



Streamline splitting between thermodynamics and hydrodynamics in a compositional gas–liquid flow through porous media

Sergey Oladyshkin, Mikhail Panfilov *

LEMMA UMR 7563, École de géologie – INPL, 2, avenue de la forêt de Haye, BP160, 54504 Vandoeuvre les Nancy, France

Received 28 November 2006; accepted 12 December 2006

Available online 8 January 2007

Presented by Charles-Michel Marle

Abstract

A method of splitting the thermodynamics and hydrodynamics in the model of two-phase compositional flow with phase transitions through porous media is proposed. This enables us to obtain effective solutions to flow problems along streamlines. The split independent thermodynamic subsystem includes new thermodynamic differential equations responsible for the mass balance in an open thermodynamic system. The split hydrodynamic subsystem contains only two flow equations with respect to pressure and saturation. The model is validated numerically. *To cite this article: S. Oladyshkin, M. Panfilov, C. R. Mecanique 335 (2007).* © 2006 Académie des sciences. Published by Elsevier Masson SAS. All rights reserved.

Résumé

Découplage de la thermodynamique et hydrodynamique suivant les lignes de courant dans un écoulement gaz–liquide compositionnel en milieux poreux. On propose une méthode de découplage de la thermodynamique et l'hydrodynamique dans le modèle compositionnel diphasique avec transition de phases, ce qui permet d'obtenir des solutions efficaces semi-analytiques des problèmes d'écoulement suivant les lignes de courant. Le modèle thermodynamique découplé est indépendant et contient des nouvelles équations différentielles thermodynamiques responsables pour le bilan de masse dans un système thermodynamique ouvert. Le modèle hydrodynamique découplé contient deux équations, pour la pression et la saturation. Le modèle est validé numériquement. *Pour citer cet article : S. Oladyshkin, M. Panfilov, C. R. Mecanique 335 (2007).*

© 2006 Académie des sciences. Published by Elsevier Masson SAS. All rights reserved.

Keywords: Porous media; Two-phase flow; Gas-condensate; Gas–liquid; Phase transition; Compositional

Mots-clés: Milieux poreux ; Écoulement diphasique ; Gaz-condensate ; Gaz–liquide ; Transition de phase ; Compositionnel

1. Introduction

Many engineering applications concerning the subsurface flow in porous reservoirs deal with the so-called two-phase gas–liquid compositional systems which consist of a large number of chemical components which determine a continuous mass exchange between the phases due to a changing dissolution degree of each component in each phase

* Corresponding author.

E-mail address: Mikhail.Panfilov@ensg.inpl-nancy.fr (M. Panfilov).

at various pressures and temperatures. The typical examples of such systems are the oil reservoirs with liberating gas, gas-condensate reservoirs, miscible enhanced oil recovery, underground storages of CO₂, methane, hydrogen and helium in aquifers, coal beds saturated with methane and water, groundwater with liberating air, and so on.

The full mathematical model of a compositional flow consists of a great system of mass and momentum balance equations coupled with the phase equilibrium and phase state relations, which global dimension is proportional to the number of chemical components. Resolution of such a hydrodynamic system requires to solve simultaneously the thermodynamic nonlinear high-order subsystem at each space point and at each time instant. The global algorithm becomes so complex that frequently can not be realized in practice, when leading to a CPU memory/lack of time, algorithm divergences or numerical instabilities. Various attempts to develop approximate simplified models may be grouped in two tendencies.

The first consists in reducing the thermodynamic block by reducing the number of chemical components up to two or three. The binary and ternary mixtures are frequently used in theoretical research, but are not very appreciated in practical studies.

The second tendency concerns the flow equations and consists in using the steady-state solutions [1]. However in [2], the steady-state solutions were shown to be unstable for compositional gas–liquid systems and were replaced by a so-called semi-stationary model.

In the present Note we show that the general compositional model may be split into an independent thermodynamic model and a significantly reduced hydrodynamic part. Due to such a splitting, we obtain an effective semi-analytical solution to the two-phase compositional problem.

2. Compositional model of two-phase flow in porous media

Let us consider a mixture consisting of N chemical components which form two various thermodynamic phases (gas and liquid) moving in porous medium.

According to the phenomenological approach [3], the general isothermal dynamic compositional model represents N equations of mass balance for each chemical component and two equations of momentum balance for each phase in a domain Ω :

$$\varepsilon \phi \frac{\partial \rho^{(k)}}{\partial t} = \text{div}(\Psi_g^{(k)} \text{grad } \mathcal{P}_g + \omega \Psi_l^{(k)} \text{grad } \mathcal{P}_l), \quad k = 1, \dots, N$$

$$\rho^{(k)} \equiv \rho_l c_l^{(k)} s + \rho_g c_g^{(k)} (1 - s), \quad \Psi_i^{(k)} \equiv \frac{K k_i \rho_i c_i^{(k)}}{\mu_i}, \quad \mathcal{P}_i \equiv P_i + \rho_i g z; \quad i = g, l \quad (1)$$

The momentum balance equations which are already substituted into (1) represent Darcy's law for each phase (V is the Darcy velocity):

$$V_l = -\omega \frac{K k_l}{\mu_l} \text{grad } \mathcal{P}_l, \quad V_g = -\frac{K k_g}{\mu_g} \text{grad } \mathcal{P}_g$$

where superscript ' k ' refers to k th chemical component ($k = 1, \dots, N$), indexes g and l to gas and liquid; ϕ is the porosity; $c_g^{(k)}$ or $c_l^{(k)}$ is the mass concentration of k th component in gas or liquid [kg/kg]; s is the volume saturation of pores by liquid; ρ_g and ρ_l are the phase densities; $\rho^{(k)}$ is the total partial density of the component k ; k_g and k_l are the relative permeabilities; μ is the phase viscosity; K is the absolute permeability; P is the phase pressures. The components are assumed to be arranged in ascending order of the molecular weight, so that the component 1 is the lightest, whilst the component N is the heaviest; z is the vertical coordinate, g is the gravity acceleration.

System (1) has a dimensionless form. The characteristic scales for the space coordinate, time, pressure, permeability, porosity, gas and liquid viscosities and phase density are defined as: the domain size L , the characteristic time of the process t_* , a characteristic pressure P^0 , the mean permeability $\langle K \rangle$, the mean porosity $\langle \phi \rangle$, the mean gas and the mean liquid viscosities μ_g^0 and μ_l^0 , and the mean gas density ρ_g^0 respectively. Due to this two dimensionless parameters arise in (1):

$$\varepsilon = \frac{t^*}{t_*} \quad \text{and} \quad \omega = \frac{\rho_l^0 \mu_g^0}{\rho_g^0 \mu_l^0} \quad (2)$$

where $t^* \equiv L^2 \langle \phi \rangle \mu_g^0 / (\langle K \rangle P^0)$ is the characteristic time of pressure stabilization in the overall domain, which is an eigen intrinsic parameter of the system.

The system of N equations (1) is formulated with respect to N basic variables:

$$P_g, \quad s, \quad \chi^{(j)} = c_g^{(j+1)} \quad (j = 1, \dots, N-2) \quad (3)$$

The $N-2$ independent concentrations $\chi^{(k)}$ will be called ‘the concentration basis’. The selection of the intermediary components as the basic ones is arbitrary.

The closure relationships for system (1) describe the locally equilibrium thermodynamic behavior. They consist of N equilibrium equations (4) for each chemical component, two equations of phase state (5), two normalizing relations (5) and the equilibrium equation for phase pressures in the capillary field:

$$v_g^{(k)}(P, c_g^{(1)}, \dots, c_g^{(N)}) = v_l^{(k)}(P, c_g^{(1)}, \dots, c_g^{(N)}), \quad k = 1, \dots, N \quad (4)$$

$$\rho_i = \rho_i(P, c_i^{(1)}, \dots, c_i^{(N)}), \quad \sum_{k=1}^N c_i^{(k)} = 1, \quad i = g, l; \quad P_g - P_l = P_c(s) \quad (5)$$

where $v_i^{(k)}$ is the chemical potential of k th component in i th phase; $P_c(s)$ is the capillary pressure function. Equations of state are written in a general form. Functions $v_g^{(k)}(P, \dots)$ are given.

We assume that the capillary pressure P_c may be neglected with respect to the characteristic pressure difference over the domain, ΔP .

Subsystem (4), (5) does not contain saturation s , as the phase densities and concentrations are independent of s . This system includes $N+4$ equations with respect to $2N+3$ variables. The difference $v = (2N+3) - (N+4) = N-1$, determines the number of independent variables in the thermodynamic part. We select the pressure and $N-2$ basic concentrations mentioned above as the set of the independent variables. Due to this, the phase densities and other concentrations which do not enter in the basis (3) depend on P and $N-2$ basic concentrations $\chi^{(q)}$, which constitutes the Gibbs rule of phase.

For a gas–liquid system, the relative phase mobility parameter ω is always small, of order 0.01–0.05. We will examine also the processes where the pressure is a slow varying function in time, so that the characteristic process time t_* is much greater than the proper system time t^* . Due to this, both parameters (2) are small.

3. Canonical form of the compositional model

A system is called to be formulated in a canonical form, if the leader derivatives of each differential equation describe the behaviour of one physical basic variable from the list (3) in such a way that all the equations describe different variables.

The transformation of (1) into the canonical form is performed while transforming each equation step-by-step, by excluding progressively the leading derivatives (or their main parts) from the preceding equations. The main part of a derivative is that which does not contain the small factors.

At the first step, the sum of all the equations provides a total mass balance equation with a leading derivative with respect to pressure:

$$\varepsilon \phi \frac{\partial \rho}{\partial t} = \text{div}([\omega \Psi_l + \Psi_g] \text{grad } \mathcal{P}) \quad (6)$$

where $\rho \equiv \rho_l s + \rho_g (1-s)$ is the total density. This equation will replace the first equation in (1).

At the second step, in all the remaining equations the main term of the second-order derivative with respect to pressure $\text{div}(\Psi_g \text{grad } \mathcal{P})$ is excluded while using (6):

$$\varepsilon \phi \left[\frac{\partial \rho^{(k)}}{\partial t} - c_g^{(k)} \frac{\partial \rho}{\partial t} \right] = \omega \Delta c^{(k)} \text{div}(\Psi_l \text{grad } \mathcal{P}) + \omega \Psi_l \text{grad } c_l^{(k)} \text{grad } \mathcal{P} + \Psi_g \text{grad } c_g^{(k)} \text{grad } \mathcal{P} \quad (7)$$

where $\Delta c^{(k)} \equiv c_l^{(k)} - c_g^{(k)}$, $k = 2, \dots, N$.

At the third step, the leading derivative $\text{div}(\Psi_l \text{grad } \mathcal{P})$ is excluded from the first $N-2$ equations (7) by subtracting the last equation taken at $k = N$:

$$\begin{aligned} & \varepsilon \phi \left[\rho_l s \frac{\partial}{\partial t} (\zeta_l^{(k)} - \zeta_l^{(N)}) + \rho_g (1-s) \frac{\partial}{\partial t} (\zeta_g^{(k)} - \zeta_g^{(N)}) \right] \\ & = \omega \Psi_l \text{grad } \mathcal{P} \cdot \text{grad} (\zeta_l^{(k)} - \zeta_l^{(N)}) + \Psi_g \text{grad } \mathcal{P} \cdot \text{grad} (\zeta_g^{(k)} - \zeta_g^{(N)}), \quad k = 2, \dots, N-1 \end{aligned} \quad (8)$$

where new functions are introduced, $\zeta_g^{(k)}$ and $\zeta_l^{(k)}$, called *the component neutralities* and defined as:

$$d\zeta_l^{(k)} \equiv \frac{1}{\Delta c^{(k)}} dc_l^{(k)}, \quad d\zeta_g^{(k)} \equiv \frac{1}{\Delta c^{(k)}} dc_g^{(k)} \quad (9)$$

Such a function describes how much the concentration variation within a fixed phase is higher/lower than the concentration variation between two phases. For a neutral component, the concentration variation in each phase is equal to the concentration difference between the phases [4], so ζ tends to a constant value of order 1.

4. HT-splitting

Subsystem (8) describing the transport of the basic concentrations can be transformed into a thermodynamic one along streamlines, if two conditions are satisfied: (i) the phases have a contrasting mobility, i.e. parameter ω is small; this is always satisfied for gas–liquid systems; (ii) the process characterized by a slow pressure variation in time is examined, i.e. parameters ω is small. Such a second condition is always satisfied for the processes of enhanced oil recovery which objective is to maintain pressure at a fixed level, as well as for any long-term quasi-stationary processes (with respect to pressure). In this case, subsystem (8) takes the form:

$$\text{grad } \mathcal{P} \cdot \text{grad} (\zeta_g^{(k)} - \zeta_g^{(N)}) = 0, \quad k = 2, \dots, N-1 \quad (10)$$

Let us rewrite (10) along a streamline y (for each $k = 2, \dots, N-1$):

$$\frac{\partial \mathcal{P}}{\partial y} \frac{\partial}{\partial y} (\zeta_g^{(k)} - \zeta_g^{(N)}) = 0 \quad \Rightarrow \quad \frac{\partial}{\partial y} (\zeta_g^{(k)} - \zeta_g^{(N)}) = 0 \quad (11)$$

This means that each function $\Phi^{(k)} \equiv (\zeta_g^{(k)} - \zeta_g^{(N)})$ (for each k) does not depend on y :

$$\Phi^{(k)}(P, \chi^{(1)}, \dots, \chi^{(N-2)}) = \Phi_0^{(k)}, \quad k = 2, \dots, N-1 \quad (12)$$

where $\Phi_0^{(k)}$ are constant values which may depend on time as on a parameter. These values should be known a priori from the boundary-value conditions imposed at a boundary point of each streamline. In the last relation we taken into account that the neutrality functions do not depend on the saturation.

Eqs. (12) represent an algebraic system of $N-2$ equations containing $N-1$ basic variables $P, \chi^{(1)}, \dots, \chi^{(N-2)}$. So this system determines all the basic concentrations as the functions of pressure only:

$$\chi^{(q)} = \chi^{(q)}(P), \quad q = 1, \dots, N-2 \quad (13)$$

Therefore the limit compositional system is monovariant along each streamline. Using property (12), we can now transform the transport equations (11) into thermodynamic differential equations:

$$0 = d\Phi^{(k)}(P, \chi^{(1)}(P), \dots, \chi^{(N-2)}(P)) = \frac{d\Phi^{(k)}}{dP} dP, \quad \frac{d\Phi^{(k)}}{dP} \equiv \frac{\partial \Phi^{(k)}}{\partial P} + \sum_{q=1}^{N-2} \frac{\partial \Phi^{(k)}}{\partial \chi^{(q)}} \frac{d\chi^{(q)}}{dP} \quad (14)$$

which yields the following thermodynamic differential subsystem in which the time and space variables are not present explicitly:

$$\frac{d}{dP} (\zeta^{(k)} - \zeta^{(N)}) \equiv \frac{1}{\Delta c^{(k)}} \frac{dc_g^{(k)}}{dP} - \frac{1}{\Delta c^{(N)}} \frac{dc_g^{(N)}}{dP} = 0, \quad k = 2, \dots, N-1 \quad (15)$$

which is called the ‘delta-law’. This system is a basic law which describes the mass balance in an open thermodynamic system.

The system of differential thermodynamic equations (15) completed with the boundary values $\Phi_0^{(k)}$ determines the basic concentrations $\chi^{(k)}$ as the functions of pressure. System (15) added to the thermodynamic relations (4)

constitutes a closed thermodynamic model with an only free parameter—the pressure. So, if the pressure is given, then this enlarged thermodynamic subsystem is totally independent of flow equations.

The remaining first two equations in the canonical form of the compositional model, (6) and (7) (taken for $k = N$) represents the split hydrodynamic model with respect to two variables: $P(x, t)$ and $s(x, t)$:

$$\varepsilon\phi \frac{\partial \rho(P, s)}{\partial t} = \text{div}([\omega\Psi_l(P, s) + \Psi_g(P, s)] \text{grad } \mathcal{P}) \tag{16a}$$

$$\varepsilon\phi \left[\frac{\partial \rho^{(k)}(P, s)}{\partial t} - c_g^{(k)}(P) \frac{\partial \rho(P, s)}{\partial t} \right] = \omega \Delta c^{(k)}(P) \text{div}(\Psi_l(P, s) \text{grad } \mathcal{P}) + \omega\Psi_l(P, s) \text{grad } c_l^{(k)}(P) \text{grad } \mathcal{P} + \Psi_g(P, s) \text{grad } c_g^{(k)}(P) \text{grad } \mathcal{P} \tag{16b}$$

which can be simplified, if necessary, while using the fact that ω and ε are small.

5. Numerical validation to the split model

To validate the delta-law (15) and the model (16), we have simulated the full non-split problem of radial flow of a three-component gas-condensate system (methane–ethane–decane) to a production well by using the simulator ECLIPSE. The system was discretized with the finite difference method, a fully implicit scheme, with refining the numerical grid in the vicinity of the well. The classic thermodynamic subsystem was based on the Peng–Robinson equation of state. The results obtained for pressure, saturation and concentrations along one streamline were treated in terms of the functions $c_g^{(k)}(P)$ and $c_l^{(k)}(P)$. Using these functions we have calculated functions $F_1 \equiv \frac{1}{\Delta c^{(k)}} \frac{dc_g^{(k)}}{dP}$ and $F_2 \equiv \frac{1}{\Delta c^{(N)}} \frac{dc_g^{(N)}}{dP}$ entering in the delta-law.

In Fig. 1(a) two curves correspond to functions F_1 and F_2 respectively. The equivalence between these two curves is more than satisfactory, which justifies the validity of the limit thermodynamic model (15).

The similar radial 1D flow problem was also simulated in an other way, by using the split hydrodynamic model (16). All the thermodynamic parameters of these two equations were calculated by using the split thermodynamic model (4) and (15) as the functions of pressure. We applied the implicit discretization in time for the first equation, and the explicit scheme for the second one. The spacial discretization was performed by volume elements with using a logarithmic grid. The initial state of the reservoir was gaseous; the condensate appears when pressure decreases.

The liquid saturation along the radial coordinate (the flow is oriented from the right to the left) is presented in Fig. 1(b) for one time instant. The comparison with the full compositional model (ECLIPSE-based simulation) illustrates a very satisfactory fit.

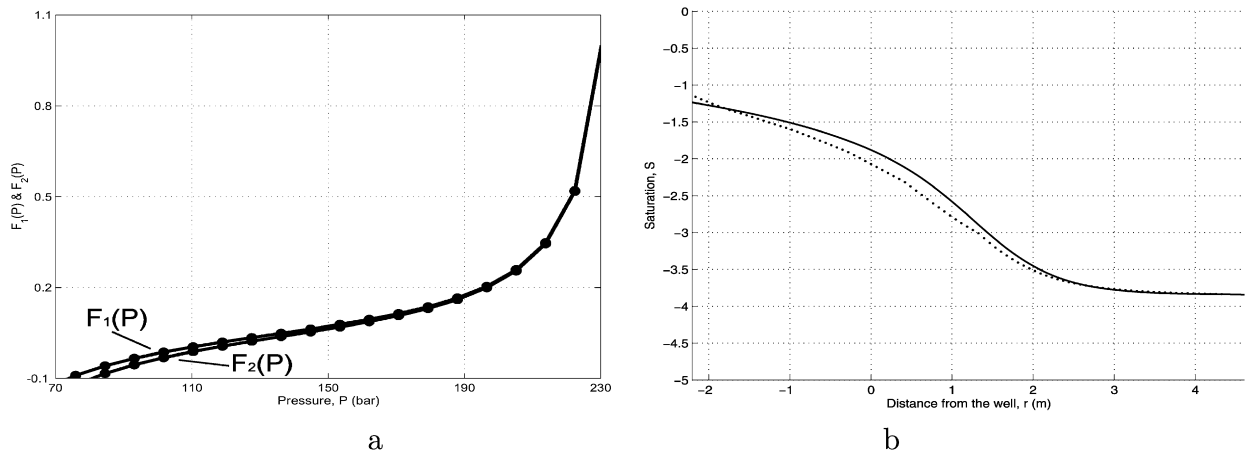


Fig. 1. Validation of the delta-law (a) and liquid saturation in the vicinity to a well (b): the full compositional model (dotted curve) and the split model (solid curve).

Due to HT-splitting we reach a significant gain in the simulation speed. In the examined case, the ratio between the simulation time needed for the split model and that for the full model is 1:1500.

Acknowledgements

The research was financed partially by the Groupement de Recherche MOMAS: Modélisation mathématique et simulations numériques liées aux études d'entreposage souterrain de déchets radioactifs (CNRS, ANDRA, EDF, CEA, BRGM) and the Schlumberger Abingdon Technology Center.

References

- [1] H.G. O'Dell, R.N. Miller, Successfully cycling a low-permeability, high-yield gas-condensate reservoir, *JTP* (January 1967) 41–47.
- [2] S. Oladyshkin, M. Panfilov, Two-phase flow with phase transitions in porous media: instability of stationary solutions and a semi-stationary model, in: *Third Biot Conference on Poromechanics*, Norman, Oklahoma, USA, 2005, pp. 529–535.
- [3] K. Aziz, A. Settari, *Petroleum Reservoir Simulation*, Applied Science Publishers, London, 1979.
- [4] I. Panfilova, M. Panfilov, Near-critical gas–liquid flow in porous media: monovariant model, analytical solutions and convective mass exchange effects, *Transport in Porous Media* 56 (1) (July 2004) 61–85.