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Use of Proper Orthogonal Decomposition for reconstructing the 3D in-cylinder mean-flow field from PIV data

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Abstract

Based on PIV measurements obtained in planes along the main flow direction, a snapshot POD based on the full velocity field is performed. The dominant POD coefficients associated to the mean flow, are interpolated in the whole space domain according to boundary flow properties. From the knowledge of these POD coefficients, the entire mean-flow field is reconstructed and a 3D mean-flow representation is obtained. An application is presented using PIV measurements performed in in-cylinder engine flow. ***To cite this article: Ph. Druault, C. Chaillou, C. R. Mecanique 335 (2007).***

Résumé

Utilisation de la POD pour reconstruire le champ moyen de vitesse 3D à partir de données PIV. A partir de mesures PIV du champ de vitesse, une nouvelle application de la snapshot POD est proposée pour reconstruire entièrement le champ moyen de vitesse. Les coefficients POD relatifs au champ moyen sont interpolés dans le domaine d'étude en respectant les propriétés de l'écoulement aux frontières de ce domaine. Connaissant ces coefficients et les fonctions propres POD, le champ de vitesse moyen est alors reconstruit. Une application est proposée à partir de données obtenues dans le cylindre d'un moteur à combustion interne. ***Pour citer cet article : Ph. Druault, C. Chaillou, C. R. Mecanique 335 (2007).***

Keywords: Turbulence; Proper Orthogonal Decomposition; PIV; Swirl; In-cylinder flow

Mots-clés : Turbulence ; Décomposition orthogonale aux valeurs propres ; PIV ; Structure tourbillonnaire de grande échelle

1. Introduction

Today, the measurement of the three velocity components (3C) in a three-dimensional (3D) complex turbulent flow field still remains difficult. Indeed, even if recent developments of High-Resolved PIV (HRPIV) and Holographic PIV (HPIV) have allowed experimenters to access the spatial and temporal details of turbulence, these measurement techniques require a considerably complex optical system and an advanced data processing technique. Furthermore, the

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use of multiple hot-wire sensors to access a 3C velocity field [1], is limited to a slice representation. Instead of using a sophisticated measurement system for a 3D flow representation, an alternative solution may consist in developing post-processing tools which reconstruct the lack of data information. For instance, Linear Stochastic Estimation and gappy Proper Orthogonal Decomposition (POD) have been used for spatial experimental data reconstruction [1,2]; the phase averaging method and POD have also been used for temporal PIV data reconstruction [3,4]. A new spatial reconstruction procedure of the mean flow has been recently achieved by using the continuity equation for incompressible flows [5]. From PIV velocity vectors recorded in parallel planes, Sousa [5] computed the third mean-velocity component (out of plane) from the time-average continuity equation. However, such a procedure brings uncertainties to the reconstructed velocity component, which is mainly related to the propagation of uncertainties in the two other velocity components and to the presence of discontinuities in the domain of PIV interrogation.

In this Note, a new application of POD is proposed for the 3D reconstruction of the mean-velocity field from velocity fields obtained in selected parallel and orthogonal planes of a turbulent flow. The methodology is demonstrated for an in-cylinder engine flow. The strong influence of aerodynamic phenomena on combustion processes and on heat transfer, [6], makes such a study essential for the investigation of engine efficiency and the reduction of pollutant emission. In engine flows, the initial flow pattern is governed by the intake process. This initial flow distribution is very important for the overall location of the mixture and the specific mixture strategy during the engine cycle. Therefore, this work focuses on the in-cylinder engine flow during the intake stroke since the analysis of the three-dimensional large-scale structures deduced from the mean-flow field reconstruction is of great interest for engine investigation.

2. Proper Orthogonal Decomposition

Proper Orthogonal Decomposition is a powerful and elegant method of data analysis for extracting coherent structures from turbulent flows. In this decomposition, the kernel is the correlation tensor. Mathematically, the basic concept of POD consists in finding among a set of realisations of the flow field, the realisation which maximizes the mean square energy. For reasons of computational efficiency, the application of the *classical* POD introduced by Lumley [7] is prohibitive due to the large number of available PIV grid points. The snapshot POD introduced by Sirovich [8] is then used to obtain the POD modes. The R velocity time correlation tensor is defined as follows:

$$[R]_{ij} = \frac{1}{n_1 n_2} \sum_{l=1}^{n_1} \sum_{m=1}^{n_2} V(X_{lm}, t_i) V(X_{lm}, t_j)$$

where $n_1 \times n_2$ is the number of the total grid points in the PIV measurement plane, V and X are the velocity vector and the space variable, respectively. The POD coefficients $a^{(n)}$ are then deduced from this correlation matrix using the following Fredholm equation: $RA = \lambda A$ where $\lambda^{(n)}$ corresponds to the occurrence and the energy contained within the nth POD mode. The POD eigenfunction $\Phi^{(n)}(X)$ is deduced from the projection of the instantaneous velocity field onto the corresponding POD coefficient. From this decomposition, instantaneous velocity field (for instance the u velocity component) is reconstructed with the following equation:

$$u(X,t) = \sum_{n=1}^{N_{\mathrm{mod}}} a^{(n)}(t) \Phi^{(n)}(X)$$

where N_{mod} is the total POD mode number. Generally, POD procedure is devoted to the detection and the analysis of the coherent structures present in turbulent flows and also to the development of low order dynamical system [9]. Recently, POD procedure has been also used for other applications like: (i) the reconstruction of spatial correlation tensor in turbulent plane mixing layer flow [10]; (ii) the spatial reconstruction of missing experimental data [2,11]; (iii) the temporal interpolation from HRPIV database [4]. In this work, a new investigation of POD procedure is proposed to access the 3D mean-velocity field from selected 2D PIV velocity fields.

The POD spatial reconstruction of the mean-flow field is detailed for one velocity component, u. A same procedure is performed for the other components. Suppose that PIV measurements are performed in N_x aligned (y,z) planes in a turbulent flow, and in each measurement plane x_k, N_t instantaneous velocity fields are obtained. A snapshot POD is performed from the total velocity field $u(x_k, y, z, t_l)$ with $k = 1, \ldots, N_x$ and $l = 1, \ldots, N_t$, providing $N_x \times N_t$ POD eigenfunctions. Suppose that the projection of the velocity field onto the first M POD modes corresponds to the mean-velocity field in each x_k plane. From each instantaneous flow realisation t_l obtained in a particular x_k plane, the *instantaneous* mean-flow field is extracted with the following equation:

$$\overline{u(x_k, y, z, t_l)} = \sum_{n=1}^{M} a^{(n)}(x_k, t_l)\Phi^{(n)}(y, z)$$

In this x_k section, the mean-flow field is deduced from

$$\overline{u(x_k, y, z)} = \sum_{n=1}^{M} \overline{a^{(n)}(x_k)}\Phi^{(n)}(y, z)$$

with

$$\overline{a^{(n)}(x_k)} = \frac{1}{N_t}\sum_{l=1}^{N_t} a^{(n)}(x_k, t_l)$$

Note that for $n > M$, $\overline{a^{(n)}(x_k)}$ is quasi equal to zero and hence, they cannot be taken into account for the calculation of the mean-flow field.

For each available measurement x_k plane, the first M POD mean coefficients $\overline{a^{(n)}(x_k)}$ are computed. The purpose of the spatial reconstruction procedure consists in interpolating and eventually extrapolating along the x direction (out of PIV measurement plane), these coefficients. Such reconstruction has to be performed according to the boundary conditions of the test-flow under investigation. At any x location of the space domain, each POD coefficient $\overline{a^{(n)\text{rec.}}(x)}$ is then accessed. Finally, the three-dimensional u-mean-component is estimated using the following equation:

$$\overline{u(x, y, z)} = \sum_{n=1}^{M} \overline{a^{(n)\text{rec.}}(x)}\Phi^{(n)}(y, z) \tag{1}$$

The interest of such reconstruction procedure is that only the mean POD coefficients are space-interpolated and original spatial POD eigenfunctions $\Phi^{(n)}(y, z)$ remain the same. Note that when N_x is sufficient to obtain a statistical convergence of the mean-velocity field correlation tensor, this POD reconstruction procedure can be directly applied to the mean-flow field.

3. Experimental set up and PIV measurements

The experimental apparatus including the in-cylinder flow configuration and PIV measurement system implementation is given on Fig. 1. A transparent quartz cylinder (diameter of 85 mm and height of 180 mm) where optical

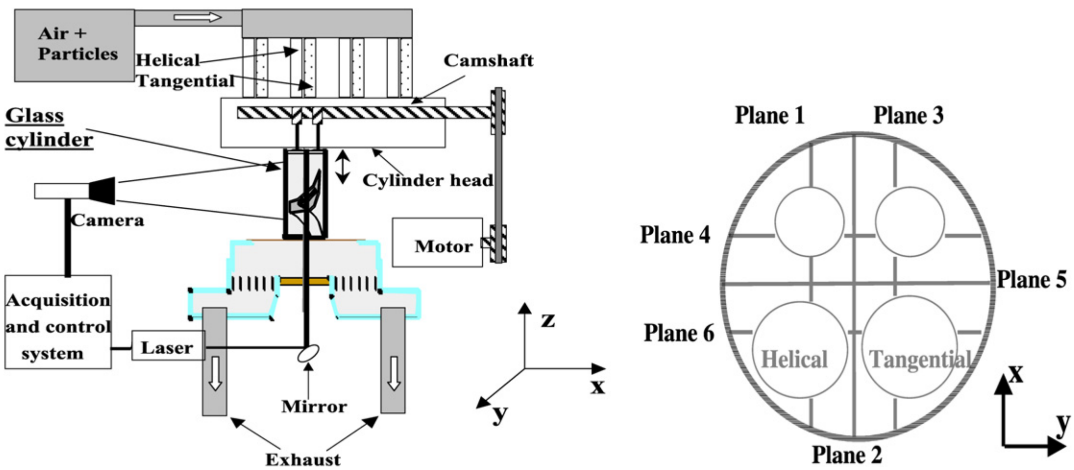

Fig. 1. Left-hand side: Experimental apparatus of in-cylinder flow configuration and PIV measurement system. Right-hand side: (xy)-plane view of the locations of the six 2D-2C PIV vertical planes.

access to the in-cylinder volume is possible allowing the PIV measurement of all velocity components is used (Fig. 1). A particular system has been implemented allowing the camera to move around the cylinder. To investigate the swirl generated by given both tangential and helical inlet ports, quasi-steady flow conditions are used. The flow analysis is then performed using a constant valve gap. Even though steady flow analysis is unable to predict well the real in-cylinder flow characteristics due to its dependence on piston interaction, it provides some important preliminary swirl investigations like the influence of both the design of the intake ports and the valve lift opening on the swirl generation [12]. The new reconstruction procedure is tested on a particular experimental flow configuration corresponding to a 5 mm valve lift for both tangential and helical inlet valves. A constant pressure drop (100 mbar) across the inlet valve is used, which is representative of in-service condition and in-cylinder turbulent flow field.

Particle Image Velocimetry (PIV) measurement technique is implemented to access the velocity field in six vertical planes (Fig. 1): (i) three (xz)-planes (planes 1 to 3 corresponding to y_1, y_2 and y_3 locations respectively); and (ii) three others in the (yz)-plane (planes 4 to 6 corresponding to x_1, x_2, x_3 locations respectively). The planes are equally spaced in both x and y-directions with a distance of 13.5 mm between two consecutive planes and $x_2 = y_2 = 0$. 600 instantaneous velocity fields are obtained in each vertical plane. A double-pulsed laser technique (YAG Spectra Physics, with a wavelength of 532 nm) is used. The positions of particles (oil droplets of 2–4 μm) are recorded using a CCD Kodak Megaplus camera with a resolution of 1008 × 1018 pixels. The average displacement for the particle pairs in each window is calculated using a cross correlation algorithm and is overlapped by 50%, producing 62 × 62 velocity vectors on a regular mesh grid with a step of 1.4 mm. The particles are seeded by aspirating air in a mixing tank upstream the inlet duct.

4. Application: 3D mean swirl flow reconstruction

The available PIV database is divided into two databases: (1) (u, w) obtained in planes 1 to 3; and (2) (v, w) obtained in planes 4 to 6. Based on both databases, two vectorial snapshot POD are performed called POD_{uw} and POD_{vw} respectively. Each POD allows the decomposition of the total velocity field into a sum of 1800 orthogonal eigenfunctions (deduced from the 600 available instantaneous velocity fields in each of the three parallel planes). Fig. 2 shows the convergence of both POD accumulated eigenvalues. The plot is limited to the first 100 modes in order to properly investigate the energy behavior of the first POD modes. A good convergence is observed for both POD due to the large number of available snapshots having a time space giving consistency in mode. The first 3 modes (corresponding to less than 0.02% of the total POD modes) capture 67% (POD_{vw}) and 70% (POD_{uw}) of the total energy. The small differences observed in these mode energy distributions are directly linked to the in-cylinder swirl flow field where (u, v) distribution is quite different.

Fig. 2 represents the second POD_{vw} coefficients $a^{(2)}(x_i, t)$ with $i = 1, \ldots, 3$. Dotted lines indicate the standard deviation of each coefficient relative to its mean value. Similar representations are obtained for $a^{(1)}(x_i, t)$ and $a^{(3)}(x_i, t)$ coefficients and also for the first three POD_{uw} coefficients.

In order to access the 3D representation of the mean-flow field, the POD cut-off number M, separating the mean and the fluctuating flow fields has to be determined. Two analyses are performed for such determination. First, from

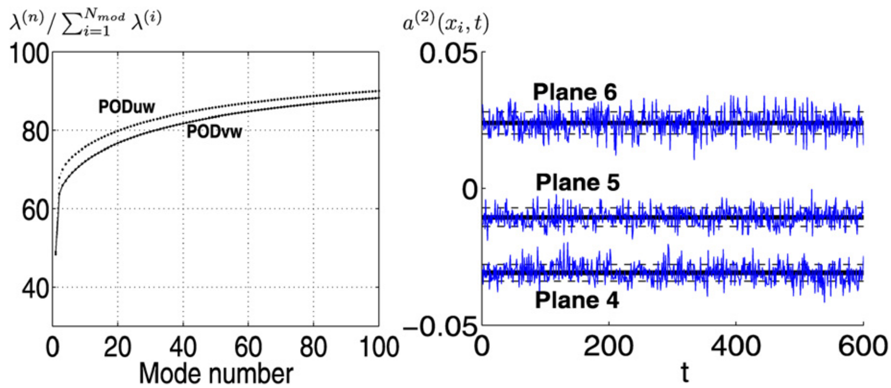

Fig. 2. Left-hand side: POD energy convergence. Right-hand side: Evolution of $a^{(2)}(x_i, t)$ for i varying from 1 to 3.

the preliminary knowledge of the mean velocity field deduced from raw PIV measurements, it is possible to compare it directly to the mean-flow field extracting from instantaneous velocity fields with the first POD modes. Second, the analysis of the standard deviation of the first POD coefficients is performed. Indeed, previous analyses of the engine-flow cyclic variability based on the POD coefficient analysis shown that the standard deviation of the POD coefficients corresponding to the mean-flow field is smaller than the one relative to other POD coefficients [13]. In this work, the remainder POD coefficients $a^{(n)}$ with $n > 3$ have higher standard deviations than the first three POD modes. Finally, the number $M = 3$ is retained. This number is coherent with current flow analysis. We need as many modes as the number of planes in each direction because each of these three modes taken separately is equivalent to the fundamental POD mode of a set of data in one of the three parallel planes. After the calculation of the mean values of the first three POD_{uw} and POD_{vw} coefficients in each corresponding plane, a spline interpolation/extrapolation is performed in the cross-section of the flow, taking into account the free slip condition on the cylinder boundary i.e. each POD coefficient tends to zero value on this boundary. Based on this spatial reconstruction, each mean-flow component is reconstructed using Eq. (1). Note that the w velocity component is reconstructed from both snapshot PODs because this velocity component is measured in each PIV measurement plane. Very little differences are observed between both spatial reconstructions. Finally, the entire mean-velocity field is reconstructed allowing an investigation of the 3D swirl flow structure.

The Q-criterion used in vortex identification technique [14] is investigated due to its ability in locating regions where rotation dominates strain in the flow. This scalar corresponds to the second invariant of the velocity gradient tensor. Based on the 3D mean-flow field, the scalar Q is computed, using a second order centered difference scheme for spatial derivatives (Fig. 3). Recall that the combination of two inlet ports (tangential and helical) supports the formation of the swirl structure. Indeed, the swirl motion is initiated in the helical port and the tangential jet (arising from the directed port) brings the swirl motion in the tangential direction. In this stationary flow test, we observe a realistic 3D representation of the spatial motion of the swirl structure along the main flow direction. Near the inlet valve, the well established swirl structure is small with weak intensity, and it is centered on the cylinder axis. During its spatial development, the vortex intensity increases and the vortex centre displaces from the cylinder centre while remaining close to the cylinder axis. We observe that the swirl structure evolution is mainly driven by the inlet tangential jet. As a consequence, the structure grows with an ellipsoidal shape. The location of the mean vortex centre as well as the swirl intensity are in an excellent agreement with previous swirl analyses performed in one cross-section of the cylinder [12], validating this new 3D reconstruction procedure. Finally, note that this POD reconstruction method has also been compared with mathematical methods like cubic spline interpolation. Due to the distance between two consecutive measurement planes, results obtained from spline reconstruction procedure differ from those deduced from POD procedure. Indeed, POD procedure based on physical concepts has the advantage of taking into account the whole available mean-flow information *via* the velocity correlation tensor.

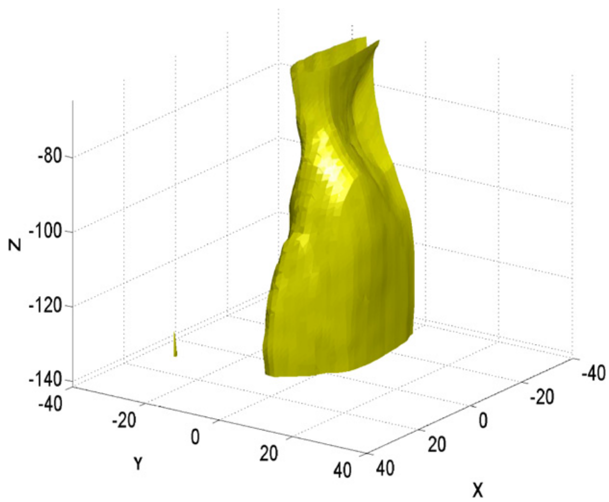

Fig. 3. Isosurface of the Q criterion ($Q = 0.02$).

5. Conclusion

A new application of the POD procedure is provided for reconstructing the 3D mean-velocity field from the knowledge of the PIV measurements performed in selected aligned or orthogonal planes. An application is proposed from PIV database obtained in in-cylinder engine flow. An analysis of the Q-criterion of the resulted 3D mean-velocity field shows the ability of such reconstruction procedure in providing realistic 3D mean swirl structure. In conclusion, the coupling PIV-POD approach constitutes a valid method for the 3D representation of the mean-velocity field. Some future applications are then conceivable. Based on classical dual PIV measurements or on instantaneous multiple simultaneous 2D-PIV measurements, it is then possible to reconstruct the entire 3D instantaneous velocity field by using the same POD reconstruction methodology.

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