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A 2D micromechanical modelling of anisotropy in granular media

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Abstract

We propose a two-dimensional modelling of the anisotropy of granular media based on the use of a second order or a fourth order fabric tensor describing the distribution of the contact probability. This fabric tensor-based approach is then combined with a new kinematic localization rule and yields an efficient homogenization scheme for anisotropic granular media. ***To cite this article: O. Millet et al., C. R. Mecanique 335 (2007).***

Résumé

Une modélisation micromécanique de l'anisotropie des milieux granulaires. On propose une modélisation bidimensionnelle de l'anisotropie des milieux granulaires basée sur l'utilisation d'un tenseur de texture d'ordre deux ou d'ordre quatre pour décrire la distribution de probabilité de contacts. Cette approche est ensuite combinée avec une nouvelle règle de localisation cinématique pour construire un schéma d'homogénéisation pertinent pour les milieux granulaires anisotropes. ***Pour citer cet article : O. Millet et al., C. R. Mecanique 335 (2007).***

Keywords: Granular media; Anisotropy; Fabric tensor; Micromechanics

Mots-clés : Milieux granulaires ; Anisotropie ; Tenseur de texture ; Micromécanique

Version française abrégée

En général, la configuration de l'assemblage granulaire est considérée comme la caractéristique essentielle de l'état interne d'un milieu granulaire [1,2]. Au niveau local, les variables pertinentes sont la coordinance (nombre de particules voisines en contact) et le tenseur de texture de contact qui permet de décrire l'anisotropie de l'assemblage granulaire.

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Il a été montré dans différents travaux antérieurs (cf. par exemple [3,4] et la Section 2.2 de cette Note) que l'utilisation d'un tenseur de texture d'ordre quatre permet d'approcher assez précisément, aussi bien qualitativement que quantitativement, l'histogramme des contacts issu de diverses expériences. En revanche, un tenseur de texture d'ordre 2 ne rend que partiellement compte des effets d'anisotropie observés. Son utilisation se trouve limitée au plus à une symétrie matérielle de type orthotrope.

Nous proposons dans cette note d'incorporer la description en terme de tenseur de texture dans un schéma d'homogénéisation des milieux granulaires à microstructure aléatoire. L'objectif est de rendre compte des effets de l'anisotropie sur les propriétés élastiques macroscopiques. On commence par établir une hypothèse de localisation cinématique (9) qui généralise celle obtenue dans [5] au cas d'une distribution de contact anisotrope décrite à l'aide d'un tenseur de texture d'ordre deux ou d'ordre quatre. On en déduit ensuite un tenseur des raideurs élastiques homogénéisé dont l'expression (13), obtenue à partir du tenseur de texture d'ordre quatre, permet de décrire une anisotropie élastique générale, alors que (12) reste limitée à une symétrie matérielle de type orthotrope.

Dans une dernière partie, on compare dans le cas particulier isotrope, les propriétés élastiques macroscopiques déduites de l'hypothèse cinématique proposée avec celles fournies par une hypothèse de localisation simple de Voigt ou de Reuss. On notera en particulier que le module de compressibilité k^{hom} est indépendant de l'hypothèse de localisation utilisée, et dépend linéairement de la rigidité normale au contact K_n. D'autre part, la variation du module de cisaillement homogénéisé μ^{hom} en fonction du rapport des rigidités au contact $\alpha = K_t/K_n$ est représentée sur la Fig. 2 : les résultats obtenus à partir de l'hypothèse de localisation cinématique proposée se trouvent en accord avec les tendances qualitatives et quantitatives mises en évidence à l'aide de calculs numériques par [6] dans un contexte 3D.

1. Introduction

It is commonly admitted that the configuration of a granular assembly is the main characteristic of its mechanical state [1,2]. The relevant microstructural variables are the coordinance (number of neighbouring particles in contact) and the fabric tensor which describes the anisotropy of the contact distribution. It has been generally argued that the use of a fourth order fabric tensor enables to represent accurately the anisotropy effects observed in various experiments [3] or with numerical simulations [7], while a second order fabric tensor is limited to an orthotropic material symmetry. In this Note we propose to implement the fabric tensor-based approach in an homogenization scheme based on a new kinematic localization rule whose relevance is discussed. We limit our analysis to a two-dimensional quasi-static granular assembly.

2. Description of the fabric of a granular medium

2.1. Second order or fourth order fabric tensors

The fabric tensor is defined from the contact orientation distribution. The second order fabric tensor is classically defined for a two-dimensional granular assembly as [3,8]:

$$\underline{\underline{D}} = \langle n \otimes n \rangle = \frac{1}{N} \sum_{k=1}^{N} n^k \otimes n^k = \frac{1}{2\pi} \int_s P(\underline{n})(\underline{n} \otimes \underline{n})\,\mathrm{d}s \tag{1}$$

in which $\underline{n}$ denotes the normal at a contact, N the number of contacts, $P(\underline{n})$ the probability of contact in the direction of the normal $\underline{n}$ and s the unit circle. The summation is carried out on the active contacts in the assembly. $\langle\,\rangle$ denotes the average over all the orientations (in fact over the unit circle) of active contacts and $\otimes$ the tensorial product. It is readily seen that this second order fabric tensor is symmetric: $D_{ij} = D_{ji}$. It can be determined from experiments [3] or from numerical discrete simulations [7].

A fourth order tensor generalization of the previous definition reads:

$$\mathbb{D} = \langle \underline{n} \otimes \underline{n} \otimes \underline{n} \otimes \underline{n} \rangle = \frac{1}{N} \sum_{k=1}^{N} n^k \otimes n^k \otimes n^k \otimes n^k = \frac{1}{2\pi} \int_s P(\underline{n})(\underline{n} \otimes \underline{n} \otimes \underline{n} \otimes \underline{n})\,\mathrm{d}s \tag{2}$$

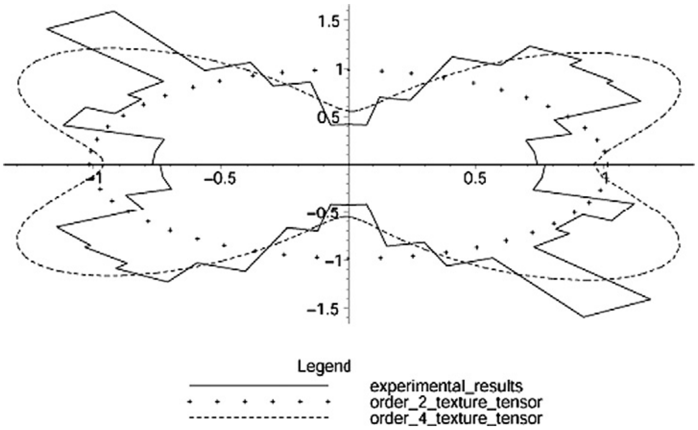

Fig. 1. Comparison of probabilities of contact.

This tensor $\mathbb{D}$ with components D_{ijkl} has the following symmetries: $D_{ijkl} = D_{jikl} = D_{klij} = D_{jilk} = D_{ijlk} = D_{ikjl} = D_{iljk}$. Moreover $\operatorname{Tr}\mathbb{D} = \underline{\underline{D}}$ where Tr denotes the contraction operator on any two indices. Finally, we note that for an anisotropic contact distribution we have:

$$\frac{1}{2\pi}\int_s P(\underline{n})\,\mathrm{d}s = 1 \tag{3}$$

whereas $P(\underline{n}) = 1$ in the case of an isotropic distribution of contacts. Combining (3) with each definition of fabric tensor, the contact probability $P(\underline{n})$ reads:

$$P(\underline{n}) = 4\left[\underline{\underline{D}} : (\underline{n}\otimes\underline{n}) - \frac{1}{4}\right] \tag{4}$$

for the second order fabric tensor-based description, whereas for a fourth order fabric tensor, one has:

$$P(\underline{n}) = 16\left[\mathbb{D} :: (\underline{n}\otimes\underline{n}\otimes\underline{n}\otimes\underline{n}) - \frac{3}{4}\underline{\underline{D}} : (\underline{n}\otimes\underline{n}) + \frac{1}{16}\right] \tag{5}$$

2.2. Comparisons and validation on a example

To compare the description given by (4) or (5), we propose to calculate the second and fourth order texture tensors, and the associated probability of contact. This comparison is made from the experimental results described in [3] (Chapter 1): filling by gravity of a sample constituted of 722 discs whose radii are 13, 18 and 28 mm. The associated contact distribution, or equivalently the probability of contact in the direction $\underline{n}$ is plotted on Fig. 1. It puts clearly in a prominent position an anisotropy induced by the filling. Let us notice that such kinds of results can also be obtained from discrete numerical simulations, performed with the software MULTICOR based on contact dynamics [3] and developed at the Laboratoire de Mécanique de Lille [7].

On the other hand, the second order and fourth order texture tensors are calculated from the distribution of contact orientations, using definitions (1), (2). The associated probability of contact $P(\underline{n})$ is then deduced from (4), (5) and plot in the same Fig. 1. We can notice that the probability of contact obtained from second order or fourth order texture tensors notably differ (see Fig. 1).

Thus the use of a fourth order texture tensor enables to represent more accurately, not only qualitatively but also quantitatively, the experimental contact histogram. In contrary, the use of a second order texture tensor takes into account very few the anisotropy effects. It is limited to an orthotropic material symmetry.

3. Homogenization of granular media: fabric-induced anisotropy effects

In order to determine the overall anisotropic elastic behavior of the granular media, we aim now at implementing the previous analysis in an homogenization scheme. For this purpose a kinematic localization assumption is made.

The starting point is the following definition of the symmetric mean stress tensor[1] which allows to compute the macroscopic stress from the contact forces $\underline{F}^c$ acting at the grain level:

$$\underline{\underline{\Sigma}} = \frac{1}{V}\left[\sum_{c=1}^{N} \underline{F}^c \otimes \underline{L}^c\right]^{\text{sym}} \tag{6}$$

in which the summation is on the contacts $c = 1, \ldots, N$ and where V denotes the volume of the granular assembly. To simplify the problem, we assume that the granular assembly is constituted of circular grains of same size. So that the branch vector reduces to $\underline{L}^c = 2r\underline{n}^c$, where r denotes the radius of the grains. Moreover, in the case of a great number of contacts, it is preferable to use the probability of contact $P(\underline{n})$ corresponding to the normal $\underline{n}$. Thus (6) becomes:

$$\underline{\underline{\Sigma}} = \frac{Nr}{\pi V}\left[\int_s P(\underline{n})(F_n^c \underline{n}^c + \underline{F}_t^c) \otimes \underline{n}^c \, \mathrm{d}s\right]^{\text{sym}} \tag{7}$$

Furthermore, we assume a linear contact law without unilateral effects and friction which reads:

$$F_n^c = K_n u_n^c \quad \text{and} \quad F_t^c = K_t u_t^c \tag{8}$$

where K_n and K_t are respectively the normal and the tangential stiffness at the contact.

3.1. General kinematic assumption

This subsection is devoted to the proposition of a general kinematic assumption linking the relative displacements at the contact $\underline{u}^c$ to the uniform strain $\underline{\underline{E}}$ imposed at the boundary of the R.E.V. (Representative Elementary Volume). Due to the linearity of the homogenization problem, $\underline{u}^c$ is a linear function of $\underline{\underline{E}}$; it also depends on $\underline{n}$ [5,10]. In the anisotropic case, according to (4), $\underline{U}^c = \frac{1}{2r}P(\underline{n})\underline{u}^c$ is a linear function of $\underline{\underline{E}}$ and depends on $\underline{n}$ and on $\underline{\underline{D}}$. The general form[2] of $\underline{U}^c$ is then established by using representation theorems [9]. Limiting here our analysis to a first order description and to only one free parameter η, the kinematic assumption leads to:

$$\underline{U}^c = \eta \underline{\underline{E}}.\underline{n} - \frac{1+\eta}{2}\left[4\underline{n}.\underline{\underline{E}}.\underline{n} - \operatorname{tr}(\underline{\underline{E}})\right]\underline{n} + 4\underline{n}.\underline{\underline{D}}.\underline{n}\,\underline{\underline{E}}.\underline{n} \tag{9}$$

Note that despite the simplification, this kinematic localization rule provides more physical predictions than the simple Voigt kinematic assumption. It is also noticeable that in the particular case of an isotropic distribution of contacts ($P(\underline{n}) = 1$ and $\underline{\underline{D}} = \frac{1}{2}\underline{\underline{\delta}}$), (9) is similar to the kinematic assumption proposed in [5]. Moreover, for $\eta = -1$ we recover Voigt kinematic assumption as a particular case. This important point was a criterion of reducing the general kinematic rule to the much simpler one (9) depending only on a free parameter η. And this by conserving a degree of generality sufficient to illustrate the homogenization scheme for anisotropic granular media which follows.

In a general anisotropic case, using a fourth order fabric tensor, the kinematic assumption (9) becomes:

$$\underline{U}^c = \eta \underline{\underline{E}}.\underline{n} - \frac{1+\eta}{2}\left[4\underline{n}.\underline{\underline{E}}.\underline{n} - \operatorname{tr}(\underline{\underline{E}})\right]\underline{n} + a\mathbb{D} :: (\underline{n} \otimes \underline{n} \otimes \underline{n} \otimes \underline{n})\underline{\underline{E}}.\underline{n} \tag{10}$$

where the constant a is determined using the fact that expressions (9) and (10) of $\underline{U}^c$ must coincide in the case of an isotropic distribution of contacts. We get $a = \frac{16}{3}$.

3.2. Determination of the macroscopic properties

In the case of a second order fabric tensor, after some lengthy but straightforward calculations, using the contact law (8) and the kinematic assumption (9), expression (7) can be written as $\underline{\underline{\Sigma}} = \mathbb{C}^{\text{hom}} : \underline{\underline{E}}$. The homogenized stiffness tensor $\mathbb{C}^{\text{hom}}$ is given by:

[1] There exists different definitions of the mean stress tensor [3]. All of these are not symmetric. We have chosen here to start from a symmetric definition of $\underline{\underline{\Sigma}}$ (Cauchy medium) in order to keep all the symmetry properties of the macroscopic stiffness tensor which will be determined from the homogenization procedure.

[2] The factor $\frac{1}{2r}$ introduced has no consequences on the final kinematic localization rules (9) and (10). It is intended for simplifying the intermediary calculations which are detailed in appendix.

$$\mathbb{C}^{\text{hom}} = \frac{4r^2N}{V}\Bigg\{K_n\Bigg[-(\eta+2)\frac{1}{2\pi}\int_s \underline{n}\otimes\underline{n}\otimes\underline{n}\otimes\underline{n}\,\mathrm{d}s + \frac{\eta+1}{2}\frac{1}{2\pi}\int_s \underline{n}\otimes\underline{n}\,\mathrm{d}s\otimes\underline{\underline{\delta}} + 4\underline{\underline{D}}:\frac{1}{2\pi}\int_s \underline{n}\otimes\underline{n}\otimes\underline{n}\otimes\underline{n}\otimes\underline{n}\otimes\underline{n}\,\mathrm{d}s\Bigg] + K_t\Bigg[\eta\frac{1}{2\pi}\int_s \underline{\underline{\underline{T}}}^T.\underline{\underline{\underline{T}}}\,\mathrm{d}s + 4\underline{\underline{D}}:\frac{1}{2\pi}\int_s (\underline{n}\otimes\underline{n})\otimes\underline{\underline{\underline{T}}}^T.\underline{\underline{\underline{T}}}\,\mathrm{d}s\Bigg]\Bigg\} \tag{11}$$

where $\underline{\underline{\delta}}$ denotes the second order identity tensor. The use of the third order tensor $\underline{\underline{\underline{T}}}^c = \underline{n}^c.\mathbb{I} - (\underline{n}^c\otimes\underline{n}^c\otimes\underline{n}^c)$ enables to simplify the notations. We recall that $\mathbb{I}$ denotes the classical fourth order symmetric identity tensor. The integrals containing $\underline{n}$ and $\underline{\underline{\underline{T}}}$ involved in (11) can be calculated by using identities provided by [11,12]. Thus the approximation of the homogenized stiffness tensor with a second order fabric tensor $\underline{\underline{D}}$ writes:

$$\mathbb{C}^{\text{hom}} = \frac{4r^2N}{V}\Bigg\{K_n\Bigg[-(\eta+2)\left(\frac{1}{8}(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}}) + \frac{1}{4}(\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}})\right) + \frac{\eta+1}{4}\underline{\underline{\delta}}\otimes\underline{\underline{\delta}} + \frac{1}{12}(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}} + 2\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}}) + \frac{1}{6}(\underline{\underline{D}}\otimes\underline{\underline{\delta}} + \underline{\underline{\delta}}\otimes\underline{\underline{D}} + 2\underline{\underline{D}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}} + 2\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{D}})\Bigg] + K_t\Bigg[\eta\left(-\frac{1}{8}(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}}) + \frac{1}{4}(\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}})\right) + \frac{1}{12}(-\underline{\underline{\delta}}\otimes\underline{\underline{\delta}} + 4\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}}) + \frac{1}{6}(-\underline{\underline{D}}\otimes\underline{\underline{\delta}} - \underline{\underline{\delta}}\otimes\underline{\underline{D}} + \underline{\underline{D}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}} + \underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{D}})\Bigg]\Bigg\} \tag{12}$$

where we have used the following notation $(\underline{\underline{a}}\,\overline{\underline{\otimes}}\,\underline{\underline{b}})_{ijkl} = \frac{1}{2}(a_{ik}b_{jl} + a_{il}b_{jk})$ for all second order tensors $\underline{\underline{a}}$ and $\underline{\underline{b}}$. Following the same procedure, the approximation of $\mathbb{C}^{\text{hom}}$ obtained from the use of the fourth order fabric tensor writes:

$$\mathbb{C}^{\text{hom}} = \frac{4r^2N}{V}\Bigg\{K_n\Bigg[-(\eta+2)\left(\frac{1}{8}(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}}) + \frac{1}{4}(\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}})\right) + \frac{\eta+1}{4}\underline{\underline{\delta}}\otimes\underline{\underline{\delta}} + \frac{1}{24}(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}} + 2\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}}) + \frac{1}{6}(\underline{\underline{D}}\otimes\underline{\underline{\delta}} + \underline{\underline{\delta}}\otimes\underline{\underline{D}} + 2\underline{\underline{D}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}} + 2\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{D}}) + \frac{1}{3}\mathbb{D}\Bigg] + K_t\Bigg[\eta\left(-\frac{1}{8}(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}}) + \frac{1}{4}(\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}})\right) + \frac{1}{24}(-\underline{\underline{\delta}}\otimes\underline{\underline{\delta}} + 6\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}}) + \frac{1}{6}(-\underline{\underline{D}}\otimes\underline{\underline{\delta}} - \underline{\underline{\delta}}\otimes\underline{\underline{D}} + 2\underline{\underline{D}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}} + 2\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{D}}) - \frac{1}{3}\mathbb{D}\Bigg]\Bigg\} \tag{13}$$

It is clear that, due to the presence of the fourth order fabric tensor $\mathbb{D}$, (13) is more relevant to describe a general elastic anisotropy than (12) which is obviously limited to an orthotropic material symmetry.

3.3. Particular case of an isotropic distribution of contacts

In the particular case of an isotropic distribution of contacts, the fabric tensors reduced to:

$$\underline{\underline{D}} = \frac{1}{2\pi}\int_s \underline{n}\otimes\underline{n}\,\mathrm{d}s = \frac{1}{2}\underline{\underline{\delta}} \quad\text{and}\quad \mathbb{D} = \frac{1}{2\pi}\int_s \underline{n}\otimes\underline{n}\otimes\underline{n}\otimes\underline{n}\,\mathrm{d}s = \frac{1}{8}(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}} + 2\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}})$$

The approximations of $\mathbb{C}^{\text{hom}}$ given by the two fabric tensors then coincide. In this case, it is interesting to compare the values of the macroscopic compressibility k^{hom} and shear modulus μ^{hom}, obtained from the kinematic assumption proposed in the present Note, to the values deduced from a simple Voigt or Reuss assumption[3] [4] (see Table 1).

Contrary to the results obtained from Voigt and Reuss assumption, in the case of the more general kinematic assumption proposed, we have a free parameter η. However, a first result is that the homogenized compressibility

[3] In the framework of Voigt and Reuss localization assumptions, a general expression of $\mathbb{C}^{\text{hom}}$ has been established for the two fabric tensors of order 2 and 4 [4]. The associated values of the homogenized Young modulus E^{hom} and Poisson coefficient ν^{hom} deduced from $\mathbb{C}^{\text{hom}}$ in the particular isotropic case, coincide with those obtained in [6] which concerns only the isotropic case. We recall that in the plane stress state considered, we have $k^{\text{hom}} = \frac{E^{\text{hom}}}{2(1-\nu^{\text{hom}})}$.

Table 1
Macroscopic compressibility k^{hom} and shear modulus μ^{hom} calculated from various models ($\alpha = \frac{K_t}{K_n}$, K_t and K_n are the tangential and normal stiffness at the contact)

	Voigt assumption	Reuss assumption	Kinematic assumption
k^{hom}	$\frac{Nr^2K_n}{V}$	$\frac{Nr^2K_n}{V}$	$\frac{Nr^2K_n}{V}$
μ^{hom}	$\frac{Nr^2K_n}{2V}(1+\alpha)$	$\frac{Nr^2K_n}{V}\frac{2\alpha}{1+\alpha}$	$\frac{Nr^2K_n}{V}(1-\frac{1}{2}(\eta+2)(1-\alpha))$

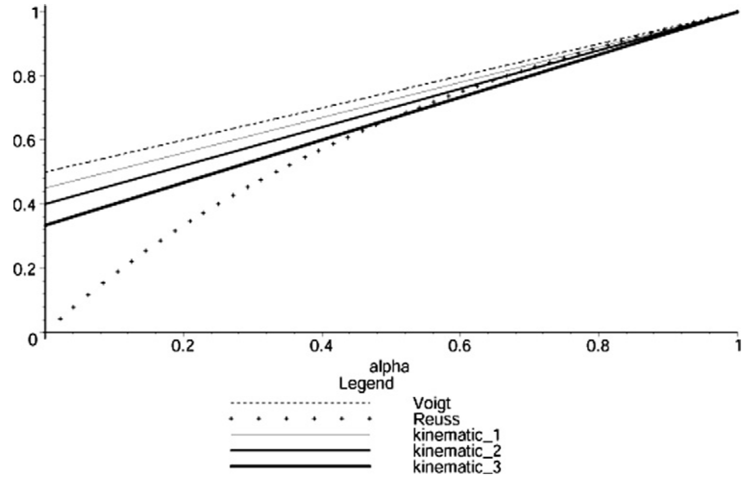

Fig. 2. Variation of $\frac{V}{Nr^2K_n}\mu^{\text{hom}}$ with respect to α. Voigt, Reuss, kinematic 1 ($\eta = -0.9$), kinematic 2 ($\eta = -0.7$) and kinematic 3 ($\eta = -0.5$) assumptions.

k^{hom} does not depend on η and on the localization rule chosen. It depends linearly on the normal stiffness K_n. On the other hand, for $\eta = -1$ and $\eta = -\frac{2\alpha}{1+\alpha}$ the value of μ^{hom} obtained from the proposed kinematic assumption corresponds to those obtained from Voigt and Reuss assumptions respectively.

The variations of $\frac{V}{Nr^2K_n}\mu^{\text{hom}}$ with respect to α are presented in Fig. 2 for three different values of η. We have limited here the analysis to $0 < \alpha < 1$ which corresponds to the classical cases encountered (Hertz contact hypothesis for example). Let us notice that for $0 < \alpha < 1$, the stability condition of the elastic homogenized material ($\mu^{\text{hom}} > 0$) implies that η must satisfy $\eta < \frac{2\alpha}{1-\alpha}$. We observe that the variations of μ^{hom} obtained from the kinematic assumption, linear with respect to α, are in good agreement with the numerical results of [6] when η is close to 0.5. In particular, the values of μ^{hom} tends to be out of the Reuss prediction when α tends towards 1.

4. Conclusion

The present Note has been devoted to the description of anisotropy effects in granular media by using second order and fourth order fabric tensors. These fabric tensors are combined with a new kinematic localization rule in order to develop an homogenization scheme. The interest of the fourth order fabric tensor is quantitatively demonstrated for the anisotropic elastic properties of the granular medium. Ongoing works concern a three-dimensional generalization of the present study.

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