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On two-scale homogenized equations of the Ishlinskii type viscoelastoplastic body longitudinal vibrations with rapidly oscillating nonsmooth data

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Abstract

We study the initial-boundary value problems for a system of operator-differential equations describing Ishlinskii type viscoelastoplastic body longitudinal vibrations with rapidly oscillating nonsmooth coefficients and initial data. The main feature is an presence of hysteresis Prandtl–Ishlinskii operator. We rigorously justify the passage to the corresponding limit initial-boundary value problems for a system of two-scale homogenized operator-integro-differential equations, including the existence theorem for the limit problems. The results are global with respect to the time interval and the data. ***To cite this article: A. Amosov, I. Goshev, C. R. Mecanique 334 (2006).***

Résumé

Sur les équations homogénéisées des oscillations longitudinales du viscoélastoplastique matériau d'Ishlinskii avec données non régulières rapidement oscillantes à deux échelles. Nous étudions l'homogénéisation du système d'équations décrivant les oscillations longitudinales du matériau viscoélastoplastique d'Ishlinskii, avec des données rapidement oscillantes. La propriété principale est la présence de l'operateur de l'hystérèse de Prandl–Ishlinskii. Nous justifions rigoureusement la convergence vers un problème limite pour un système d'équations operateur-intégro-différentielles homogéneisées à deux échelles, pour lequel nous prouvons un théorème d'existence. Les résultats sont globaux par rapport au temps et aux données. ***Pour citer cet article : A. Amosov, I. Goshev, C. R. Mecanique 334 (2006).***

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Nous étudions l'homogénéisation du système d'équations décrivant les oscillations longitudinales du matériau viscoélastoplastique d'Ishlinskii, avec des données rapidement oscillantes (voir les Éqs. (3)–(5) de la version anglaise). Dans ces équations, $e(x,t)$, $u(x,t)$, $\sigma(x,t)$ représentent la déformation, la vitesse et les contraintes, $\rho(x)$ est la densité du corps indéformé, $g(x,t)$ est la densité des forces de masse, ν est le coefficient de la viscosité, σ_{el} correspond à la composante élastique des contraintes, $\mathcal{F}$ est l'operateur hystérésique de Prandl–Ishlinskii, utilisé pour la description de la propriété de la élastoplasticité. Nous utiliserons les notations suivantes : $\Omega=(0,X)$, $Q=\Omega\times(0,T)$, $D=\partial/\partial x$, $D_t=\partial/\partial t$, ainsi que : $\nu_\varepsilon[\eta_\varepsilon](x,t)=\nu(\eta_\varepsilon(x,t),\xi,x)|_{\xi=x^\varepsilon}$, $\sigma_{\mathrm{el},\varepsilon}[e_\varepsilon](x,t)=\sigma_{\mathrm{el}}(e_\varepsilon(x,t),\xi,x)|_{\xi=x^\varepsilon}$ où on pose x^ε étant la partie fractionnaire de $x/\varepsilon-a_\varepsilon$, ε étant un paramètre positif tendant vers zéro. Le système (3)–(5) est complété par l'une des conditions aux limites (7_m) et par des conditions initiales $e^0_\varepsilon(x)=e^0(\xi,x)|_{\xi=x^\varepsilon}$, $u^0_\varepsilon(x)=u^0(\xi,x)|_{\xi=x^\varepsilon}$.

Nous montrons, que le problème limite correspondant à (3)–(7_m) est un problème aux limites d'évolution pour le système d'équations quasi linéaire operateur-intégro-différentiel homogénéisé à deux échelles (12)–(14). Dans ces équations, $J=(0,1)$ est l'intervalle de périodicité, $\langle\psi\rangle=\int_J\psi(\xi)\,\mathrm{d}\xi$. En se basant sur le théorème d'existence globale des solutions faibles du problème (3)–(7_m) avec des estimations uniformes en ε, nous montrons un résultat analogue pour le problème (12)–(15), (7_m), et un théorème précisant la relation entre ces deux problèmes, incluant la convergence forte $e_\varepsilon-e^\varepsilon\to 0$ (où $e_\varepsilon(x,t)=e(\xi,x,t)|_{\xi=x^\varepsilon}$), $u_\varepsilon\to u$, $\sigma_\varepsilon\to\sigma$ dans les espaces de Lebesgue. Ces résultats sont obtenus sous des conditions trés générales sur les données (elles ne sont pas supposées petites et peuvent être discontinues en x et en ξ).

1. We consider the homogenization problem [1,2] for a system of quasilinear operator-differential equations describing Ishlinskii type viscoelastoplastic body longitudinal vibrations with rapidly oscillating data. The formal homogenization was accomplished in [3] (see also [1]). We rigorously justify the passage to the corresponding limit system of two-scale homogenized [4] operator-integro-differential equations, including the existence theorem for the limit system, in the case of nonsmooth data. The results are global with respect to the time interval as well as the data. A similar problems were already studied for the equations of a viscous barotropic medium [5] (see also [6]), of a viscous heat-conducting gas [7] and of a nonlinear thermoviscoelastic body of Voight type [8,9].

The homogenization problem for the system of equations describing longitudinal vibrations of an elastoplastic rod (without viscosity) with more smooth data was considered in [10].

If B is a Banach space, $M\subset B$ and E is a measurable set then by $L^\infty(E;M)$ we denote the set of functions $f:E\to M$ such that $f\in L^\infty(E;B)$.

Let $J=(0,1)$, $\Omega=(0,X)$, $Q=\Omega\times(0,T)$, $\mathbb{R}^+=(0,+\infty)$, $\overline{\mathbb{R}}^+=[0,+\infty)$. We will write $D=\partial/\partial x$, $D_t=\partial/\partial t$ and $I_t w(x,t)=\int_0^t w(x,t')\,\mathrm{d}t'$.

We introduce the class $\mathcal{N}_{-1}(Q)$ of functions $e\in C([0,T];L^\infty(\Omega))$ such that $D_t e\in L^2(Q)$ and $\operatorname{ess\,inf}_{(x,t)\in Q}e(x,t)>-1$. Let $\mathcal{N}^{\langle\rangle}_{-1}(J\times Q)$ be a similar class of functions $e\in C([0,T];L^\infty(\Omega;L^\infty(J)))$ such that $D_t e\in L^2(Q;L^\infty(J))$ and $\operatorname{ess\,inf}_{(\xi,x,t)\in J\times Q}e(\xi,x,t)>-1$. We also use the Banach spaces $W(Q)$ and $V_2^{\langle 1,1/2\rangle}(Q)$ with norms

$$\|w\|_{W(Q)}=\|w\|_{L^2(Q)}+\|D_t w\|_{L^2(Q)}+\|Dw\|_{L^\infty(0,T;L^2(\Omega))}$$

$$\|w\|_{V_2^{\langle 1,1/2\rangle}(Q)}=\|w\|_{L^\infty(0,T;L^2(\Omega))}+\|Dw\|_{L^2(Q)}+\sup_{0<\tau<T}\tau^{-1/2}\big\|\Delta^{(\tau)}w\big\|_{L^2(Q_{T-\tau})}$$

where $\Delta^{(\tau)}w(x,t)=w(x,t+\tau)-w(x,t)$. Let $C(\overline{\mathbb{R}}^+)$ be the Banach space of bounded continuous functions $s^0:\overline{\mathbb{R}}^+\to\mathbb{R}$ with a norm $\|s^0\|_{C(\overline{\mathbb{R}}^+)}=\sup_{r\in\overline{\mathbb{R}}^+}|s^0(r)|$ and

$$\mathrm{Lip}_1\big(\overline{\mathbb{R}}^+\big)=\big\{s^0\in C\big(\overline{\mathbb{R}}^+\big)\mid s^0(0)=0,\ \big|s^0(r_1)-s^0(r_2)\big|\leqslant|r_1-r_2|\ \forall r_1,r_2\in\mathbb{R}^+\big\}$$

Let $M_1(\overline{\mathbb{R}}^+)$ be a set of nondecreasing right-continuous functions $\mu:\overline{\mathbb{R}}^+\to\mathbb{R}$ such that $\mu\in L^1(\mathbb{R}^+)$ and

$$M_{1,N}\big(\overline{\mathbb{R}}^+\big)=\big\{\mu\in M_1\big(\overline{\mathbb{R}}^+\big)\mid\big|\mu(0)\big|\leqslant N,\ \|\mu\|_{L^1(\mathbb{R}^+)}\leqslant N\big\}$$

Let $BV[0,T]$ be the space of functions of bounded variation.

2. We shall use the classical concept of an approximate limit. For functions $\varphi:J\to\mathbb{R}^m$, see Section 1.7.2 in [11]. This concept remains valid for functions $\varphi:J\to B$ if one replaces $\|\cdot\|_{\mathbb{R}^m}$ by $\|\cdot\|_B$.

Let x^ε be the fractional part of $x/\varepsilon - a_\varepsilon$, where a_ε is an arbitrary function of the parameter $\varepsilon > 0$ (for instance, $a_\varepsilon \equiv 0$). The following result allows us to consider a widely using in homogenization theory composite function $f^\varepsilon(x) = f(x^\varepsilon, x)$ in the sense of formula

$$f^\varepsilon(x) = \operatorname*{lim\,ap}_{\xi \to x^\varepsilon} f(\xi, x) \tag{1}$$

Theorem 1. *Assume that $f : J \times \Omega \to B$, $f \in L^q(\Omega; L^\infty(J; B))$ with some $q \in [1, \infty]$ and $\varepsilon > 0$. Then the function $f^\varepsilon : \Omega \to B$ may be defined by formula* (1) *for almost all $x \in \Omega$ and $f^\varepsilon \in L^q(\Omega; B)$.*

This result follows from [9] (in the case $B = \mathbb{R}$, it was proved in [12,7]).

The following result concerning weak convergence of $f^\varepsilon(x)$ defined by formula (1) to $\langle f \rangle(x) \equiv \int_J f(\xi, x)\,\mathrm{d}\xi$ is valid (in fact, it was proved in [12,7]).

Theorem 2. *Assume that $f : J \times \Omega \to \mathbb{R}$, $f \in L^q(\Omega; L^\infty(J))$ with some $q \in [1, \infty]$. Then $f^\varepsilon \to \langle f \rangle$ weakly in $L_q(\Omega)$ if $q \in [1, \infty)$ or weakly-star in $L^\infty(\Omega)$ if $q = \infty$.*

3. Let $r \in \overline{\mathbb{R}}^+$ be a fixed parameter, value $s^0 \in [-r, r]$ and function $e \in W_1^1(0, T)$ are given. Consider the following problem: to find a function $s : [0, T] \to [-r, r]$, $s \in W_1^1(0, T)$ satisfying the conditions

$$\big(s'(t) - e'(t)\big)\big(\phi - s(t)\big) \geqslant 0 \quad \forall \phi \in [-r, r] \text{ for almost all } t \in (0, T), \quad s(0) = s^0$$

It is known (see [13–16]) that this problem has unique solution s. So there exists operator $\mathcal{S}_r : [-r, r] \times W_1^1(0, T) \to W_1^1(0, T)$ such that $s = \mathcal{S}_r[s^0, e]$. Operator $\mathcal{S}_r$ is known as *stop-operator*. It is one of the basic elements of hysteresis operators theory and describes the relation $\sigma = E\mathcal{S}_r[s^0, e]$ between strain e and stress σ in Prandtl model of elastoplastic element. Here E is the Young module, r is a yield point, and s^0 is an initial residual stress. This model was introduced by Prandtl [17]. It is known also (see [13–16]) that stop-operator may be extended as operator $\mathcal{S}_r : [-r, r] \times C[0, T] \to C[0, T]$.

The following model was offered by Prandtl [17] and Ishlinskii [18] for describing strain-stress relation in real elastoplastic materials:

$$\sigma = E_\infty e + \sum_{i=1}^{n} E_i \mathcal{S}_{r_i}\big[s_i^0, e\big] \tag{2}$$

where $0 < r_1 < \cdots < r_n$ and $0 \leqslant E_\infty$, $0 < E_i$, $|s_i^0| \leqslant r_i$, $1 \leqslant i \leqslant n$.

Operator $\mathcal{F} : \mathrm{Lip}_1(\overline{\mathbb{R}}^+) \times C[0, T] \times M_1(\overline{\mathbb{R}}^+) \to C[0, T]$, defined by the Stieltjes integral

$$\mathcal{F}\big[s^0, e, \mu\big](t) = \int_0^\infty \mathcal{S}_r\big[s^0(r), e\big](t)\,\mathrm{d}\mu(r) \quad \text{for } t \in [0, T]$$

extends the second summand of formula (2). It is called a Prandtl–Ishlinskii operator of stop-type.

4. We study the following system of equations describing longitudinal vibrations of Ishlinskii type viscoelastoplastic body with rapidly oscillating data:

$$D_t e_\varepsilon = D u_\varepsilon \quad \text{in } Q \tag{3}$$

$$\rho_\varepsilon D_t u_\varepsilon = D\sigma_\varepsilon + \rho_\varepsilon g \quad \text{in } Q \tag{4}$$

$$\sigma_\varepsilon = \frac{\nu_\varepsilon[\eta_\varepsilon]}{\eta_\varepsilon} D u_\varepsilon + \sigma_{\mathrm{el},\varepsilon}[e_\varepsilon] + \mathcal{F}\big[s_\varepsilon^0, e_\varepsilon, \mu_\varepsilon\big], \quad \eta_\varepsilon = e_\varepsilon + 1 \quad \text{in } Q \tag{5}$$

where $\varepsilon \in (0, 1]$ is a parameter. The triple $z_\varepsilon(x, t) = (e_\varepsilon(x, t), u_\varepsilon(x, t), \sigma_\varepsilon(x, t))$ of unknown functions is defined for $(x, t) \in Q$. We write $\nu_\varepsilon[\eta_\varepsilon](x, t) = \nu_\varepsilon(\eta_\varepsilon(x, t), x)$, $\sigma_{\mathrm{el},\varepsilon}[e_\varepsilon](x, t) = \sigma_{\mathrm{el},\varepsilon}(e_\varepsilon(x, t), x)$.

Recall the physical meaning of the values included into the equations: x is Lagrange coordinate, t is time, ρ_ε is density of strainless material, e_ε is strain (by physical meaning $e_\varepsilon > -1$), u_ε is velocity, σ_ε is stress, $\sigma_{\mathrm{el},\varepsilon}[e_\varepsilon]$ and

$\mathcal{F}[s_\varepsilon^0, e_\varepsilon, \mu_\varepsilon]$ describe elastic and plastic components of stress, respectively; ν_ε is viscosity coefficient and g is density of external forces.

We supplement the system (3)–(5) with initial conditions

$$e_\varepsilon|_{t=0} = e_\varepsilon^0(x), \quad u_\varepsilon|_{t=0} = u_\varepsilon^0(x) \quad \text{in } \Omega \tag{6}$$

and one of pairs of boundary conditions

$$u_\varepsilon|_{x=0} = u_0(t), \quad u_\varepsilon|_{x=X} = u_X(t) \quad \text{on } (0,T) \tag{7_1}$$

$$\sigma_\varepsilon|_{x=0} = \sigma_0(t), \quad u_\varepsilon|_{x=X} = u_X(t) \quad \text{on } (0,T) \tag{7_2}$$

$$\sigma_\varepsilon|_{x=0} = \sigma_0(t), \quad \sigma_\varepsilon|_{x=X} = \sigma_X(t) \quad \text{on } (0,T) \tag{7_3}$$

Problem (3)–(6), (7_m) will be denoted by $\mathcal{P}_m^\varepsilon$, $m = 1, 2, 3$.

A triple $z_\varepsilon = (e_\varepsilon, u_\varepsilon, \sigma_\varepsilon) \in \mathcal{N}_{-1}(Q) \times V_2(Q) \times L^2(Q)$ of functions will be called *a weak solution to problem* $\mathcal{P}_m^\varepsilon$ if the following properties are valid: (a) Eqs. (3) and (5) are satisfied in $L^2(Q)$; (b) the integral identity

$$-\int_Q \rho_\varepsilon u_\varepsilon D_t\varphi \,\mathrm{d}x\,\mathrm{d}t + \int_Q \sigma_\varepsilon D\varphi \,\mathrm{d}x\,\mathrm{d}t = \int_\Omega \rho_\varepsilon u_\varepsilon^0 \varphi|_{t=0} \,\mathrm{d}x + \int_Q \rho_\varepsilon g\varphi \,\mathrm{d}x\,\mathrm{d}t + \int_0^T \sigma_X \varphi|_{x=X} \,\mathrm{d}t - \int_0^T \sigma_0 \varphi|_{x=0} \,\mathrm{d}t$$

holds for all $\varphi \in C^1(\bar{Q})$ such that $\varphi|_{t=T} = 0$ and $\varphi|_{x=0,X} = 0$ if $m = 1$, $\varphi|_{x=X} = 0$ if $m = 2$; (c) initial condition $e_\varepsilon|_{t=0} = e_\varepsilon^0$ is satisfied in the sense of the space $C([0,T]; L^\infty(\Omega))$; (d) boundary conditions $u_\varepsilon|_{x=0} = u_0$ if $m = 1$ and $u_\varepsilon|_{x=X} = u_X$ if $m = 1, 2$ are satisfied in the sense of the space $C(\overline{\Omega}; L^2(0,T))$.

Let us state conditions on the data. All the properties in conditions C_1, C_2 are supposed to be valid for any $a > 1$, for almost all $\xi \in J$ and almost all $x \in \Omega$. Here $1 < N$ is an arbitrary large parameter, c_1, c_2 are positive constants, $\bar{c}_0, \bar{c}_1, \bar{c}_2$ are some positive functions of the parameter a.

C_1: The functions $\nu : \mathbb{R}^+ \times J \times \Omega \to \mathbb{R}^+$ and $\sigma_{\mathrm{el}} : (-1, +\infty) \times J \times \Omega \to \mathbb{R}$ satisfy the properties $\nu \in L^\infty(\Omega; L^\infty(J; C[a^{-1}, a]))$, $\sigma_{\mathrm{el}} \in L^\infty(\Omega; L^\infty(J; C[-1 + a^{-1}, a]))$ and the inequalities

$$0 < \bar{c}_0(a)^{-1} \leqslant \nu(\eta, \xi, x) \leqslant \bar{c}_0(a) \quad \forall \eta \in [a^{-1}, a]$$

$$|\nu(\eta_1, \xi, x) - \nu(\eta_2, \xi, x)| \leqslant \bar{c}_1(a)|\eta_1 - \eta_2| \quad \forall \eta_1, \eta_2 \in [a^{-1}, a]$$

$$\underline{\Lambda}(\eta) \leqslant \Lambda(\eta, \xi, x) \equiv \int_1^\eta \nu(\zeta, \xi, x)/\zeta \,\mathrm{d}\zeta \leqslant \overline{\Lambda}(\eta) \quad \forall \eta \in \mathbb{R}^+$$

where $\lim_{\eta \to 0^+} \overline{\Lambda}(\eta) = -\infty$ and $\lim_{\eta \to +\infty} \underline{\Lambda}(\eta) = +\infty$,

$$\nu(\eta, \xi, x)\eta \leqslant c_1 \big(L^2(\eta, \xi, x) + \eta + 1\big) \quad \forall \eta \in \mathbb{R}^+ \text{ if } m = 1$$

$$\eta/\nu(\eta, \xi, x) \leqslant c_1 \big(L^2(\eta, \xi, x) + 1\big) \quad \forall \eta \in \mathbb{R}^+ \text{ if } m = 2, 3$$

$$\sigma_{\mathrm{el}}(e, \xi, x) \leqslant 0 \quad \forall e \in (-1, 0], \qquad \sigma_{\mathrm{el}}(e, \xi, x) \geqslant 0 \quad \forall e \in [0, +\infty)$$

$$|\sigma_{\mathrm{el}}(e_1, \xi, x) - \sigma_{\mathrm{el}}(e_2, \xi, x)| \leqslant \bar{c}_2(a)|e_1 - e_2| \quad \forall e_1, e_2 \in [-1 + a^{-1}, a]$$

$$|\sigma_{\mathrm{el}}(e, \xi, x)|(e + 1) \leqslant c_2 \big(E(e, \xi, x) + 1\big)$$

where $L(\eta, \xi, x) = \int_1^\eta \sqrt{\nu(\zeta, \xi, x)/\zeta} \,\mathrm{d}\zeta$, $E(e, \xi, x) = \int_0^e \sigma_{\mathrm{el}}(\zeta, \xi, x) \,\mathrm{d}\zeta$.

C_2: The functions

$$\rho \in L^\infty\big(\Omega; L^\infty(J)\big), \qquad e^0 \in L^\infty\big(\Omega; L^\infty(J)\big), \qquad u^0 \in L^2\big(\Omega; L^\infty(J)\big)$$

and

$$u_0, u_X \in BV[0, T], \qquad \sigma_0, \sigma_X \in L^2(0, T)$$

satisfy the inequalities

$$N^{-1} \leqslant \rho(\xi, x) \leqslant N, \qquad N^{-1} \leqslant e^0(\xi, x) + 1 \leqslant N$$

$$\|u^0\|_{L^2(\Omega; L^\infty(J))} \leqslant N, \qquad \|u_0\|_{BV[0,T]} + \|u_X\|_{BV[0,T]} \leqslant N$$

$$\|\sigma_0\|_{L^2(0,T)} + \|\sigma_X\|_{L^2(0,T)} \leqslant N$$

In addition, $N^{-1} \leqslant \|\langle e^0\rangle + 1\|_{L^1(\Omega)} + I_t(u_X - u_0)$ on $(0, T)$ if $m = 1$. The functions $s^0 : J \times \Omega \times \overline{\mathbb{R}}^+ \to \mathbb{R}$, $\mu : J \times \Omega \times \overline{\mathbb{R}}^+ \to \mathbb{R}$ satisfy the properties

$$s^0 \in L^\infty\big(\Omega; L^\infty\big(J; \mathrm{Lip}_1\big(\overline{\mathbb{R}}^+\big)\big)\big), \qquad \mu \in L^\infty\big(\Omega; L^\infty\big(J; M_{1,N}\big(\overline{\mathbb{R}}^+\big)\big)\big)$$

The free term satisfies $g \in L^1(0, T; L^2(\Omega))$ and $\|g\|_{L^1(0,T;L^2(\Omega))} \leqslant N$.

By assumptions C_1, C_2 and Theorem 1 the following functions may be defined for all $\varepsilon > 0$:

$$\begin{aligned}
&\nu_\varepsilon(\eta, x) = \operatorname*{lim\,ap}_{\xi\to x^\varepsilon} \nu(\eta, \xi, x), \qquad \sigma_{\mathrm{el},\varepsilon}(e, x) = \operatorname*{lim\,ap}_{\xi\to x^\varepsilon} \sigma_{\mathrm{el}}(e, \xi, x), \qquad \rho_\varepsilon(x) = \operatorname*{lim\,ap}_{\xi\to x^\varepsilon} \rho(\xi, x)\\
&s^0_\varepsilon(x, r) = \operatorname*{lim\,ap}_{\xi\to x^\varepsilon} s^0(\xi, x, r), \quad \mu_\varepsilon(x, \cdot) = \operatorname*{lim\,ap}_{\xi\to x^\varepsilon} \mu(\xi, x, \cdot) \quad \text{in } L_1\big(\mathbb{R}^+\big)\\
&e^0_\varepsilon(x) = \operatorname*{lim\,ap}_{\xi\to x^\varepsilon} e^0(\xi, x), \qquad u^0_\varepsilon(x) = \operatorname*{lim\,ap}_{\xi\to x^\varepsilon} u^0(\xi, x)
\end{aligned} \tag{8}$$

We base on the following global existence and uniqueness theorem for a weak solution to problem $\mathcal{P}^\varepsilon_m$ with uniform estimates with respect to ε (it follows from [19,20]).

Theorem 3. *Suppose that conditions C_1, C_2 hold and functions $\nu_\varepsilon, \sigma_{el,\varepsilon}, \rho_\varepsilon, s^0_\varepsilon, \mu_\varepsilon, e^0_\varepsilon, u^0_\varepsilon$ are defined by* (8)*. Then problem $\mathcal{P}^\varepsilon_m$ has an unique weak solution, and it satisfies the estimates*

$$K(N)^{-1} \leqslant e_\varepsilon + 1 \leqslant K(N), \qquad \|D_t e_\varepsilon\|_{L_2(Q)} \leqslant K(N) \tag{9}$$

$$\|u_\varepsilon\|_{V_2^{(1,1/2)}(Q)} \leqslant K(N), \qquad \|\sigma_\varepsilon\|_{L_2(Q)} \leqslant K(N), \qquad \|I_t\sigma_\varepsilon\|_{W(Q)} \leqslant K(N) \tag{10}$$

Here $K(N)$ are some positive nondecreasing functions of N; they may depend on X, T, c_i, $\bar{c}_i$, $\underline{\Lambda}$ and $\overline{\Lambda}$.

5. Consider the Bakhvalov–Eglit type system of two-scaled homogenized equations

$$D_t e = \frac{\eta}{\nu[\eta]}\big(\sigma - \sigma_{\mathrm{el}}[e] - \mathcal{F}\big[s^0, e, \mu\big]\big), \quad \eta = e + 1 \text{ in } J \times Q \tag{11}$$

$$\langle\rho\rangle D_t u = D\sigma + \langle\rho\rangle g \quad \text{in } Q \tag{12}$$

$$\sigma = \left\langle \frac{\eta}{\nu[\eta]} \right\rangle^{-1} \left(Du + \left\langle \frac{\sigma_{\mathrm{el}}[e]\eta}{\nu[\eta]} \right\rangle + \left\langle \frac{\mathcal{F}[s^0, e, \mu]\eta}{\nu[\eta]} \right\rangle \right) \quad \text{in } Q \tag{13}$$

We write $\nu[\eta](\xi, x, t) = \nu(\eta(\xi, x, t), \xi, x)$, $\sigma_{\mathrm{el}}[e](\xi, x, t) = \sigma_{\mathrm{el}}(e(\xi, x, t), \xi, x)$. The unknown triple $z(\xi, x, t) = (e(\xi, x, t), u(x, t), \sigma(x, t))$ of functions is defined for $(\xi, x, t) \in J \times Q$. Remember that $\langle f\rangle(\cdot) = \int_J f(\xi, \cdot)\,\mathrm{d}\xi$.

We supplement system (11)–(13) with the initial conditions

$$e|_{t=0} = e^0(\xi, x) \quad \text{in } J \times \Omega, \qquad \langle\rho\rangle u|_{t=0} = \big\langle\rho u^0\big\rangle(x) \quad \text{in } \Omega \tag{14}$$

and one of the pairs of boundary conditions (7_1)–(7_3).

We denote problem (11)–(14), (7_m) by $\mathcal{P}^{\langle\rangle}_m$. The definition of weak solution to problem $\mathcal{P}^{\langle\rangle}_m$ is quite similar to that for problem $\mathcal{P}^\varepsilon_m$, but now $e \in \mathcal{N}^{\langle\rangle}_{-1}(J \times Q)$ and Eq. (13) is satisfied in $L^2(Q; L^\infty(J))$.

Theorem 4. *Suppose that conditions C_1, C_2 are valid. Then problem $\mathcal{P}^{\langle\rangle}_m$ has an unique weak solution and it satisfies the estimates*

$$K(N)^{-1} \leqslant e + 1 \leqslant K(N), \qquad \|D_t e\|_{L_2(Q;L^\infty(J))} \leqslant K(N) \tag{15}$$

$$\|u\|_{V_2^{(1,1/2)}(Q)} \leqslant K(N), \qquad \|\sigma\|_{L_2(Q)} \leqslant K(N), \qquad \|I_t\sigma\|_{W(Q)} \leqslant K(N) \tag{16}$$

6. The following theorem about the limit connection between problems $\mathcal{P}_m^\varepsilon$ and problem $\mathcal{P}_m^{\langle\rangle}$ ($m = 1, 2, 3$) is the main result of this Note.

Theorem 5. *Suppose that conditions C_1, C_2 are valid. Then the weak solutions z_ε to problems $\mathcal{P}_m^\varepsilon$ converge to the weak solution z to problem $\mathcal{P}_m^{\langle\rangle}$ as $\varepsilon \to 0$ in the following sense:*

$$e_\varepsilon - e^\varepsilon \to 0 \quad \text{strongly in } L^\infty\big(\Omega; C[0,T]\big),\ \text{where } e^\varepsilon(x,t) = \lim_{\xi \to x^\varepsilon} \operatorname{ap} e(\xi, x, t) \tag{17}$$

$$u_\varepsilon \to u \quad \text{strongly in } L^s\big(0,T; L^q(\Omega)\big) \quad \forall q \in [1,\infty],\ s \in [1,\infty),\ (2q)^{-1} + s^{-1} > 1/4 \tag{18}$$

$$u_\varepsilon \to u \quad \text{weakly star in } L^\infty\big(0,T; L^2(\Omega)\big), \qquad Du_\varepsilon \to Du \quad \text{weakly in } L^2(Q) \tag{19}$$

$$\sigma_\varepsilon \to \sigma \quad \text{weakly in } L^2(Q), \qquad I_t\sigma_\varepsilon \to I_t\sigma \quad \text{strongly in } C\big(\overline{Q}\big) \tag{20}$$

The proof comprises four steps. First, using Theorem 3 and compact embeddings of corresponding spaces we ensure properties (18)–(20) and estimates (16) for some functions u, σ. Second, we construct the function e as the unique solution in the class $\mathcal{N}_{-1}^{\langle\rangle}(J \times Q)$ to the Cauchy problem for Eq. (11) with the first of initial conditions (14). For e we also prove the estimates (15) and property (17). Third, by passing to a weak limit (using Theorem 2) we obtain that Eq. (13) holds. Finally, we prove that triple $z = (e, u, \sigma)$ is indeed a weak solution to problem $\mathcal{P}_m^{\langle\rangle}$.

The full proofs are presented in [21] (under wider conditions on data); moreover Eulerian coordinate is included into consideration.

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References

[1] N. Bakhvalov, G. Panasenko, Homogenization: Averaging Processes in Periodic Media, Kluwer, Dordrecht/Norwell, MA, 1990.
[2] E. Sanchez-Palencia, Non-Homogeneous Media and Vibration Theory, Springer-Verlag, Berlin/New York, 1980.
[3] N.S. Bakhvalov, M.E. Eglit, Processes in periodic media which are not describable by averaged characteristics, Soviet Phys. Dokl. 28 (1983) 125–127.
[4] G. Allaire, Homogenization and two-scale convergence, SIAM J. Math. Anal. 23 (1992) 1482–1518.
[5] A.A. Amosov, A.A. Zlotnik, Quasi-averaged equations of the one-dimensional motion of a viscous barotropic medium with rapidly oscillating data, Comp. Math. Math. Phys. 36 (1996) 203–220.
[6] D. Serre, Variations de grande amplitude pour la densite d'un fluide visqueux compressible, Physica D 48 (1991) 113–128.
[7] A.A. Amosov, A.A. Zlotnik, Quasi-averaging of the system of equations of one-dimensional motion of a viscous heat-conducting gas with rapidly oscillating data, Comp. Math. Math. Phys. 38 (1998) 1152–1167.
[8] A. Amosov, A. Zlotnik, On two-scale homogenized equations of one-dimensional nonlinear thermoviscoelasticity with rapidly oscillating nonsmooth data, C. R. Acad. Sci. Paris, Ser. IIb 329 (2001) 169–174.
[9] A.A. Amosov, A.A. Zlotnik, Substantiation of two-scale homogenization of one-dimensional nonlinear thermoviscoelasticity equations with nonsmooth data, Comp. Math. Math. Phys. 41 (2001) 1713–1733.
[10] J. Franku, P. Krejci, Homogenization of scalar wave equations with hysteresis, Cont. Mech. & Ther. 11 (1999) 371–391.
[11] L.C. Evans, R.F. Gariepi, Measure Theory and Fine Properties of Functions, CRC Press, Boca Raton, FL, 1992.
[12] A.A. Amosov, Weak convergence for a class of rapidly oscillating functions, Math. Notes 62 (1997) 122–126.
[13] M. Brokate, J. Sprekels, Hysteresis and Phase Transitions, Appl. Math. Sci., vol. 121, Springer-Verlag, New York, 1996.
[14] M.A. Krasnoselskii, A.V. Pokrovskii, Systems with Hysteresis, Springer-Verlag, Berlin, 1989; Russian edition: Nauka, Moscow, 1983.
[15] P. Krejci, Hysteresis, Convexity and Dissipation in Hyperbolic Equations, Gakuto Int. Series Math. Sci. Appl., vol. 8, Gakkotosho, Tokyo, 1996.
[16] A. Visintin, Differential Models of Hysteresis, Springer-Verlag, Berlin/Heidelberg, 1994.
[17] L. Prandtl, Ein Gedankenmodell zur knetischen Theorie der festen Körper, Z. Angew. Math. Mech. 8 (1928) 85–106.
[18] A.Yu. Ishlinskii, Some applications of statistical methods to describing deformations of bodies, Izv. AN SSSR. Tech. Ser. 9 (1944) 580–590.
[19] A.A. Amosov, I.A. Goshev, Existence and uniqueness of global weak solutions to the equations describing the longitudinal oscillations of a viscoelastoplastic Ishlinskii material, Dokl. Math. 74 (2006) 623–627.
[20] A.A. Amosov, I.A. Goshev, Global one-valued solvability to the system of equations describing Ishlinskii body longitudinal vibrations, Differential Equations, submitted for publication.
[21] A.A. Amosov, I.A. Goshev, Substantiation of two-scale homogenization of the system of equation of viscoelastoplastic Ishlinskii body longitudinal vibrations, Comp. Math. Math. Phys., submitted for publication.