

# Strength of a granular medium reinforced by cement grouting

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## Abstract

The present Note describes an experimental study devoted to the strength of a sand reinforced by cement grouting. Through grouting, the granular medium gains a cohesion without significant change of the friction angle. The most significant experimental feature is that the cohesion is proportional to the volume fraction of cement in the grouted material. This result is interpreted within the framework of a periodic homogenization applied to yield design. *To cite this article: Y. Maalej et al., C. R. Mecanique 335 (2007).*

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## Résumé

**Résistance d'un milieu granulaire renforcé par injection de ciment.** On présente une étude expérimentale consacrée aux capacités de résistance d'un sable injecté par un coulis de ciment. L'effet principal de l'injection est un gain de cohésion sans modification significative de l'angle de frottement. La principale observation expérimentale réside dans le fait que la cohésion est proportionnelle à la fraction volumique de ciment réalisée dans le matériau injecté. On interprète ce résultat dans le cadre de l'homogénéisation périodique en Calcul à la Rupture. *Pour citer cet article : Y. Maalej et al., C. R. Mecanique 335 (2007).*

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*Mots-clés:* Milieux granulaires ; Injection de ciment ; Critère de rupture ; Calcul à la rupture ; Homogénéisation

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## 1. Introduction

Cement grouting by impregnation in granular media is a widely used technique in civil engineering, applied in order to improve the mechanical characteristics of soils [1]. The idea consists in incorporating a pressurized cement grout in the pore space of the soil. The setting of cement in the pore space increases both the stiffness and the strength [2,3].

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The resulting microstructure is a heterogeneous material made up of sand grains, cement and pores. Note that the injection by impregnation method does not modify the structure of the granular assembly.

Several experimental studies on reference sand have been devoted to the increase of the strength due to cement grouting [3,4]. These works show that the grouted material remains a frictional one, the strength of which is correctly modelled by Mohr–Coulomb criterion. Grouting is mainly responsible for the cohesion gained by the material and only slightly affects the friction angle.

The present Note describes the results of an experimental study devoted to the strength of a grouted Fontainebleau sand, with special focus on the influence of the cement volume fraction. It confirms the qualitative trends already reported on the nature of the failure criterion of grouted materials. The main contribution of the present work is the experimental and theoretical analysis of the influence of the cement volume fraction on the cohesion gained through grouting.

**2. Experimental set up**

The samples of grouted sand are obtained from the grouting set up described in [5,6]. A Plexiglas column (height 108 cm, diameter 8 cm) is first filled with Fontainebleau sand by a pluviation technique at a density index  $I_d = 0.9$ , corresponding to an initial porosity  $\phi_0 = 0.37$ . After water saturation of the column, a fine cement grout (Spinor A12) is then injected from the bottom to the top. The cement/water mass ratio, denoted by  $C/E$ , is controlled during injection. In the present study, the values  $C/E = 0.1$  and  $0.2$  have been considered.

After injection of the grout, the column is kept under wet conditions during 28 days. It is finally cut into five  $160 \times 80$  mm samples, numbered from 1 to 5 from the bottom to the top (see Fig. 1(a)).

The samples are then water saturated and subjected to a confining pressure in a triaxial apparatus. Drained triaxial compression tests under constant confining pressure are then performed. The limit load is measured as a function of the confining pressure. In addition, triaxial tests have been performed on pure Fontainebleau sand samples prepared at the same density index  $I_d = 0.9$ .

**3. Experimental results**

The purpose of the experimental study is to evaluate the influence of the cement volume fraction  $\phi_c$  on the strength. The final porosity  $\phi_f$  is determined by mercury porosimetry for each level in the column [2]. Clearly enough, the cement volume fraction  $\phi_c$  is equal to the decrease  $\phi_0 - \phi_f$  of the porosity.  $\phi_c$  decreases as the distance to the injection source increases (Fig. 1(b)).

Identical triaxial tests have been performed for each level in the column. Fig. 2(a) presents the results of five triaxial compression tests corresponding to five different confining pressure  $p_c$  ranging from 100 to 1000 kPa. These

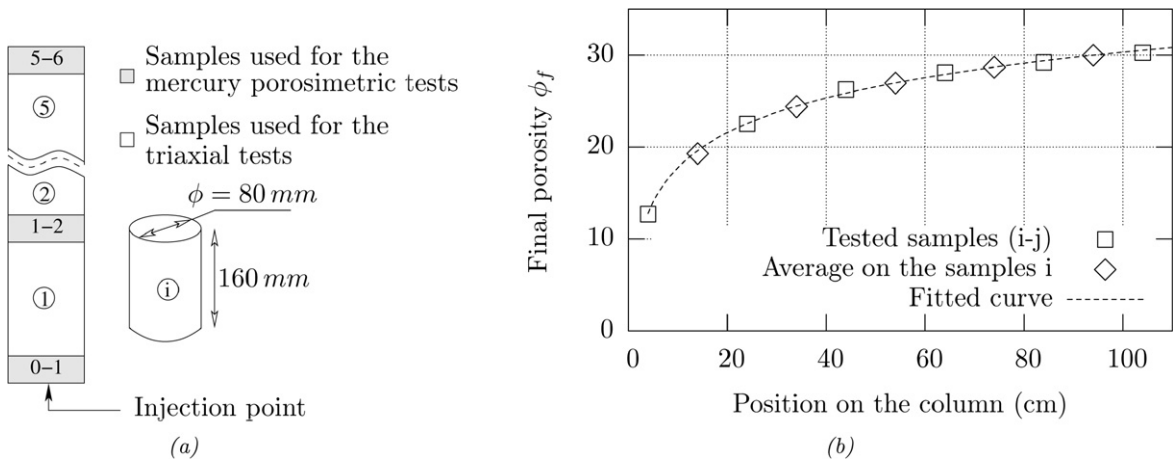


Fig. 1. (a) Injected sand column with numbered samples; (b) final porosity  $\phi_f$  as function of the location of sample along the column.

Fig. 1. (a) Schéma de découpage de la colonne ; (b) porosité finale  $\phi_f$  en fonction de la position dans la colonne.

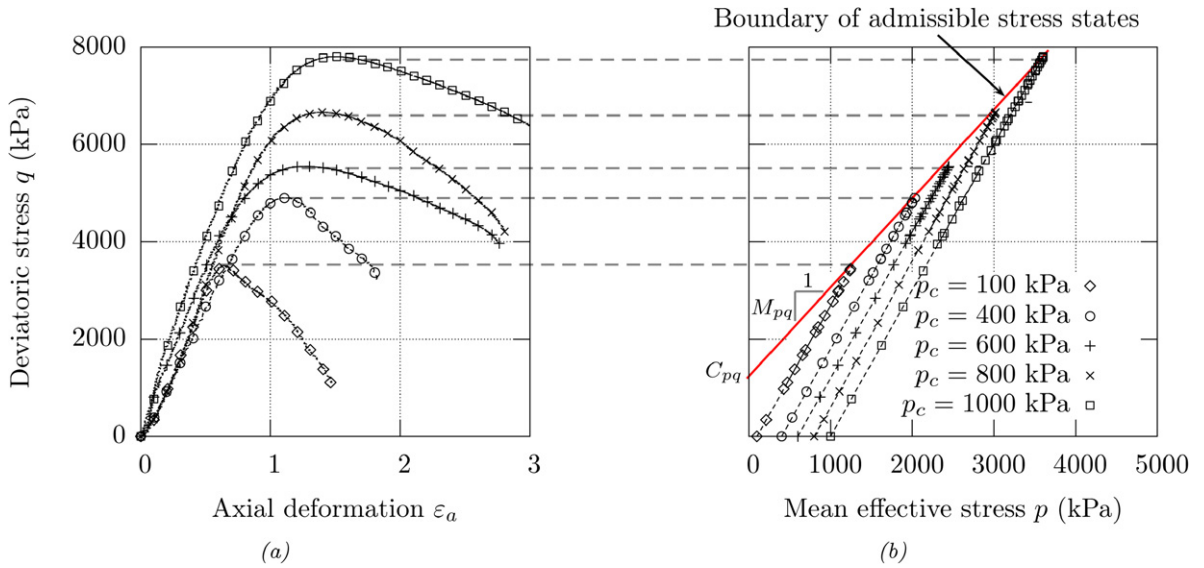


Fig. 2. Triaxial tests results (a) and failure criterion (b) ( $p_c$  is the initial confining pressure applied in each tests).

Fig. 2. Essais triaxiaux et détermination du critère de rupture, (a) plan du déviateur de contrainte ; (b) plan de contrainte moyenne,  $p_c$  désigne la pression de confinement initiale correspondante à chaque essai.

tests have been performed on samples of level 2 taken from five different columns prepared identically. Fig. 2(b) plots the stress paths in the classical  $(p, q)$  plane, where  $p = -\frac{1}{3} \text{tr} \sigma$  is the mean effective stress and  $q$  is the difference between the axial compression stress and the confining pressure. The boundary of the admissible stress states in this plane is found to be a straight line.

Disregarding the possible role of the intermediate principal stress (which cannot be investigated with the classical triaxial apparatus), this result confirms that the strength of a grouted sand under compressive loading is correctly described by a Mohr–Coulomb criterion (friction angle  $\varphi^{\text{inj}}$ , cohesion  $C^{\text{inj}}$ ). These two parameters are classically related to the slope  $M_{pq}$  and to the ordinate at the origin  $C_{pq}$  by:

$$\varphi^{\text{inj}} = \arcsin\left(\frac{3M_{pq}}{6 + M_{pq}}\right) \quad \text{and} \quad C^{\text{inj}} = \frac{3 - \sin \varphi^{\text{inj}}}{6 \cos \varphi^{\text{inj}}} C_{pq} \quad (1)$$

Although the tensile strength of the injected material was not investigated into detail, it seems reasonable to neglect the latter as compared to the cohesion  $C^{\text{inj}}$  itself. As a consequence, the injected material should be regarded as a Mohr–Coulomb material with no tensile strength.

The data plotted in Fig. 3 concerning the influence of  $\phi_c$  on  $C^{\text{inj}}$  and  $\varphi^{\text{inj}}$  represent the original experimental contribution of the present Note. Each point has required five triaxial tests of the kind presented in Fig. 2. It corresponds to a specific cement volume fraction, associated with  $C/E = 0.1$  or  $0.2$  and with a particular level in the column (levels 1, 3 and 5 for  $C/E = 0.1$ , and levels 1 to 5 for  $C/E = 0.2$ ).

The most significant conclusions that can be drawn from Fig. 3 are: (i) the linearity of the variations of  $C^{\text{inj}}$  with respect to  $\phi_c$  (see Fig. 3(a)); and (ii) the order of magnitude of the cohesion gained by grouting (several hundred of kPa). Fig. 3(b) also reveals that the friction angle  $\varphi^{\text{inj}}$  is a slightly increasing function of  $\phi_c$ , the maximum discrepancy with respect to the friction angle  $\varphi^s$  of pure sand being of the order of 5 degrees. The influence of this phenomenon on the strength of the grouted sand is however much weaker than the increase of the cohesion. Indeed, for typical values of the mean stress  $p$ , say 100 kPa, the deviatoric stress  $q = pM_{pq}$  varies from  $q = 168$  kPa for  $\varphi^s = 41^\circ$  to  $q = 189$  kPa for  $\varphi^{\text{inj}} = 46^\circ$ , that is, increases by 21 kPa. This increase is negligible with respect to that induced by the cohesion, which can be as high as 900 kPa.

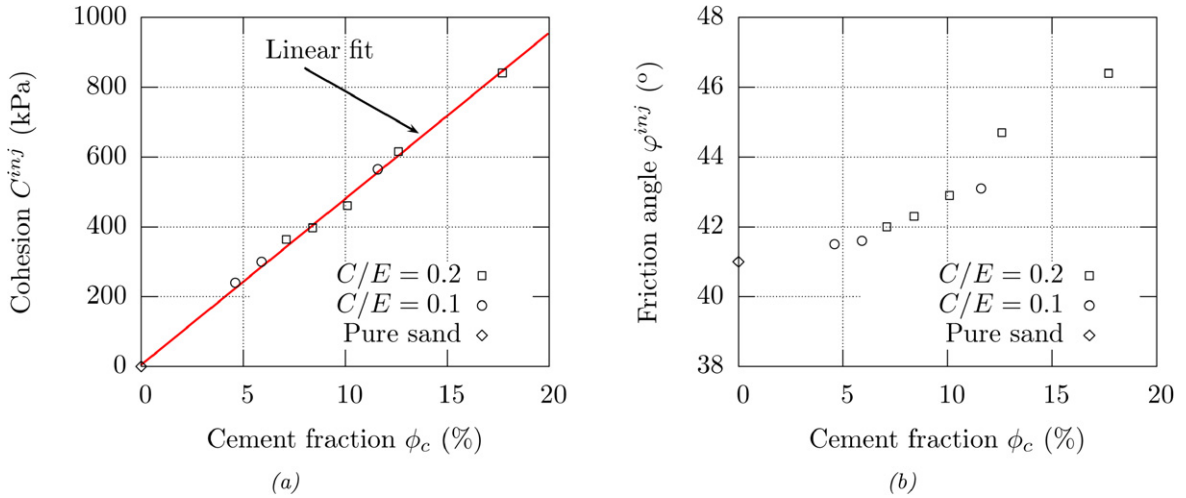


Fig. 3. The variation of the cohesion  $C^{inj}$  (a) and the friction angle  $\varphi^{inj}$  (b) as function of the cement fraction  $\phi_c$ .  
 Fig. 3. (a) Variation de la cohésion  $C^{inj}$  en fonction de  $\phi_c$  ; (b) variation de l'angle de frottement  $\varphi^{inj}$  en fonction de  $\phi_c$ .

**4. Micromechanics interpretation**

The aim of this section is to provide a micromechanics basis to the proportionality observed between the cohesion  $C^{inj}$  and the cement volume fraction  $\phi_c$ . To this end, a periodic model of the microstructure is considered. The latter is characterized by the elementary cell  $\mathcal{E}$ . From a geometrical point of view, the pure sand is described as a set of spheres with identical radius  $a$ , in pointwise contact with one another. This kind of simplification is reasonable for a Fontainebleau sand. Moreover, it is assumed that the hydrated cement appears as a layer of thickness  $e \ll a$  covering the whole boundary of the grain (Fig. 4). Neglecting the second order in  $e/a$ , it is readily seen that this geometrical model implies

$$\phi_c = 3 \frac{e}{a} (1 - \phi_0) \tag{2}$$

It is emphasized that the reasoning presented hereafter can be implemented for any geometry of the set of spherical grains making up the elementary cell  $\mathcal{E}$ , whatever its complexity. Fig. 4 only describes the model adopted for the contact between two spherical grains, and is obviously not an elementary cell by itself.

Following the remark at the end of the previous section, the variation of the friction angle induced by cement grouting is neglected, so that  $\varphi^{inj}$  is taken equal to the friction angle  $\varphi^s$  of the pure sand.

Considering a macroscopic strain rate  $\mathbf{D}$ , the support function  $\Pi(\mathbf{D})$  of the homogenized criterion is derived by means of two different approaches. First, taking advantage of the experimental observations concluding that the

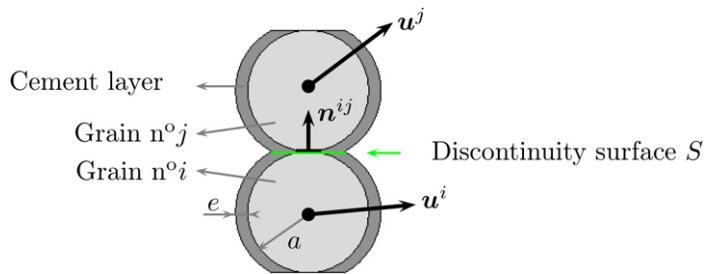


Fig. 4. Geometrical model and failure mechanism.  
 Fig. 4. Géométrie adoptée et mécanismes de rupture.

strength of the grouted sand can be described by a Mohr–Coulomb criterion with no tensile strength, the macroscopic derivation of  $\Pi(\mathbf{D})$  [7] reads

$$\Pi(\mathbf{D}) = C^{\text{inj}} \tan\left(\frac{\pi}{4} + \frac{\varphi^{\text{inj}}}{2}\right) \left(\sum_i |\mathbf{D}_i| - \text{tr } \mathbf{D}\right) \tag{3}$$

provided that  $\text{tr } \mathbf{D} \geq (\sum_i |\mathbf{D}_i|) \sin \varphi^{\text{inj}}$ , where  $D_i$  denote the eigenvalues of the strain rate  $\mathbf{D}$ .

Let us now examine the micromechanics interpretation to  $\Pi(\mathbf{D})$ . The first step consists in modelling the strength of the components of the microstructure. It is assumed that the strength of the grains is infinite and that the intergranular forces obey a cohesionless Mohr–Coulomb criterion, with friction angle  $\varphi^{\text{int}}$ . More precisely, let  $p^{ij}$  denote the contact point between grains  $n^\circ i$  and  $n^\circ j$ , and  $\mathbf{n}^{ij}$  be the unit normal to the contact plane, oriented from grain  $n^\circ i$  toward grain  $n^\circ j$ . The normal and tangential components to the force  $\mathbf{F}^{ij}$  applied on grain  $n^\circ i$  by grain  $n^\circ j$  read

$$F_n^{ij} = \mathbf{F}^{ij} \cdot \mathbf{n}^{ij}; \quad F_t^{ij} = |\mathbf{F}^{ij} - F_n^{ij} \mathbf{n}^{ij}| \tag{4}$$

and are subjected to the condition  $F_t^{ij} + \tan \varphi^{\text{int}} F_n^{ij} \leq 0$ . A Tresca criterion (cohesion  $C^c$ ) without tensile strength is adopted for the hydrated cement phase. Taking into account some non-vanishing tensile strength of the cement phase would clearly yield a tensile strength at the macroscopic scale. However, the latter has already been said to be negligible.

Let  $\mathcal{K}$  denote the set of velocity fields defined on  $\mathcal{E}$  at the microscopic scale, (i) which are kinematically admissible with  $\mathbf{D}$  and (ii) for which the maximum resisting work is finite for the given strength properties of the microstructure. Since the strength of the grains is infinite, this condition (ii) implies that the restriction of such a velocity field  $\mathbf{u}$  to a given grain is a rigid body motion. It can be shown [8] that it is possible, without loss of generality, to neglect the rotation component of the grains velocity, so that they are subjected to translations. Let  $\mathbf{u}^{ij}$  denote the discontinuity  $\mathbf{u}^j - \mathbf{u}^i$  at the contact point  $p^{ij}$ . The normal and tangential components of  $\mathbf{u}^{ij}$  read

$$u_n^{ij} = \mathbf{u}^{ij} \cdot \mathbf{n}^{ij}; \quad u_t^{ij} = |\mathbf{u}^{ij} - u_n^{ij} \mathbf{n}^{ij}| \tag{5}$$

The velocity field  $\mathbf{u}$  must be extended to the cement layers surrounding the grains in a way which must be compatible with the velocity discontinuities at the contact points. The simplest model consists in assuming that the cement phase surrounding grain  $i$  (resp.  $j$ ) is subjected to the same translation as this grain, so that we have to introduce a velocity discontinuity equal to  $\mathbf{u}^{ij}$ , located in the plane tangent to these grains at the point  $p^{ij}$  (Fig. 4). This model is relevant with respect to condition (ii) provided that  $u_n^{ij} \geq 0$ , which means that the contribution of this discontinuity to the resisting work is finite. The area  $S$  of the discontinuity surface in the tangent plane is  $\pi((a + e)^2 - a^2)$ , that is, neglecting the second order in  $e/a$  and using (2):

$$S = \frac{2\pi a^2}{3(1 - \phi_0)} \phi_c \tag{6}$$

The condition (ii) to be satisfied by the velocity fields of  $\mathcal{K}$  reduces to:

$$(\forall \{i, j\}) \quad u_n^{ij} \geq u_t^{ij} \tan \varphi^{\text{int}} \tag{7}$$

Since these conditions are not affected by the presence of cement in the microstructure, the set  $\mathcal{K}$  is itself independent of the cement volume fraction  $\phi_c$ . According to the micromechanics definition of  $\Pi(\mathbf{D})$  [9],  $\Pi(\mathbf{D})$  appears to be the sum of the contributions of each discontinuity zone in the cement phase:

$$\Pi(\mathbf{D}) = C^c \frac{S}{|\mathcal{E}|} \mathcal{U} \quad \text{with } \mathcal{U} = \inf_{\mathbf{u} \in \mathcal{K}} \left( \sum_{\{i,j\}} u_t^{ij} \right) \tag{8}$$

where  $|\mathcal{E}|$  denotes the volume of the elementary cell. The quantity  $\mathcal{U}$  involved in (8) only depends on the geometry of the pure sand microstructure, and is therefore independent of  $\phi_c$ . Comparing (3) and (8) reveals the existence of a coefficient  $\lambda(\mathcal{E})$ , purely geometrical in nature, having the dimension of a length, such that  $\mathcal{U} = \lambda(\mathcal{E}) (\sum_i |\mathbf{D}_i| - \text{tr } \mathbf{D})$ . This coefficient depends on the morphology of the elementary cell  $\mathcal{E}$ . Owing to (6), we eventually obtain

$$C^{\text{inj}} = c \phi_c \quad \text{with } c = \frac{C^c}{\tan(\frac{\pi}{4} + \frac{\varphi^{\text{inj}}}{2})} \frac{2\pi a^2 \lambda(\mathcal{E})}{3(1 - \phi_0) |\mathcal{E}|} \tag{9}$$

Within the framework of the approximation in which the variation of  $\varphi^{\text{inj}}$  is neglected, the above result establishes the proportionality between the cohesion of the grouted sand and the volume fraction of the cement paste introduced in the pore space. Once the morphology of the granular network is specified, that is, for a specific choice of the elementary cell,  $\lambda(\mathcal{E})$  can be determined and (9) can be used for predicting the cohesion gained through grouting.

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