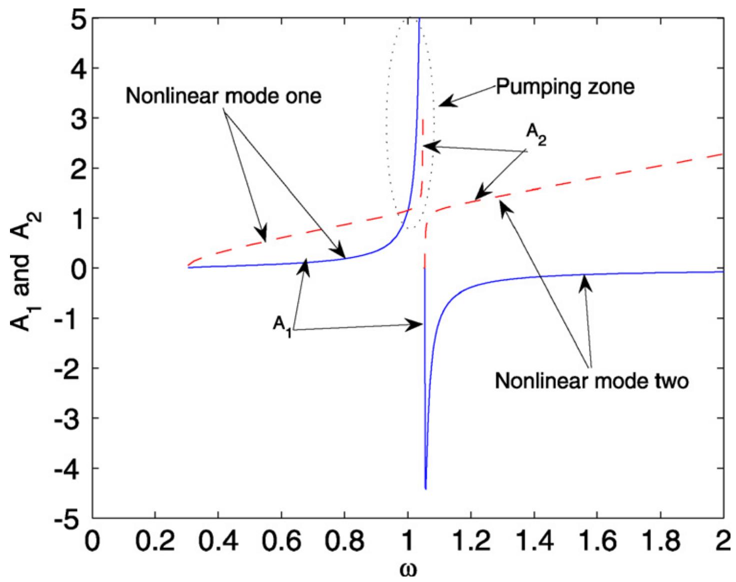

Fig. 3. Periodic solutions of the undamped system: solution curves $A_1(\omega)$ and $A_2(\omega)$ for $\beta = 0.1$, $\gamma = 1$, $\alpha_3 = 1$.
Fig. 3. Solutions périodiques du système non amorti : courbes de solutions $A_1(\omega)$ et $A_2(\omega)$ pour $\beta = 0{,}1$, $\gamma = 1$, $\alpha_3 = 1$.

linear with respect to time, the nonlinear mass keeps an almost constant amplitude, and the two masses oscillate (almost) in phase. A one-way irreversible transfer of energy occurs from the linear system to the nonlinear one, where it is finally dissipated. The pumping phenomenon can only occur when the initial velocity is over a threshold value which is found to be around $\dot{u}(0) = 3.1$ in this example. We want to insist on the unusual fact that a bigger impulse (above the threshold) is more rapidly cancelled than a smaller one (below the threshold). The pumping phenomenon is caused by a 1:1 resonance capture [2]. It can be enlighted by looking at the structure of the nonlinear modes of vibrations of the system [1]. Hereafter, we look at the periodic solutions of the system (1) when the damping coefficient μ is null. Following the harmonic balance method, we seek the motion under the form $u_1(t) = A_1\cos(\omega t)$ and $u_2(t) = A_2\cos(\omega t)$. Introducing into (1) and neglecting the higher harmonics, we get the following algebraic system for the amplitudes A_1 and A_2. It can be easily solved in closed-form.

$$\begin{aligned} &\left(1+\beta-\omega^2\right)A_1 + (-\beta)A_2 = 0 \\ &(-\beta)A_1 + \left(\beta-\gamma\omega^2\right)A_2 + \frac{3}{4}\alpha_3 A_2^3 = 0 \end{aligned} \tag{2}$$

The solution curves $A_1(\omega)$, $A_2(\omega)$ are reported in Fig. 3. The two branches correspond to the (approximated) undamped nonlinear modes of the two d.o.f. systems. It can be seen that the two masses move in phase on the first nonlinear mode and out of phase on the second one. It should be noticed that when $\beta = 0$ (no coupling), the solution curves are the straight lines $\omega = 1$ for the linear system and $A_2 = \pm 2\sqrt{\frac{\gamma}{3\alpha_3}}\omega$ for the nonlinear one (an essentially nonlinear oscillator has no natural frequency: it can oscillate at any frequency). If we now account for a small damping μ, the free motion of the system occurs in the neighborhood of these nonlinear modes, but with a slow decrease of the amplitude A_1 and A_2 with respect to time.

The numerical simulation shown in Fig. 2 can now be reconsidered. All the free motions occur on the branches near the natural frequency of the linear oscillator. The (almost) out of phase motion with a slow decrease of amplitude corresponds to the second nonlinear modes at low amplitude. The (almost) in phase motion, where the pumping effect occurs, corresponds to the first nonlinear mode at large amplitude. A minimal initial velocity (or energy) has to be given for the motion to take place on this first mode. The second transient zone in the simulation with $\dot{u}(0) = 4$ corresponds to a jump from the first mode to the second one.

An insight of the irreversible transfer of energy and of the linear decrease of the linear mass amplitude that occur in the pumping zone can be illustrated by considering a linear oscillator submitted to a sinusoidal force acting at the natural frequency of the system. The solution of $\ddot{u} + \omega^2 u = f \sin(\omega t)$ with the initial conditions $u(0) = A$ and $\dot{u}(0) = 0$, is

$$u(t) = \left(A - \frac{f}{2\omega} t\right) \cos(\omega t) + \left(\frac{f}{2\omega^2}\right) \sin(\omega t)$$

The velocity is $\dot{u}(t) = (-A\omega + \frac{f}{2} t) \sin(\omega t)$ and the power of the applied forces is always negative (one-way transfer). It is this situation that occurs in the first equation of the system (1) during the pumping. The nonlinear mass oscillation $u_2(t)$ is synchronized with $u_1(t)$, but with a small phase lag which produces the sinusoidal forcing mentioned above.

3. Design of the experimental set-up

The experimental set-up shown on Fig. 4 aims to reproduce the pumping phenomenon between an acoustic medium and a thin visco-elastic membrane. The linear oscillator is here the first acoustic mode of the air contained in a tube. The nonlinear damped oscillator is a thin membrane that performs large amplitude oscillations. The damping in the nonlinear oscillator is due to the viscosity of the membrane. The two systems are slightly coupled by the air contained in the box between the tube and the membrane.

A continuous model of this set up can be obtained by taking Helmholtz equation for the air contained in the tube, and the nonlinear plate equation of the Von Kármán type for the thin membrane. A Kelvin–Voigt model is adopted to account for the viscosity in the membrane: the stress tensor is related to the strain tensor and the strain velocity tensor by $\sigma = D(E, \nu) : (\varepsilon + \eta\dot{\varepsilon})$, where D is a classical fourth order elastic tensor. As a first approximation, the continuous model can be reduced to a two degrees of freedom system by taking $u(x, t) = u(t) \cos(\frac{\pi x}{L})$ for the air in the tube (first undamped mode), and $w(r, t) = q(t)(1 - (\frac{r}{R})^2)$ for the transversal displacement of the circular membrane (parabolic shape function). The two coordinates $u(t)$ and $q(t)$ correspond to the displacement at the end of the tube and at the center of the membrane. In the box, the acoustic velocity is supposed to be negligible and the pressure is thus considered as spatially uniform. It is related to the volume variation by $p = \rho_{\mathrm{air}} c^2 \frac{\Delta V}{V}$ with $\Delta V = u(t) S_t - q(t) \frac{h S_m}{2}$.

Applying a classical Galerkin method, we get the following reduced system

$$\begin{aligned} &\ddot{u} + u + \beta(u - q) = 0 \\ &\gamma\ddot{q} + \alpha_3\left(2\eta\omega q^2\dot{q} + q^3\right) + \beta(q - u) = 0 \end{aligned} \tag{3}$$

which looks like (1) except for the dissipative term which is here nonlinear, because of the geometrical nonlinearity in the membrane. Since the membrane performs large amplitude oscillations, the linear stiffness of the membrane has been neglected in (3). This model reduction yields to explicit formulas for the coefficients β, γ and α_3 with respect to the physical characteristics of the set-up. The natural frequency of the linear oscillator being $\omega = \frac{\pi c}{L}$, we have

$$\beta = \frac{2}{\pi^2} \frac{S_t L}{V}, \qquad \gamma = \frac{8}{3} \frac{h}{L} \frac{S_t}{S_m} \frac{\rho_m}{\rho_{\mathrm{air}}}, \qquad \alpha_3 = \frac{32}{3(1-\nu^2)\pi^2} \frac{E}{\rho_{\mathrm{air}} c^2} \frac{h^3 L}{R^4} \frac{S_t}{S_m} \tag{4}$$

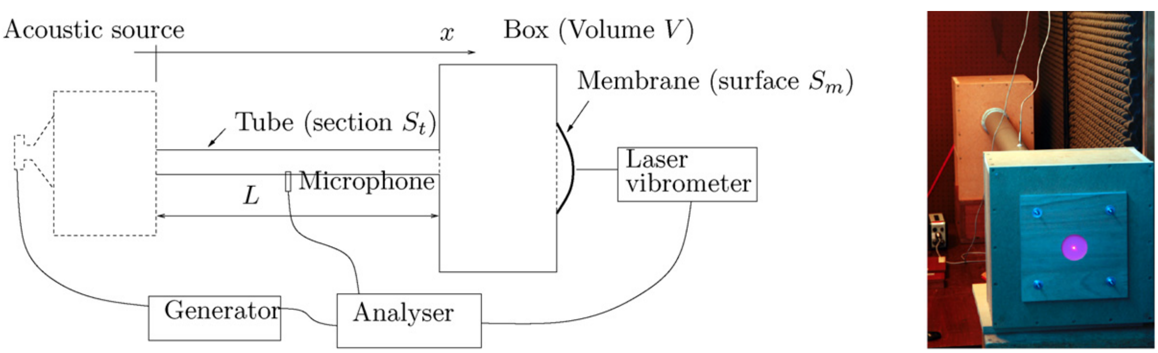

Fig. 4. The experimental set-up consists in a tube, a box and a visco-elastic membrane.

Fig. 4. Le montage expérimental est constitué d'un tube, d'une boite, et d'une membrane visco-élastique.

where h, R, S_m, ρ_m are the membrane thickness, radius, section and density, L, S_t are the tube length and section, V is the volume of the coupling box, E the Young modulus of the membrane, ρ_{air} the density of the air and c the sound speed.

The set-up has been designed to get $\omega \approx 580$ rd/s (≈ 92 Hz) $\beta \approx 0.1$, $\gamma \approx 1$, $\alpha_3 \approx 10^{-2}$. With these values, and with $\eta = 0.002$, the threshold on the initial velocity (initial energy) is found to be compatible with the experimental conditions. The physical parameters are $L = 2$ m, $R = 0.03$ m, $S_t = 0.0069$ m^2, $h = 0.00046$ m, $E = 1.3 \times 10^6$ Pa, $\nu = 0.49$, $\rho_m = 1000\ \text{kg}\,\text{m}^{-3}$, $\rho_{\text{air}} = 1.2\ \text{kg}\,\text{m}^{-3}$, $c = 340\ \text{m}\,\text{s}^{-1}$, $V = 0.028$ m^3.

4. Results and comments

To start the motion in the system, we use a generator and a loudspeaker connected to the tube by mean of a coupling box. Experimentally, we cannot use a single impulse to start the motion. Instead, we use a sinusoidal forcing at the natural frequency of the set-up. This signal is suddenly turned off, and from that time, we observe the free oscillations of the system. The curve in Fig. 5 has been obtained for an initial oscillating pressure of 380 Pa in the middle of the tube. In that case, we are below the energy threshold for the pumping, and we can observe a classical exponential decrease of the pressure in the tube. It should be noticed that the decrease is more pronounced than that in Fig. 2 because, experimentally, the linear oscillator is damped. For an initial value of 680 Pa, we have obtained the pumping phenomenon as it can be seen in Fig. 6. We can see a linear decrease of the pressure amplitude in the tube, whilst

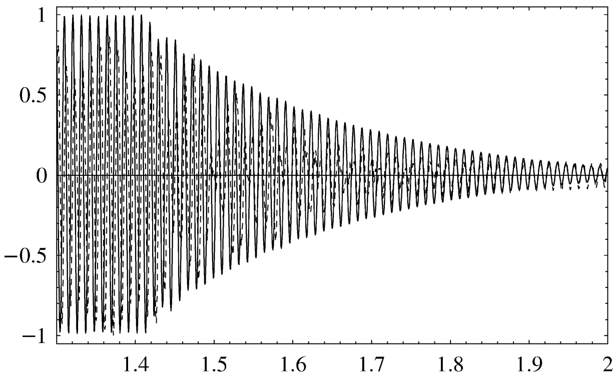

Fig. 5. Experimental results for an initial pressure of 380 Pa in the tube, showing a classical exponential decay: pressure in the tube (line); velocity of the center of the membrane (dash). Both signals have been normalized to unity.

Fig. 5. Résultats expérimentaux pour une pression initiale de 380 Pa dans le tube : pression dans le tube (ligne continue) ; vitesse au centre de la membrane (pointillé). Les signaux ont été normalisés a un.

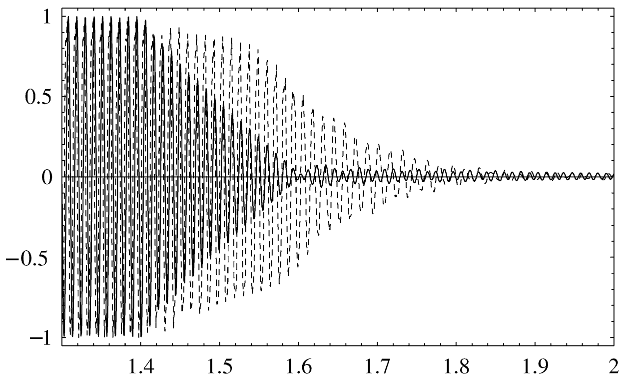

Fig. 6. Experimental results showing the pumping effect. The initial pressure amplitude is 680 Pa. The pressure amplitude (continuous line) shows a rapid linear decrease with respect to time, whilst the membrane keeps a constant velocity amplitude.

Fig. 6. Résultats expérimentaux montrant le phénomène de pompage. L'amplitude de la pression (ligne continue) présente une décroissance linéaire rapide, pendant que la vitesse de la membrane garde une amplitude quasi-constante.

the velocity of the membrane is almost constant. It should be noticed that the pumping occurs until the complete cancellation of the oscillations in the tube (this is not the case in the simulation shown in Fig. 2).

This experimental device permits to show an irreversible transfer of energy from an acoustic medium to a visco-elastic membrane, acting as an energy sink. It is a first step toward a more complete understanding of low frequency noise reduction by using essentially nonlinear oscillators. Parametric optimisation is now scheduled, together with an insight of potential acoustic applications.

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