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Macroscale turbulence modeling for flows in media laden with solid structures

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Abstract

The spatially averaged balance equation of the turbulent kinetic energy is derived for flows in media laden with solid structures. Two additional contributions are highlighted: sub-filter production and dispersion. The sub-filter production is tightly linked to the drag force and to the wake dissipation. Within the framework of the plane channel flow, we then analyse magnitudes of the various contributions to the spatially averaged equation of the turbulent kinetic energy. We postulate a balance equation for the spatially averaged turbulent dissipation rate. Finally, we propose a model for the characteristic time scale of the sub-filter production. ***To cite this article: F. Pinson et al., C. R. Mecanique 335 (2007).***

Résumé

Modélisation macroscopique de la turbulence pour les écoulements en milieux encombrés par des structures solides. L'équation de bilan de l'énergie cinétique turbulente spatialement moyennée est établie pour un écoulement dans un mileu encombré de structures solides. Deux termes supplémentaires sont mis en évidence dans cette équation : la production de sous-filtre et la dispersion. La production de sous-filtre est étroitement liée à la force de trainée et à la dissipation de sillage. Nous étudions ensuite les différentes contributions au bilan d'énergie cinétique turbulente spatialement moyennée, pour un écoulement dans un canal plan. Le temps caractéristique associé à la production de sous-filtre est étudié en postulant une équation bilan du taux de dissipation moyenné spatialement. ***Pour citer cet article : F. Pinson et al., C. R. Mecanique 335 (2007).***

Keywords: Fluid mechanics; Turbulent flows; k–ε model; Ordered porous media; Plane channel

Mots-clés : Mécanique des fluides ; Écoulements turbulents ; Modèle k–ε ; Milieux poreux ordonnés ; Canal plan

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La modélisation globale des écoulements turbulents traversant de grands systèmes fluidiques concerne de nombreuses applications pratiques comme la simulation des échangeurs de chaleur, des cœurs de réacteurs nucléaires ou encore des écoulements dans les milieux urbains. Dans de tels écoulements, de nombreuses échelles interagissent, qu'elles soient liées à la turbulence ou à la structure géométrique du milieu. Elles s'étendent sur une large gamme et ne sont donc pas toutes accessibles à une modélisation détaillée. L'objectif de la modélisation macroscopique de la turbulence n'est donc pas de restituer finement les détails de ces écoulements mais de rendre compte de leur action sur les échelles les plus grandes.

Dans ce but nous mettons en œuvre simultanément deux moyennes distinctes : la moyenne statistique (moyenne de Reynolds) et la moyenne spatiale [1–3]. La première moyenne est couramment employée dans le cadre de la modélisation locale de la turbulence. Elle donne accès à une représentation relativement structurée des écoulements turbulents. Selon cette moyenne, chaque quantité statistique ζ peut se décomposer en une valeur moyenne, notée $\overline{\zeta}$, et une fluctuation ζ'. La moyenne spatiale est classiquement mise en œuvre dans la modélisation des écoulements laminaires en milieux poreux [4,5]. De nouveau, une quantité quelconque ζ peut se décomposer en une partie moyenne $\langle\zeta\rangle_{\mathtt{f}}$ et une déviation spatiale $\delta\zeta$. Il a été montré que cette moyenne est idempotente sous réserve de séparation entre l'échelle des variations macroscopiques et l'échelle du filtre. Nous nous plaçons dans ce cadre et nous utiliserons donc par la suite la propriété d'idempotence pour les deux moyennes statistique et spatiale. Par ailleurs, si sur le plan mathématique les deux moyennes commutent strictement [2], chaque application de l'un ou l'autre opérateur de moyenne introduit des approximations qui rendent caduque la propriété de commutativité [6,7]. Dans ce papier, nous nous plaçons dans le cadre d'une double décomposition : statistique puis spatiale.

Il est classique dans l'étude de la turbulence de caractériser son intensité par l'énergie cinétique du mouvement fluctuant : k. C'est pourquoi, l'action de la matrice solide sur l'écoulement turbulent est tout d'abord modélisée au travers de $\langle k\rangle_{\mathtt{f}}$. L'équation de bilan pour cette quantité est formellement dérivée et des termes supplémentaires, intrinsèquement liés à l'encombrement du milieu, sont mis en évidence. Ce sont : la production de sous-filtre et la dispersion. Notons que, de par les conditions aux limites en paroi de k : $k=0$ et $\vec{\nabla}k=0$, il n'y pas de contribution de tortuosité.

De la même manière que dans les modèles locaux de turbulence où le terme source par cisaillement est un terme d'échange depuis l'énergie cinétique du mouvement moyen vers le mouvement fluctuant, nous sommes tentés de modéliser la production supplémentaire d'énergie cinétique turbulente (précédemment appelée production de sous-filtre) en fonction de la traînée produite par les structures solides. À l'aide d'une analyse à deux échelles, nous montrons que la traînée contribue effectivement à transférer l'énergie cinétique depuis le mouvement macroscopique vers le mouvement microscopique (inférieur à la taille du filtre). Cette analyse nous conduit à écrire la production de sous-filtre comme la puissance macroscopique de la force de traînée déduite d'une dissipation de sillage.

Dans le cadre de l'analyse de l'établissement d'un écoulement turbulent dans un canal plan, nous étudions les amplitudes respectives des différentes contributions : dispersion, diffusion, production de sous-filtre et dissipation visqueuse. De plus, sur la base d'une équation modèle postulée pour la moyenne spatiale de la dissipation visqueuse, $\langle\varepsilon\rangle_{\mathtt{f}}$, nous proposons une expression modèle pour l'échelle de temps caractéristique associée à la production de sous-filtre.

1. Introduction

The macroscopic modeling of turbulent flows passing through ordered media laden with solid structures concerns many practical applications such as nuclear reactors, heat exchangers or urban flows. In such flows, various scales coexist. The challenge of the macroscopic modeling is not to reproduce the fine structure dynamics of the flow but to take into account phenomena, such as turbulent heat exchange and friction embedded in smaller scales, for large scale modelization. With this aim we develop a macroscopic turbulence model that is based upon the use of two average operators: the statistical average (or Reynolds average) that is practical for turbulence study, and the spatial average, well adapted for the porous media approach.

In the Reynolds average framework, any statistical quantity ζ may be split up into: $\zeta \equiv \overline{\zeta} + \zeta'$ where $\overline{\zeta}$ is called the mean value and ζ' its fluctuation. We also introduce the spatial average upon a Representative Elementary Volume (REV). We shall assume that the REV is well adapted to the geometrical characteristics of the media under study following Refs. [4,5] recommendations. Let us call ΔV the volume of the REV and ΔV_f the volume of fluid embedded within ΔV. We define the spatial average

$$\langle\zeta\rangle_{\mathrm{f}}(\vec{x},t) \equiv \frac{1}{\Delta V_f(\vec{x})} \int\limits_{\Delta V_f(\vec{x})} \zeta(\vec{y},t)\,\mathrm{d}V_{\vec{y}} \quad \text{where } \Delta V_f = \Delta V \cap V_f$$

Thus, any quantity ζ may also split up into: $\zeta \equiv \langle\zeta\rangle_{\mathrm{f}} + \delta\zeta$ where $\delta\zeta$ is called the deviation of ζ from the spatial average. The porosity of the media is defined by: $\phi \equiv \Delta V_f/\Delta V$.

If variation length scales of the macroscopic quantities are large with respect to the filter size then the spatial average can be assumed idempotent [4,5]. Under this assumption, spatially averaged quantities are called macro-scale quantities while deviations, whose variation length scale is smaller than the REV size, are called micro-scale quantities. Furthermore, in a strict mathematical way, both averages commute [2]. However, each modeling step related to an average application involves simplifications. Hence, the macroscopic turbulence modeling necessarily depends on the order of application of the averages [6,7]. Following the pioneering works of [1], [2] and [3], we choose first to apply the statistical average in order to get a structured picture of the turbulent flow and to benefit from the large amount of knowledge available for RANS turbulence modeling. We then apply the spatial average.

In this Note, we formally derive the balance equation for the macroscopic turbulent kinetic energy (TKE) and we highlight the specific contributions induced by the action of the solid matrix on turbulence (Section 2): sub-filter production and dispersion. Since at walls we have $k=0$ and $\vec{\nabla}k=0$, there is no tortuosity contribution. Next, we focus on the analysis of the plane channel flow in order to evaluate magnitudes of the various contributions to the $\langle k\rangle_{\mathrm{f}}$ balance equation (Section 3). Since in such flows several length/time scales compete, we also postulate a modeled balance equation for the dissipation of the macroscopic TKE (Section 4). We then study the characteristic time scale associated to the sub-filter production.

2. Derivation of the $\langle k\rangle_{\mathrm{f}}$ balance equation

We introduce χ_f the distribution function of solid embedded within the REV. By definition, we have $\langle\chi_f\rangle = \phi$ and $\vec{\nabla}\chi_f \equiv -\vec{n}\delta_w$ where $\vec{n}$ is the outward normal from fluid and δ_w is the Dirac function associated to walls. In what follows we will use Einstein's notations for subscripts. A practical way to highlight the solid matrix action on turbulence at a macroscopic scale is to study the double averaged balance equation of the TKE for an incompressible fluid. With this aim, we introduce the TKE balance equation

$$\frac{\partial k}{\partial t} + \frac{\partial k\bar{u}_i}{\partial x_i} = -\frac{\partial \overline{u_i'k'}}{\partial x_i} - \frac{1}{\rho}\frac{\partial \overline{u_i'P'}}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\nu\frac{\partial k}{\partial x_i}\right) - R_{ij}\frac{\partial \bar{u}_i}{\partial x_j} - \varepsilon \tag{1}$$

where R_{ij} is the so-called Reynolds tensor. Turbulent fluxes in Eq. (1) and viscous diffusion are generally modeled together

$$\frac{\partial}{\partial x_i}\left(\nu\frac{\partial k}{\partial x_i}\right) - \frac{\partial \overline{u_i'k'}}{\partial x_i} - \frac{1}{\rho}\frac{\partial \overline{u_i'P'}}{\partial x_i} = \frac{\partial}{\partial x_i}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right] \tag{2}$$

We multiply Eq. (1) by χ_f and we take the spatial average of the resulting equation. This yields

$$\begin{aligned}\phi\frac{\partial\langle k\rangle_{\mathrm{f}}}{\partial t} + \frac{\partial}{\partial x_i}\phi\langle k\rangle_{\mathrm{f}}\langle\bar{u}_i\rangle_{\mathrm{f}} = {} & \underbrace{-\frac{\partial}{\partial x_i}\phi\langle\delta k\,\delta\bar{u}_i\rangle_{\mathrm{f}}}_{\text{Dispersion}} + \frac{\partial}{\partial x_i}\phi\left\langle\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right\rangle_{\mathrm{f}} + \underbrace{\phi\left\langle\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}n_i\delta_w\right\rangle_{\mathrm{f}}}_{=0 \text{ since } \vec{\nabla}k\cdot\vec{n}=0 \text{ at walls}} \\ & - \underbrace{\langle R_{ij}\rangle_{\mathrm{f}}\frac{\partial\phi\langle\bar{u}_i\rangle_{\mathrm{f}}}{\partial x_j}}_{\text{Macroscopic shear production}} - \underbrace{\phi\left\langle\delta R_{ij}\frac{\partial\delta\bar{u}_i}{\partial x_j}\right\rangle_{\mathrm{f}}}_{\text{Sub-filter production}} - \underbrace{\phi\langle\varepsilon\rangle_{\mathrm{f}}}_{\text{Dissipation}}\end{aligned} \tag{3}$$

The use of the spatial average lets the dispersion term appear. This term describes, at macro-scale, the convective transport due to the local velocity heterogeneities of the quantity under study. It is generally modeled by a diffusivity tensor [8].

Using the spatial averaging theorem [8] and the averaging idempotence property, the shear production splits up into two contributions. The first contribution is the production term induced by the macro-scale shear. The second contribution, that we call sub-filter production (see Eq. (3)), is induced by velocity gradients at small scales. Since in flows under study, these types of velocity gradients are mainly induced by walls, sub-filter production is tightly linked to boundary layers and therefore, to the drag force in the macro-scale framework. In clear (free) flows, the TKE production induced by the shear is an energy transfer from the mean kinetic energy $\bar{u}_i\bar{u}_i/2$ towards the TKE. Intuitively, we are tempted to follow the same reasoning for the sub-filter production and to model this contribution proportionally to the drag. However, since velocity is zero at walls, it is easy to show that the drag is not a sink term in the balance equation for the mean macroscopic kinetic energy: $K = \langle\bar{u}_i\bar{u}_i\rangle_{\mathrm{f}}/2$. In order to highlight energy transfers between scales in such media, we have performed a two scale analysis [9]. We consider an incompressible flow inside a medium with a constant porosity. We introduce the macro-scale and micro-scale mean kinetic energies: $K^M = \langle\bar{u}_i\rangle_{\mathrm{f}}\langle\bar{u}_i\rangle_{\mathrm{f}}/2$ and $K^m = \delta\bar{u}_i\,\delta\bar{u}_i/2$ respectively. These equations can be written:

$$\frac{\partial K^M}{\partial t} + \langle\bar{u}_i\rangle_{\mathrm{f}}\frac{\partial K^M}{\partial x_i} = -\frac{\langle\bar{u}_i\rangle_{\mathrm{f}}}{\rho}\frac{\partial\langle\overline{P}\rangle_{\mathrm{f}}}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\nu\frac{\partial K^M}{\partial x_i}\right) - \nu\frac{\partial\langle\bar{u}_i\rangle_{\mathrm{f}}}{\partial x_j}\frac{\partial\langle\bar{u}_i\rangle_{\mathrm{f}}}{\partial x_j} - \frac{\partial}{\partial x_j}\langle R_{ij}\rangle_{\mathrm{f}}\langle\bar{u}_i\rangle_{\mathrm{f}} + \langle R_{ij}\rangle_{\mathrm{f}}\frac{\partial\langle\bar{u}_i\rangle_{\mathrm{f}}}{\partial x_j} - \langle\bar{u}_i\rangle_{\mathrm{f}}\frac{\partial}{\partial x_j}\langle\delta\bar{u}_i\delta\bar{u}_j\rangle_{\mathrm{f}} - \langle\bar{u}_i\rangle_{\mathrm{f}}\overline{F}_{\phi_i} \tag{4}$$

$$\frac{\partial\langle K^m\rangle_{\mathrm{f}}}{\partial t} + \langle\bar{u}_i\rangle_{\mathrm{f}}\frac{\partial\langle K^m\rangle_{\mathrm{f}}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial\langle\delta\overline{P}\delta\bar{u}_i\rangle_{\mathrm{f}}}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\nu\frac{\partial\langle K^m\rangle_{\mathrm{f}}}{\partial x_i}\right) - \left\langle\nu\frac{\partial\delta\bar{u}_i}{\partial x_j}\frac{\partial\delta\bar{u}_i}{\partial x_j}\right\rangle_{\mathrm{f}} - \frac{\partial}{\partial x_j}\langle\delta R_{ij}\delta\bar{u}_j\rangle_{\mathrm{f}} + \left\langle\delta R_{ij}\frac{\partial\delta\bar{u}_i}{\partial x_j}\right\rangle_{\mathrm{f}} - \langle\delta\bar{u}_i\delta\bar{u}_j\rangle_{\mathrm{f}}\frac{\partial\langle\bar{u}_i\rangle_{\mathrm{f}}}{\partial x_j} - \frac{\partial}{\partial x_i}\langle\delta K^m\delta\bar{u}_i\rangle_{\mathrm{f}} + \langle\bar{u}_i\rangle_{\mathrm{f}}\overline{F}_{\phi_i} \tag{5}$$

where $\overline{P}$ stands for the Reynolds averaged pressure and $\overline{F}_{\phi_i} = -\langle(-\delta\overline{P}/\rho\delta_{ij} + \nu\partial\bar{u}_i/\partial x_j)n_j\delta_w\rangle_{\mathrm{f}}$ is the specific mean drag. From Eqs. (4) and (5), we note that drag transfers energy from large to small scales. In canopy turbulence models, some authors considered that the sub-filter production is conventionally the work performed by the mean macroscopic flow against the drag force minus a sink term. This term is supposed to represent the accelerated spectral cascade of TKE due to the plant foliage elements [10]. Based on this reasoning, we define the wake dissipation ($\langle\varepsilon_w\rangle_{\mathrm{f}}$), for flows without macro-scale shear, as the difference between the sub-filter production (P_{SF}) and the drag work against the macroscopic flow

$$\langle\varepsilon_w\rangle_{\mathrm{f}} = \overline{F}_{\phi_i}\langle\bar{u}_i\rangle_{\mathrm{f}} - P_{\mathrm{SF}} \quad \text{with } P_{\mathrm{SF}} = -\langle\delta R_{ij}\partial\delta\bar{u}_i/\partial x_j\rangle_{\mathrm{f}} \tag{6}$$

Let us note that, in the limit of the homogeneous macroscopic flow with no macroscopic shear, we deduce from Eq. (5): $\langle\varepsilon_w\rangle_{\mathrm{f}} = \langle\nu\partial\delta\bar{u}_i/\partial x_j\partial\delta\bar{u}_i/\partial x_j\rangle_{\mathrm{f}}$. One can summarize this result by saying that the net TKE sub-filter production is equal to the 'work' of the bulk flow against the mean specific drag minus a supplementary dissipation. Let us emphasize that for a steady laminar flow the wake dissipation is strictly equal to the work performed by the mean macroscopic flow against the drag force such that the sub-filter production is zero. In general, $\langle\varepsilon_w\rangle_{\mathrm{f}}$ depends on the sub-filter flow imbalance compared with its steady state.

3. Analysis of $\langle k\rangle_{\mathrm{f}}$ balance equation for the plane channel flow

We choose to analyse one of the simplest media representative of heat exchangers, say the plate heat exchanger. In such media, the REV for fluid flow is the plane channel flow. We perform this analysis in order to get a better understanding of the four unknown contributions of the $\langle k\rangle_{\mathrm{f}}$ balance equation. A description of the geometric configuration is given in Fig. 1. The hydraulic diameter D_H is twice the clearance between the plates and the Reynolds number of the flow is defined by $Re = \langle\bar{u}_z\rangle_{\mathrm{f}}D_H/\nu$. No velocity gradient exists within the x direction. Hence, macroscopic shear in Eq. (3) vanishes.

In order to evaluate magnitudes of the various contributions in Eq. (3), fine scale simulations have been performed with the CAST3M CFD code [11]. We use the standard k–ε model, where R_{ij} is modeled through the Boussinesq approximation [12], with Van Driest wall functions. Validations have been carried out with respect to the experimental results of Comte-Bellot [13]. At the inlet, velocity, TKE and viscous dissipation profiles are flat in the central part of

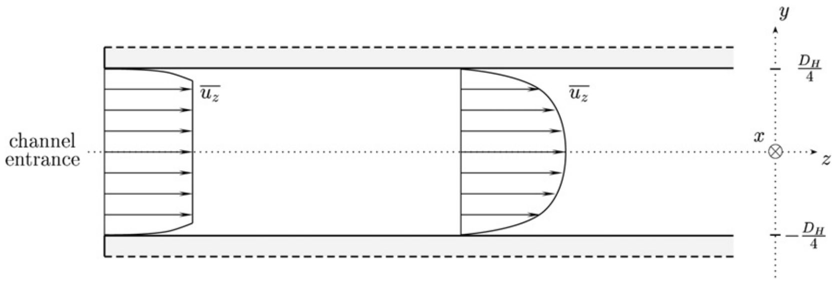


Fig. 1. Sketch of the considered plane channel configuration.

Fig. 1. Description géométrique du canal plan.

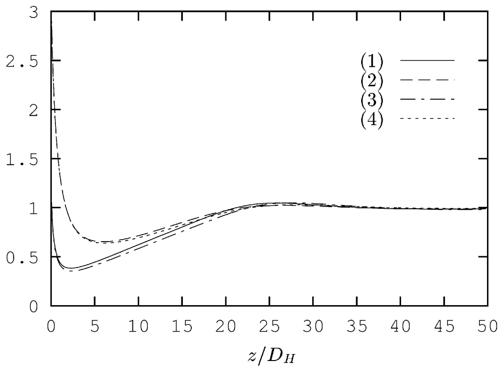


Fig. 2. Evolution of $\langle k\rangle_f$ along z. (1): $Re = 2.28 \times 10^5$, CL1. (2): $Re = 2.28 \times 10^5$, CL2. (3): $Re = 4.8 \times 10^5$, CL1. (4): $Re = 4.8 \times 10^5$, CL2.

Fig. 2. Evolution suivant z de $\langle k\rangle_f$ pour différents nombres de Reynolds et conditions d'entrée.

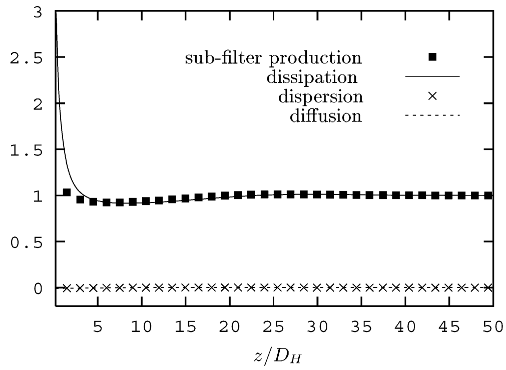


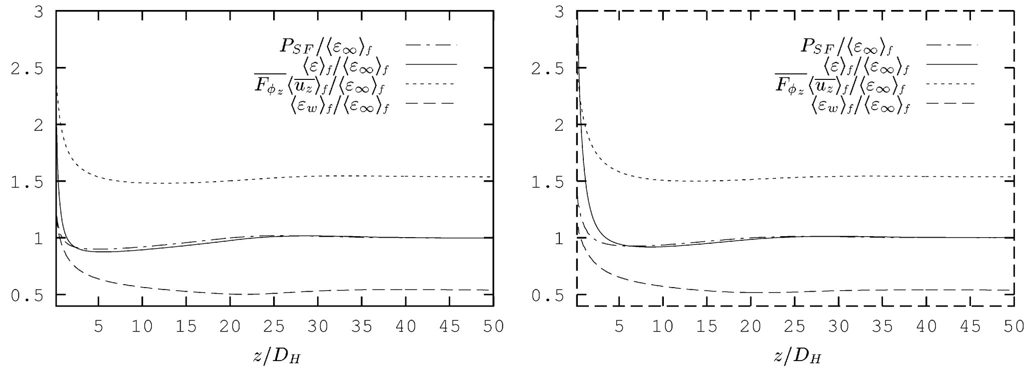
Fig. 3. Dimensionless contributions in the $\langle k\rangle_f$ balance equation (Eq. (3)) for plane channel flows. All quantities are scaled by $\langle \varepsilon_\infty\rangle_f$. $Re = 4.8 \times 10^5$, CL2.

Fig. 3. Contributions (adimensionnées par $\langle \varepsilon_\infty\rangle_f$) de l'équation bilan de $\langle k\rangle_f$ (Éq. (3)).

the channel and they follow wall functions in the near wall region. Each contribution of the $\langle k\rangle_f$ balance equation is directly calculated by applying the spatial average to the fine scale simulation results. We are aware that simulation results depends upon the turbulence model we use and especially to wall functions we use. However, we have checked that this dependency remains low within the RANS model framework. Notably, we have checked that the use of low Reynolds models does not significantly modify the phenomenology.

We introduce the asymptotic values (far from the inlet) of TKE and dissipation: $\langle k_\infty\rangle_f$ and $\langle \varepsilon_\infty\rangle_f$ respectively. Four simulations with two Reynolds numbers [13]: $Re = 2.28 \times 10^5$ and 4.8×10^5 and two couples of inlet boundary conditions: CL1 ($\langle k_0\rangle_f/\langle k_\infty\rangle_f = 1$, $\langle \varepsilon_0\rangle_f/\langle \varepsilon_\infty\rangle_f = 2$) and CL2 ($\langle k_0\rangle_f/\langle k_\infty\rangle_f = 3$, $\langle \varepsilon_0\rangle_f/\langle \varepsilon_\infty\rangle_f = 3$) have been performed. Profiles of $\langle k\rangle_f/\langle k_\infty\rangle_f$ are displayed in Fig. 2. One can notice that these profiles exhibit an oscillation a few hydraulic diameters downstream the inlet. Let us emphasize that the oscillation existence, magnitude and location is strongly sensitive to boundary conditions at the inlet. This dependency is only slightly sensitive to Reynolds number within the range under study. Hence, in what follows, we shall focus on boundary conditions sensitivity and perform this analysis for $Re = 4.8 \times 10^5$ (CL2). Four contributions remain on the right hand side of Eq. (3): dispersion, diffusion, sub-filter production and dissipation. Fig. 3 highlights that, for this particular flow, dispersion and diffusion of $\langle \bar{k}\rangle_f$ remain negligible in front of sub-filter production and dissipation.

Following Eq. (6) we split up P_{SF} into drag contribution and $\langle \varepsilon_w\rangle_f$. Profiles of the wake dissipation and the drag contribution scaled by the averaged dissipation are displayed in Fig. 4 for both inlet boundary conditions. Near the inlet, dissipation is uniformly distributed along the channel width while production is concentrated in the very thin boundary layer. This results in $P_{SF} < \langle \varepsilon\rangle_f$ in the vicinity of the inlet ($z/D_H \lesssim 5$). Turbulence imbalance is not only induced by level imbalance but also because profiles near the inlet are far from equilibrium. After turbulence has decreased due to dissipation, the stiffness of velocity gradient at walls produces a large amount of TKE that diffuses towards the bulk flow. We emphasize that since $\langle k\rangle_f$ is very sensitive to the imbalance between production and



(a) CL1. (b) CL2.

Fig. 4. Dimensionless contributions in Eq. (6) for plane channel flows: all quantities are scaled by $\langle\varepsilon_\infty\rangle_{\mathfrak{f}}$ and $Re = 4.8 \times 10^5$.

Fig. 4. Contributions de l'Éq. (6) adimensionnées par $\langle\varepsilon_\infty\rangle_{\mathfrak{f}}$: canal plan, $Re = 4,8 \times 10^5$.

dissipation this imbalance is the key feature of the macroscopic turbulence modeling. As turbulent boundary layer thickens, production region spreads and is no more concentrated at walls. Consequently, a few hydraulic diameters downstream the inlet, P_{SF} becomes greater than dissipation. This explains oscillations of the $\langle k\rangle_{\mathfrak{f}}$ profiles noticed in Fig. 2.

4. Analysis of the sub-filter production time scale

In order to analyse the time scale associated with the sub-filter production, we choose to propose a balance equation for the macroscopic dissipation, $\langle\varepsilon\rangle_{\mathfrak{f}}$ for channel flows. Rather than to directly average the microscopic ε balance equation, we choose to proceed by analogy with the $\langle k\rangle_{\mathfrak{f}}$ balance equation. This method is commonly used within the RANS framework. Each production/dissipation phenomenon is scaled by a characteristic time scale. Diffusion and dispersion of $\langle\varepsilon\rangle_{\mathfrak{f}}$ are assumed negligible. The $\langle\varepsilon\rangle_{\mathfrak{f}}$ balance equation then reads

$$\phi\frac{\partial\langle\varepsilon\rangle_{\mathfrak{f}}}{\partial t} + \frac{\partial}{\partial z}\phi\langle\varepsilon\rangle_{\mathfrak{f}}\langle\bar{u}_z\rangle_{\mathfrak{f}} = C_{\varepsilon_2}\phi\frac{P_{\mathrm{SF}}}{\tau_s} - C_{\varepsilon_2}\phi\frac{\langle\varepsilon\rangle_{\mathfrak{f}}}{\tau_t} \tag{7}$$

In order to recover the clear flow limit, we choose $\tau_t \equiv \langle k\rangle_{\mathfrak{f}}/\langle\varepsilon\rangle_{\mathfrak{f}}$. Let us emphasize that, although the asymptotic state must achieve $\lim_{x\to\infty}\tau_s = \tau_{t_\infty} \equiv \langle k_\infty\rangle_{\mathfrak{f}}/\langle\varepsilon_\infty\rangle_{\mathfrak{f}}$, it is not possible to set $\tau_s \equiv \tau_t$ because this choice would imply that one cannot achieve asymptotic values independent from the inlet boundary conditions. Now, the asymptotic state is obviously independent from the inlet conditions. This mathematical condition is verified by the model proposed in Ref. [1] but not in [2].

Thanks to the spatially averaging of the fine scale simulation results, it is possible to evaluate the advective and dissipative contributions and the sub-filter production in Eq. (8). The wake dissipation is thus given by (6). We are aware that those results formally depend upon the microscale turbulence model we use. However, we shall infer that, once spatially averaged, those results only slightly depend upon the microscale turbulence model used. The unsteady contribution in (8) is zero. We then perform a priori tests, by comparing the 'reference' time τ_s coming from Eq. (8):

$$\tau_s = C_{\varepsilon_2}P_{\mathrm{SF}}\left(\langle\bar{u}_z\rangle_{\mathfrak{f}}\frac{\partial\langle\varepsilon\rangle_{\mathfrak{f}}}{\partial z} + C_{\varepsilon_2}\frac{\langle\varepsilon\rangle_{\mathfrak{f}}^2}{\langle\bar{k}\rangle_{\mathfrak{f}}}\right)^{-1} \tag{8}$$

Hence it is possible to plot profiles of the reference time τ_s (Fig. 5). By comparing these results with various time scales built from model quantities, it appears that expression

$$\tau_s = \tau_t \times \sqrt{f_{p_\infty}/f_p} \tag{9}$$

satisfactorily recovers reference results (Fig. 5). Let us note that according to the a priori tests procedure the friction factor f_p and its equilibrium value f_{p_∞} in Eq. (9) result from the fine scale simulations. We thus suggest that the friction factor is one of the major parameters in order to model non-equilibrium turbulent flows in organized media like heat exchangers or nuclear reactor cores.

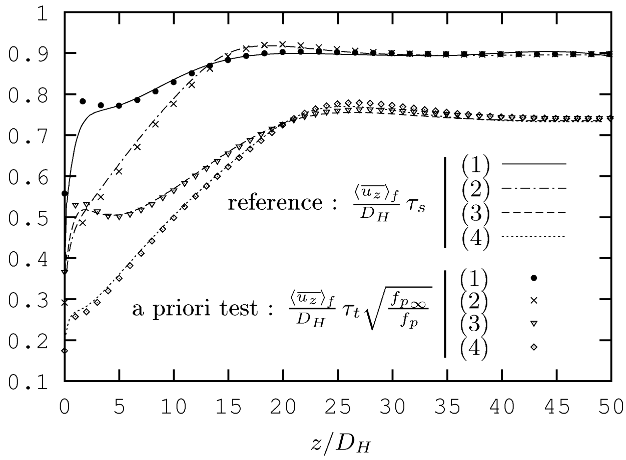


Fig. 5. Comparison between the reference time scale τ_s and the modeled time scale defined in Eq. (9). Each time is divided by the macroscopic convection time scale $D_H/\langle \bar{u}_z \rangle_f$. (1): $Re = 10^5$, CL1. (2): $Re = 10^5$, CL2. (3): $Re = 4.8 \times 10^5$, CL1. (4): $Re = 4.8 \times 10^5$, CL2.

Fig. 5. Comparaison entre le temps τ_s de référence et le temps proposé dans l'équation (9). Les temps sont adimensionnés par l'échelle de temps convective : $D_H/\langle \bar{u}_z \rangle_f$.

5. Conclusion

The spatial average usually applied for laminar flows in porous media is extended here to turbulent flows in organized laden media. Within the double averaging framework (statistical and spatial) a formal derivation of the double averaged turbulent kinetic energy is presented. New terms coming from the spatial averaging are highlighted. In order to derive closures for those contributions, we carried out a two-scale analysis. By doing this, we exhibit energy transfers between the mean motion and turbulence embedded in a laden medium. The sub-filter production is shown to derive from the specific mean drag contribution and a supplementary dissipation, defined as the wake dissipation. Due to k boundary conditions at walls, tortuosity contribution cancels. For the plane channel flow, we show that diffusion and dispersion of $\langle k \rangle_f$ along the flow direction are negligible and that sub-filter production and viscous dissipation compete. We propose a strategy to evaluate the sub-filter production time scale. It is based upon a postulated equation for the macroscopic dissipation. Finally, a model is proposed to approximate this time scale. In forthcoming works, we will focus on the modeling of the sub-filter production, the wake dissipation and the friction factor.

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