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An interpretative model of the pedestrian fundamental relation

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Abstract

The relation between density and velocity of pedestrian movement has so far mainly been analysed using an empirical approach and fundamental relations found from the fitting of experimental measurements of the main quantities. The present study proposes a phenomenological model that is able to distinguish and take into account various factors that can affect the density–velocity relation by means of the induced microscopic walking phenomena. In particular, the effect of the lateral vibrations of the platform on which pedestrians walk is considered in the light of the use of the fundamental diagrams within a crowd–structure interaction model applied to lively footbridges. A comparison with some of the empirical fundamental laws shows an excellent agreement, demonstrating that the main walking phenomena are correctly assumed in the present model. ***To cite this article: F. Venuti, L. Bruno, C. R. Mecanique 335 (2007).***

Résumé

Un modèle interpretatif de la loi fondamentale pour les piétons. La relation entre la densité et la vitesse des piétons a été principalement étudiée jusqu'au présent à travers une approche empirique et les lois fondamentales obtenues à partir de mesures expérimentales directes des grandeurs concernées. Cette étude propose un modèle phénoménologique capable de distinguer et de prendre en compte plusieurs facteurs qui peuvent affecter la relation densité–vitesse par le biais des phénomènes microscopiques engendrés. En particulier, l'effet des vibrations latérales de la plate-forme sur laquelle le piétons marche a été introduit en vue de l'utilisation de la loi fondamentale dans le cadre d'un modèle d'interaction piétons–structure appliqué aux passerelles piétonnes flexibles. La comparaison avec certaines des lois fondamentales empiriques montre un excellent accord et démontre que les principaux phénomènes sont correctement pris en compte par le modèle proposé. ***Pour citer cet article : F. Venuti, L. Bruno, C. R. Mecanique 335 (2007).***

Keywords: Dynamical systems; Pedestrian flow; Fundamental relations; Footbridges

Mots-clés : Systèmes dynamiques ; Traffic pieton ; Lois fondamentales ; Passerelles piétonnes

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1. Introduction

Since the 1960s, many studies have been dedicated to the determination of a law that links walking velocity to crowd density. Most of these studies belong to the transportation research field and have the aim of controlling the layout and dimensions of pedestrian walking facilities (e.g. [1,2]). In recent years, research has been directed towards the study of crowd flow patterns under emergency situations (e.g. [3]), and increasing attention has been devoted to the effects of crowd behaviour on the dynamics of structures in the civil engineering field (e.g. [4]). Because of the great number of factors that can affect pedestrian walking behaviour (age, culture, gender, travel purpose, type of infrastructure, walking direction), rather different fundamental diagrams can be found in literature. A complete review of the speed–density relations proposed so far can be found in [2] and [5]: most of them are based on fitting to experimental or observation data. All the proposed speed–density relations have some common features, concerning the monotonic decreasing trend of the velocity v with increasing density u and the identification of some critical points [2]: the free speed v_M, which corresponds to the mean maximum velocity; the critical density u_c, corresponding to the lower bound for unconstrained free walking; the jam density u_M, that is, the maximum admissible density corresponding to null speed and flow; the capacity speed v_{ca} and density u_{ca}, corresponding to the maximum flow $q_{ca} = u_{ca} v_{ca}$. The region of density under u_{ca} is called free flow region, while the congestion region corresponds to a higher density than the capacity. These relations have the advantage of being expressed through very compact formulas; on the other hand, they do not distinguish the effects that each factor has on the walking behaviour. The resulting lack of generality makes them difficult to extend from experimental conditions to different cases.

One of the above mentioned factors is the motion of the platform on which pedestrians walk. To the authors' knowledge, the sensitivity of the walking speed to the lateral motion of the platform has not be taken into account so far. The influence of platform motion on walking behaviour becomes very important when modelling crowd–structure interaction phenomena. This topic has come to public attention in recent years, after a famous footbridge, the London Millennium Bridge [6], was closed the day it was opened because of excessive lateral vibrations induced by the walking crowd. In situ observations on actual footbridges allow some modelling assumptions to be introduced: the flow is one-directional and its value never exceeds the capacity flow; the walking surface is smooth and generally free from obstacles. Under such assumptions, a phenomenological relation that links the walking speed to the crowd density and the platform motion can be used within a coupled crowd–structure interaction model, where the crowd is described by a first order hydrodynamic model [4].

The present study proposes a speed–density relation based on phenomenological considerations, which takes into account the influence of the platform lateral motion. The general aim of the Note is to outline a open modelling framework that can take into account, in each of its parts, the macroscopic effects of different factors that affect the microscopic properties of pedestrian movement. In particular, each parameter of interest has to be represented by one coefficient, whose value can be determined by means of ad hoc experiments. In the following, the fundamental relations are expressed in their dimensional form in order to better discuss the physical meaning of the above mentioned factors.

The proposed model is described in Section 2; the model is compared to other laws found in literature and revisited in Section 3.

2. Proposed model

The fundamental relation that links crowd density to walking velocity is usually expressed in the direct form $v = v(u)$. In the following, a phenomenological relation that links pedestrian velocity v to crowd density u and platform acceleration $\ddot{z}$ is proposed in the inverse form $u = u(v, \ddot{z})$. Bearing in mind that crowd density u [ped/m^2] is a scarcely intuitive quantity, its reciprocal Pedestrian Area Module (PAM) [7], i.e. the surface S occupied by a pedestrian, is used as a more manageable unit [m^2]. Hence, the proposed relation is based on some phenomenological considerations about $S(v, \ddot{z})$. Obviously, once the latter is obtained, the density can be calculated as the reciprocal of $S(v, \ddot{z})$, and the speed–density relation can be recovered as an inverse relation.

In the following, among the various factors that can affect walking behaviour, two are specifically retained, that is, the geographic area and the travel purpose: the related coefficients are respectively named with the subscripts G and T. These factors are assumed to affect both free speed v_M and PAM S through the coefficients α and β, respectively.

Table 1
Coefficients of geographic area and travel purpose that affect the free speed v_M

Geographic area α_G			Travel purpose α_T		
Europe	USA	Asia	Rush hour/Business	Commuters/Events	Leisure/Shopping
1.05	1.01	0.92	1.20	1.11	0.84

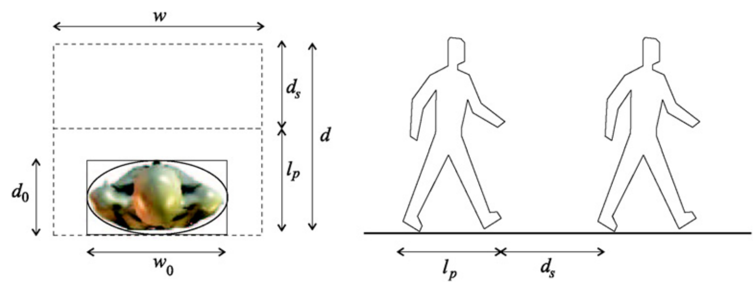

Fig. 1. Human body dimensions: motionless and walking pedestrian.

The free speed is expressed in the general form:

$$v_M = \bar{v}_M \alpha_G \alpha_T g(\ddot{z}) \tag{1}$$

where $\bar{v}_M = 1.34$ m/s is the average free speed [5]; the coefficients α_G and α_T were determined analyzing the data reported in [5] as the ratio between the proposed free speeds and $\bar{v}_M$, and they are reported in Table 1. The corrective factor $g(\ddot{z})$ takes into account the sensitivity of v to the platform acceleration $\ddot{z}$ and it has a qualitative trend because of the lack of experimental data: $g(\ddot{z}) = 1$ for $\ddot{z} \leqslant \ddot{z}_c$, $g(\ddot{z}) = 0$ for $\ddot{z} \geqslant \ddot{z}_M$ and $g(\ddot{z}) = (\ddot{z}_M - \ddot{z})/(\ddot{z}_M - \ddot{z}_c)$ for $\ddot{z}_c < \ddot{z} < \ddot{z}_M$, where $\ddot{z}_c \cong 0.2$ m/s^2 [8] corresponds to the threshold of motion perception, while $\ddot{z}_M = 2.1$ m/s^2 [9] is the maximum acceptable acceleration above which pedestrians stop walking.

The PAM occupied by a motionless pedestrian can be calculated considering an elliptical or a rectangular form. According to the latter hypothesis, the average surface occupied by a motionless pedestrian is $S_0 = w_0 d_0$, where w_0 and d_0 are the average lateral width and depth of a human body (Fig. 1). When a pedestrian is walking, a greater surface is required, that is, $S = wd$ (Fig. 1). Both the terms w and d can be expressed as a function of the walking velocity v. In addition, the lateral width could be made sensitive to the platform acceleration $\ddot{z}$, since pedestrians tend to walk with more spread legs when the surface is laterally moving, as stressed by various authors [6,10]. The required forward distance d can be expressed as the sum of two terms: l_p, the step length, which is a function of the walking velocity and step frequency f_p; d_s, the sensory distance, which is defined by Fruin as "*the area required by the pedestrian for perception, evaluation and reaction*" [7]. While the former can be physically measured, the latter depends to a great extent on cultural and psychological factors and is, therefore, hard to evaluate. As for the lateral width w, even though the same distinction can be made between pacing and sensory zones, a unique width is retained because of the lack of available data. Therefore, S takes the form

$$S(v, \ddot{z}) = S_p + S_s = w(v, \ddot{z}) l_p(v) + w(v, \ddot{z}) d_s(v) \tag{2}$$

where surfaces S_p and S_s are the pacing zone and the sensory zone, respectively. It is supposed that β_G only affects the pacing zone, since the geographic area is mainly related to the body dimensions (width and step length), while β_T only concerns the sensory zone, since the travel purpose can be connected to psychological factors. Therefore, Eq. (2) can be rewritten as

$$S = w(\beta_G l_p + \beta_T d_s) \tag{3}$$

It is worthwhile recalling that the value of the maximum admissible density can be calculated as $u_M = 1/S(0, 0)$, while the critical density is $u_c = 1/S(v_M, 0)$.

All the parameters introduced in the modelling framework are characterized in the following on the basis of several experimental data coming from various research fields such as transportation, biomechanics, safety and structural engineering.

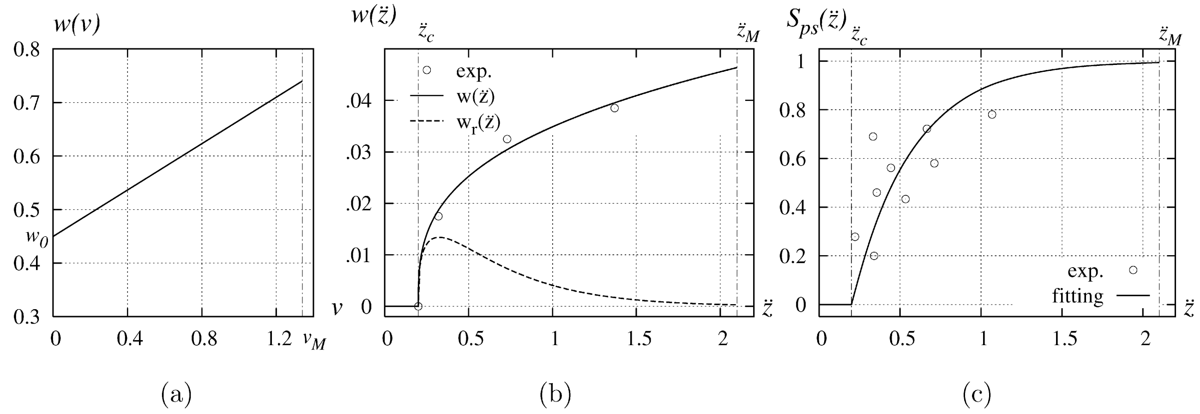

Fig. 2. $w(v)$, $w(\ddot{z})$ [m] and $S_{ps}(\ddot{z})$ laws.

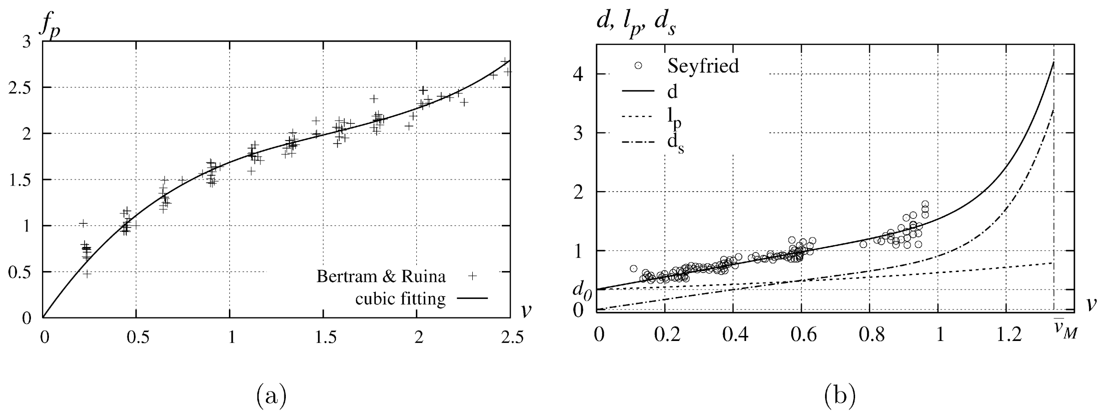

Fig. 3. Proposed f_p [Hz]-v [m/s] (a) and d, l_s, d_b [m]-v (b) relations.

The lateral width $w(v, \ddot{z})$ is expressed in the additive form $w(v, \ddot{z}) = w(v) + w_r(\ddot{z})$, where $w_r(\ddot{z})$ is the peak-to-peak value of the relative lateral displacement of the torso between adjacent pedestrians. According to [5], in a free walking regime, a pedestrian requires a lateral additional space equal to about 62% of his average width. Therefore, the simplest expression is the linear relation

$$w(v) = w_0\left(1 + 0.62\frac{v}{\bar{v}_M}\right) \tag{4}$$

where $w_0 = 0.45$ m [5] (Fig. 2(a)). The relative lateral displacement of the torso considers the fact that, the higher the platform motion, the higher the synchronization between pedestrians and the platform and therefore, the lower the relative displacement among pedestrians. Hence, the relative displacement can be expressed as $w_r(\ddot{z}) = w(\ddot{z})(1 - S_{ps})$, where the relation between w and $\ddot{z}$ can be inferred by fitting experimental data recorded on actual lively footbridges (Fig. 2(b)) [10,9]:

$$w(\ddot{z}) = 0.0375(\ddot{z} - \ddot{z}_c)^{1/3}, \quad \ddot{z} \geqslant \ddot{z}_c \tag{5}$$

while $S_{ps}(\ddot{z})$ is the pedestrian-platform synchronization coefficient (Fig. 2(c)) introduced by [6,10]. From Figs. 2(a), (b) it is clear that the value of the term $w_r(\ddot{z})$ is two orders of magnitude less than $w(v)$. Therefore, it is not retained in the following.

The relation between the step length l_s and the velocity v can be derived from the relation between the step frequency f_p and v, since $l_p = v/f_p$ (Fig. 3(b)). The frequency–speed relation is derived from a cubic fitting to the Bertram and Ruina [11] (laboratory tests on a treadmill with fixed velocity) experimental data (Fig. 3(a)): $l_p(v) =$

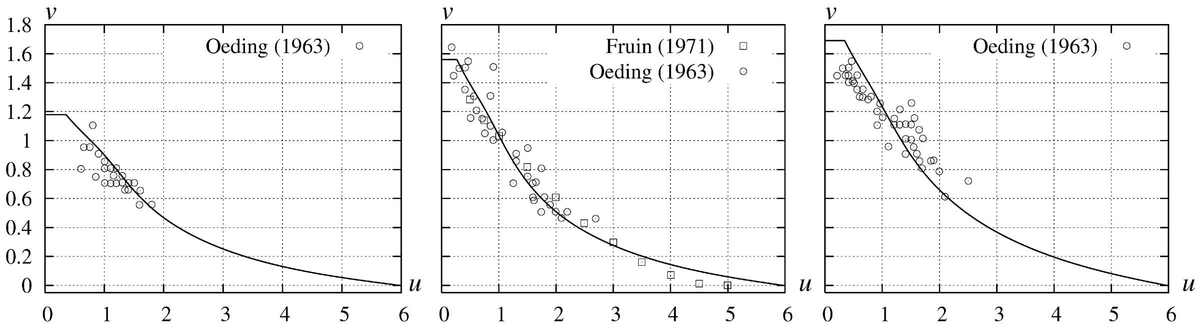

Fig. 4. Fitting to the experimental data: leisure/shopping (left); commuters/events (centre); business/rush hour (right).

Table 2
Coefficients β_T

	Leisure/Shopping	Commuters/Events	Rush hour/Business
v_M [m/s] (Europe)	1.18	1.56	1.69
β_T	1.07	0.93	0.55

$v/f_p = v/(0.35v^3 - 1.59v^2 + 2.93v)$. Fig. 3(b) shows that, when a pedestrian is motionless ($v = 0$), the body depth d_0, whose value is set equal to $d_0 = 0.36$ m according to Seyfried et al. [12], takes the place of the step length.

The sensory distance d_s (Fig. 3(b)) can be obtained as $d - l_p$, since some experimental data concerning the $d - v$ relation are available in literature [12]. The data have been fitted according to the law $d(v) = d_0 + 1.06v + bv^{10}$, where $b = (2.08v_M - d_0)/v_M^{10}$, so that $d(v_M)$ takes the value for which $u_c \cong 0.3$ ped/m^2, as proposed by Oeding [13]. It should be pointed out that, according to Seyfried [12], the almost linear trend of the $d - v$ relation at low velocities is due to synchronization phenomena among pedestrians (e.g. marching in lock-step) which reduce the sensory distance d_s.

The geographic area coefficient β_G is derived, considering the dimension occupied by the human body in different countries (Table 1 in [5]), as the ratio between the surface averaged per geographic area and the mean surface S_0. It shows that β_G equals 1.075 for the European and American cases, while it equals 0.847 for Asian countries. The travel purpose coefficient β_T is determined by fitting the reciprocal of Eq. (3) to the experimental data reported by Oeding [13] and Fruin [7] (Fig. 4 and Table 2). It can be noticed that β_T monotonically decreases for increasing free speeds v_M associated with different travel purposes.

In order to evaluate the sensitivity of the model to the different parameters of interest, the proposed model has been applied to different combinations of the latter. Some results are represented in Fig. 5 in terms of fundamental diagrams. In Fig. 5(a), the model is applied with a fixed geographic area (Europe) and a motionless platform, varying the travel purpose; in Fig. 5(b), the travel purpose is fixed (commuters) with a motionless platform and the geographic area varies; finally, in Fig. 5(c) the sensitivity to the platform acceleration [m/s^2] is explored, fixing the geographic area (Europe) and the travel purpose (commuters).

The following conclusions can be drawn: (i) the effects of the travel purpose can be seen above all in the variation of v_M and β_T, which determines different values of u_c and u_{ca}. It should be noticed that neither u_c nor u_{ca} grow monotonically with the free speed; (ii) the geographic area has the effect of varying the jam density u_M. While Europe and the USA have practically the same diagram, Asia shows a higher u_M and capacity flow q_{ca}, which is in agreement with data in literature [5]; (iii) the platform motion causes a linear decrease in v_M and q_{ca}.

3. Comparison with fundamental laws revisited

The proposed fundamental relation is compared to two laws previously proposed in literature in the direct form $v = v(u)$: the first is the Kladek formula, proposed by Weidmann [1], while the second is recovered from vehicular traffic theory [14]. Both are reported in their original-dimensional form in the first column of Table 3.

In order to make these relations sensitive to the walking features considered in this work, the two laws are rewritten in a revisited form (second column of Table 3), where v_M, u_c and u_M are determined according to the principles

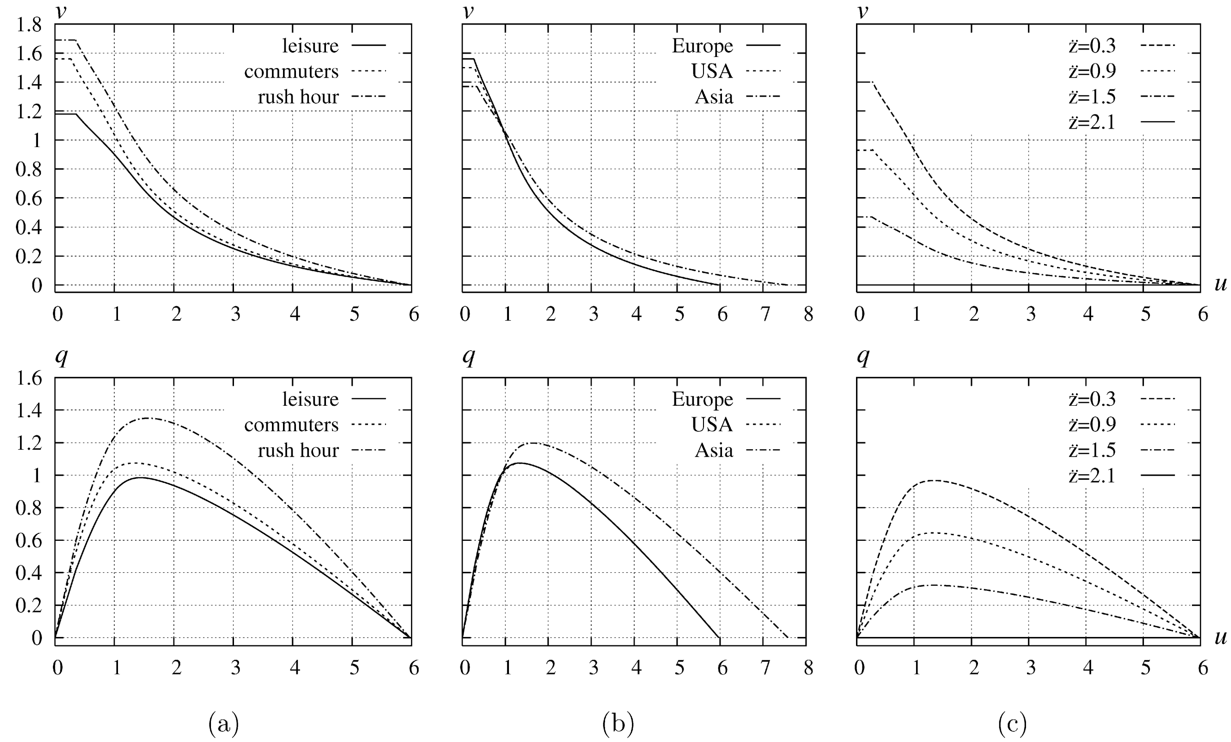

Fig. 5. Fundamental diagrams varying the travel purpose (a), the geographic area (b) and the platform acceleration $\ddot{z}$ (c).

Table 3
Original and revisited fundamental laws proposed in literature

	Original form	Revisited form
Kladek formula	$v = v_M\{1 - \exp[-1.913(\frac{1}{u} - \frac{1}{5.4})]\}$	$v = v_M\{1 - \exp[-\gamma(\frac{1}{u} - \frac{1}{u_M})]\}$
Vehicular law	$v = v_M \exp(-\alpha \frac{u}{u_M - u})$	$v = \begin{cases} v_M & u \leqslant u_c \\ v_M \exp(-\gamma \frac{u-u_c}{u_M-u}) & u > u_c \end{cases}$

Table 4
Coefficients γ in the revisited fundamental laws (Table 3)

γ	Leisure/Shopping	Commuters/Events	Rush hour/Business
Kladek revisited	$0.245u_M$	$0.214u_M$	$0.273u_M$
Vehicular law	2.191	2.340	1.788

explained in the previous section (see Eqs. (1), (3)) and γ is the fitting free parameter determined referring to the same experimental data [13,7] selected on the basis of travel purpose. The results of the fitting are reported in Table 4: it should be pointed out that the coefficients γ do not show a monotonic trend for increasing free speeds. The revisited laws and the interpretative model are plotted in Fig. 6 in the case of Europe and leisure/shopping.

The vehicular traffic law shows a good correspondence with the other two for densities lower than 2 ped/m^2, while it underestimates the velocities for higher densities: this behaviour is stressed in the flow diagram. It can be explained recalling that, in the case of vehicular traffic, the driver tends to reduce speed more quickly for high densities, in order to avoid crashes. As can be noticed, the proposed model shows a surprising agreement with the revisited Kladek formula, even though the same results are obtained through different approaches, i.e. based on microscopic interpretative modelling and fitting to macroscopic observation data, respectively. The physical-based model represents a complementary tool to shed some light on the role that microscopic walking phenomena play in the macroscopic fundamental

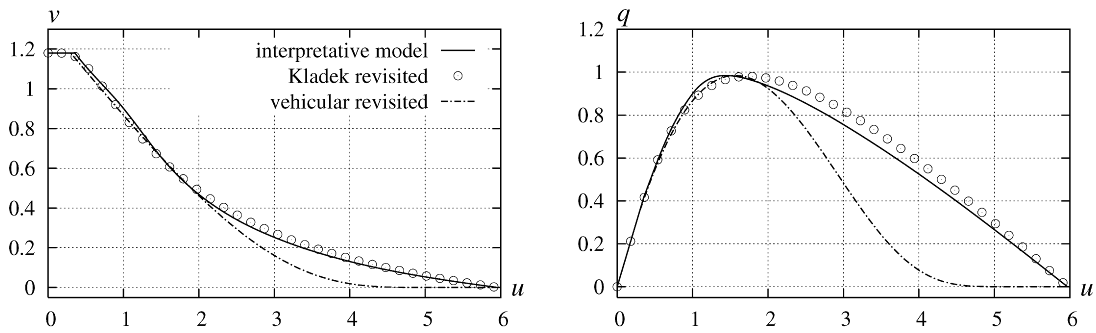

Fig. 6. Velocity and fundamental diagrams for the three laws in the case of Europe and leisure/shopping.

relation, while the revisited Kladek formula, because of its continuity and compact direct form, is more suitable for practical use. From this point of view, the physical-based model can be used to generate data from numerical experiments, to be further best-fitted with the revisited Kladek formula by tuning the single free parameter γ.

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