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Leak identification in porous media by solving the Cauchy problem

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Abstract

We propose in this Note a method of identifying leak zones in a saturated and homogeneous porous domain by solving a Cauchy problem. The method is based on the minimisation of an energy-like error functional procedure developed in 2006 by Andrieux and Baranger. ***To cite this article: X. Escriva et al., C. R. Mecanique 335 (2007).***

Résumé

Identification de fuites dans un milieu poreux par résolution du problème de Cauchy. Nous proposons dans ce travail, l'identification de fuites dans un domaine poreux homogène et saturé, en résolvant un problème de Cauchy. Cette résolution est effectuée au moyen d'une procédure de minimisation d'une fonctionnelle d'erreur énergétique développée en 2006 par Andrieux et Baranger. ***Pour citer cet article : X. Escriva et al., C. R. Mecanique 335 (2007).***

Keywords: Porous media; Leak identification; Data completion; Cauchy problem; Darcy law

Mots-clés : Milieux poreux ; Identification de fuites ; Complétion de données ; Problème de Cauchy ; Loi de Darcy

1. Introduction

This Note deals with the identification of leaks at a boundary of a homogeneous porous media saturated with fluid. The identification is performed by exploiting overspecified measured data: the pressure and the pressure gradient on a part of the boundary. This problem can be stated as a Cauchy problem, where overspecified boundary conditions are known on a part of the boundary and unknown on the remaining parts. Hence, solving the Cauchy problem leads one to identify the missing boundary conditions, and then the leak zones are characterised by null pressure values.

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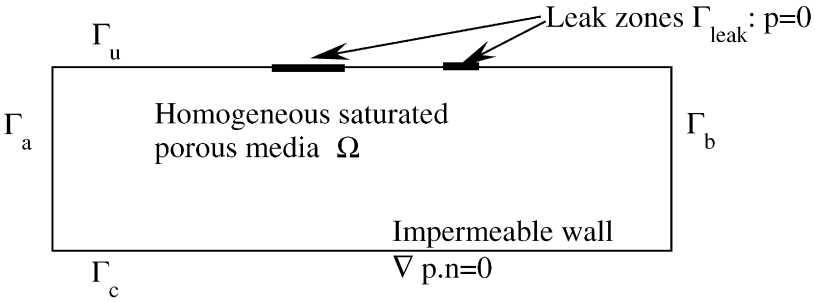

Fig. 1. Geometry and boundary conditions.

Fig. 1. Géométrie et conditions aux limites.

Leak identification may have several fields of application in petroleum engineering, hydrogeology, agriculture and environmental operating research where boundary data at some edges of an oil or water reservoir are required, without the possibility of probing. Also, leak identification is an important matter when the contamination of aquifers by pollutants is concerned. In the field of leak identification in a saturated and homogeneous porous medium, significative work has been proposed by Belhachmi et al. [1]. They defined an inverse problem based on the method introduced by Santosa et al. [2] to detect interior cracks in a medium by using measurements of electrostatic field at the boundaries. Although the method shows successful results, important assumptions are made by Belhachmi et al. [1]: the length and position of each leak are considered as parameters of the problem. This assumption makes their method expensive and tedious.

In this Note, we propose to identify multiple leaks on some part of the boundary, without assumptions on leak position and length, by solving the equivalent Cauchy problem. This last step is based on a variational formulation firstly introduced by Kohn and Vogelius [3], and recently updated by Andrieux et al. [4,5] for the Laplacian and elasticity operators. They introduced an error functional which depends on the two kinds of data lacking (Neumann and Dirichlet boundary conditions). This functional reaches its minimum at the pairs of data which solve the Cauchy problem.

The outline of this Note is as follows: in Section 2 we define the direct Darcy problem, then in Section 3 the equivalent Cauchy problem is derived. In Section 4, we define the error functional and present the numerical procedure used to minimize this functional. In Section 5, we show some results about leak identification with different sizes and multiplicity and its sensitivity to noisy data.

2. Direct Darcy problem

We consider a homogeneous saturated porous media domain denoted by Ω as shown in Fig. 1. The direct problem is defined by the following incompressible Darcy equations:

$$\begin{cases} \mathbf{u} + \dfrac{k}{\mu}\nabla p = \mathbf{f} \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega \\ p = p_m \quad \text{on } \Gamma_m, \qquad \mathbf{u}\cdot\mathbf{n} = q_m \quad \text{on } \Gamma_m \end{cases} \tag{1}$$

where p is the pressure field, $\mathbf{u}$ is the velocity field, k is the homogeneous and isotropic permeability coefficient and μ the dynamic viscosity of the fluid. Without loss of generality, we assume $\mathbf{f} = 0$. We denote by Γ_m the boundary where the data are overspecified: $\Gamma_m = \Gamma_a \cup \Gamma_b \cup \Gamma_c$, and by Γ_u the boundary where the leaks have to be identified.

3. Derived Cauchy problem

The direct problem can be transformed into a classical Laplacian Cauchy problem for the pressure field p, by applying the divergence operator to the first equation of (1) which becomes $\nabla \cdot (\frac{k}{\mu}\nabla p) = 0$. The corresponding Neumann boundary condition $-\frac{k}{\mu}\frac{\partial p}{\partial n} = q_m$ is obtained by substituting the field $\mathbf{u}$ in $\mathbf{u}\cdot\mathbf{n} = q_m$ with its expression from the first equation of (1). The Dirichlet boundary condition is expressed by $p = p_m$. Hence the problem (1) can be stated as follows:

$$\begin{cases} \nabla \cdot \left(\dfrac{k}{\mu} \nabla p \right) = 0 \quad \text{in } \Omega, \\ p = p_m, \quad -\dfrac{k}{\mu} \dfrac{\partial p}{\partial n} = q_m \quad \text{on } \Gamma_m \end{cases} \tag{2}$$

This problem above is known as the Cauchy problem with overspecified data on Γ_m. It is known since Hadamard [6] to be ill-posed in the sense that the dependence of p on the data (q_m, p_m) is known to be not continuous. Solving the Cauchy problem can be stated as follows: for all compatible pairs data (q_m, p_m), find (q_u, p_u) on Γ_u such that:

$$\begin{cases} \nabla \cdot \left(\dfrac{k}{\mu} \nabla p \right) = 0 \quad \text{in } \Omega, \\ p = p_m, \quad -\dfrac{k}{\mu} \dfrac{\partial p}{\partial n} = q_m \quad \text{on } \Gamma_m, \quad \text{and} \quad p = p_u, \quad -\dfrac{k}{\mu} \dfrac{\partial p}{\partial n} = q_u \quad \text{on } \Gamma_u \end{cases} \tag{3}$$

4. Solving Cauchy problem by minimizing an error functional

We propose now to use the method developed by Andrieux et al. [4] to solve the Cauchy problem (3) defined on the pressure field p. The idea is to use simultaneously the overspecified data by defining two well posed problem as follows:

$$\begin{cases} \nabla \cdot \left(\dfrac{k}{\mu} \nabla p_1 \right) = 0 & \text{in } \Omega \\ p_1 = p_m & \text{on } \Gamma_m \\ -\dfrac{k}{\mu} \dfrac{\partial p_1}{\partial n} = \eta & \text{on } \Gamma_u \end{cases} \qquad \begin{cases} \nabla \cdot \left(\dfrac{k}{\mu} \nabla p_2 \right) = 0 & \text{in } \Omega \\ p_2 = \tau & \text{on } \Gamma_u \\ -\dfrac{k}{\mu} \dfrac{\partial p_2}{\partial n} = q_m & \text{on } \Gamma_m \end{cases} \tag{4}$$

Then, an error functional is defined as a function of the pressure fields $p_1(p_m, \eta)$ and $p_2(\tau, q_m)$ as follows:

$$E(p_1, p_2) = \int_\Omega \nabla(p_1 - p_2) \cdot \frac{k}{\mu} \nabla(p_1 - p_2) \, \mathrm{d}\Omega \tag{5}$$

Using the properties of the p_i fields, we easily derive an alternate expression of E:

$$E(\eta, \tau) = \int_{\Gamma_u} (\eta - \nabla p_2.n) \frac{k}{\mu} (p_1 - \tau) + \int_{\Gamma_m} (\nabla p_1.n - q_m) \frac{k}{\mu} (p_m - p_2) \tag{6}$$

The functional E is always positive and expresses an energy-like error between the two fields p_1 and p_2. Assuming that the data q_m and p_m are compatible, the functional E vanishes when the pair (η, τ) meets the real data (q_u, p_u). Then $p_1 = p_2 + Cst$ and the Cauchy problem (3) is solved; for more details see Andrieux et al. [4]. To conclude, in order to identify leak zones, a Cauchy problem is derived from the Darcy equations, which is solved by minimizing an energy-like error functional. The optimization problem can be written as follows:

$$\begin{cases} (q_u, p_u) = \arg\min\limits_{\eta, \tau} E\big(p_1(\eta, p_m), p_2(q_m, \tau)\big) \\ \text{with } p_1, p_2 \text{ solutions of (4) and } (\eta, \tau) \in H^{-1/2}(\Gamma_u) \times H^{1/2}(\Gamma_u) \end{cases} \tag{7}$$

5. Numerical experiments

In this section, we apply the present method to a hydrogeology problem. We consider an underground aquifer where there flows a liquid saturating a porous media. The goal is to identify leaks on an inaccessible part of the boundary by exploiting overspecified measurements on the remaining parts. We assume, for sake of simplicity, that the domain is a rectangle with constant pressure levels imposed on the left and right sides. The lower side is considered as an

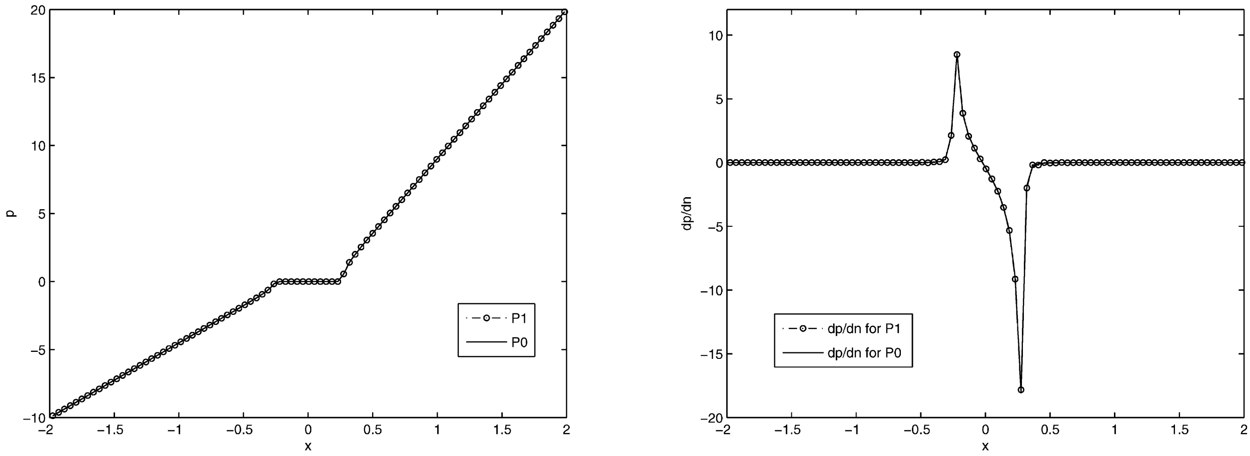

Fig. 2. Left side: pressure for direct problem (P0) and data completion problem (P1) with medium gap on Γ_u with $h = 0.5$. Right side: normal pressure gradient for the direct problem (P0) and data completion problem (P1) with medium gap on Γ_u with $h = 0.5$.

Fig. 2. A gauche : la pression sur Γ_u du problème direct (P0) et celle du problème inverse (P1) pour une largeur de fuite moyenne : $h = 0{,}5$. A droite : le gradient de pression normal sur Γ_u du problème direct (P0) et celui du problème inverse (P1) pour une largeur de fuite moyenne : $h = 0{,}5$.

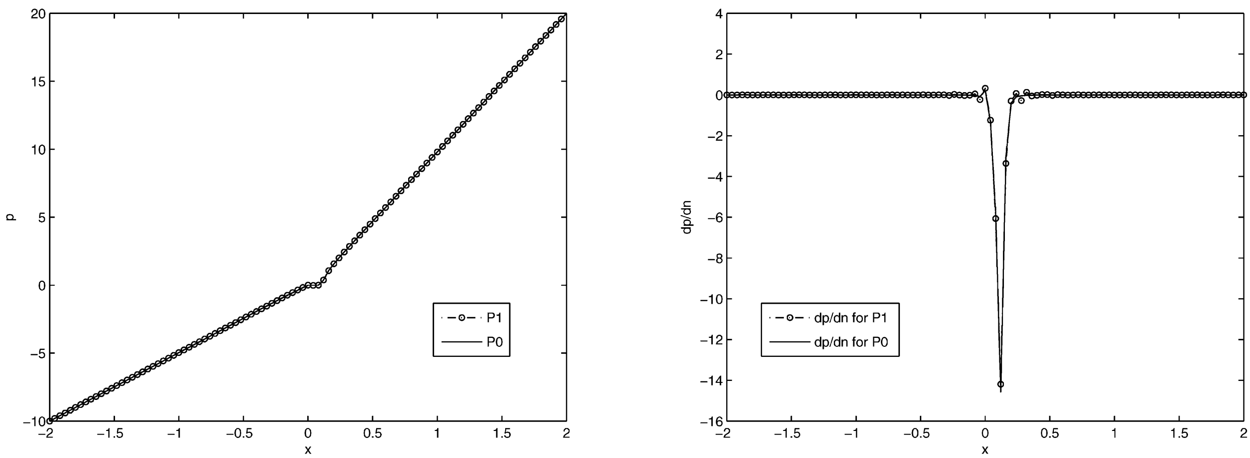

Fig. 3. Left side: pressure for direct problem (P0) and data completion problem (P1) with small gap on Γ_u with $h = 0.1$. Right side: normal pressure gradient for the direct problem (P0) and data completion problem (P1) with small gap on Γ_u with $h = 0.1$.

Fig. 3. A gauche : la pression sur Γ_u du problème direct (P0) et celle du problème inverse (P1) pour une fuite de petite largeur : $h = 0{,}1$. A droite : le gradient de pression normal sur Γ_u du problème direct (P0) et celui du problème inverse (P1) pour une fuite de petite largeur : $h = 0{,}1$.

impermeable wall and the inaccessible upper side may contain one or various leaks (see Fig. 1). We assume $k/\mu = 1$. All numerical experiments are performed by using Comsol Multiphysics [7].

The data completion methodology consists in solving the Cauchy problem (7) where the overspecified data (q_m, p_m) on the boundary Γ_m are extracted from the direct problem. The domain is meshed uniformly with a resolution of 80 by 16 cells. To show the efficiency of the method, three cases have been studied. The first and second cases have only one leak of different sizes, the third case has two leaks with different sizes. All figures show, the identified pressure and the normal gradient of the pressure. In Figs. 2 and 3, notice that the leak is identified through a zero pressure plateau on the pressure curve, and a sharp peak on the normal pressure gradient plot. The velocity through the leak can easily be computed with Darcy law ($\mathbf{u} \cdot \mathbf{n} = -\frac{k}{\mu}\frac{\partial p}{\partial n}$) as can the volumic flow rate ($Q_v = \int_{\Gamma_u} \mathbf{u} \cdot \mathbf{n}\, \mathrm{d}s$). A positive pressure gradient peak means inward flow, and conversely, a negative peak an outward flow. In both cases, data recovered from the data completion problem are in very good agreement with the direct problem solution.

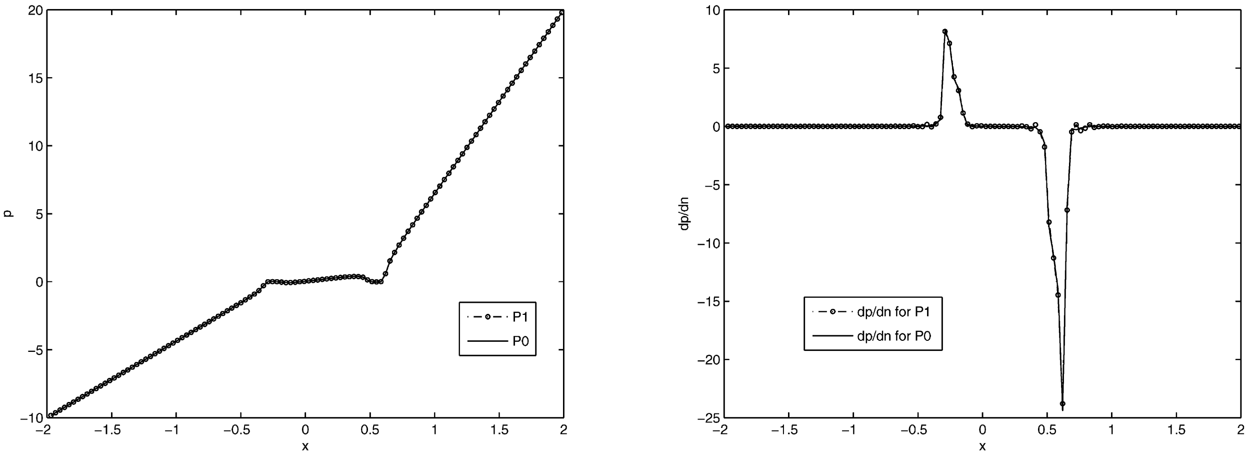

Fig. 4. Left side: pressure for direct problem (P0) and data completion problem (P1) with two gaps on Γ_u with $h = 0.1$. Right side: normal pressure gradient for the direct problem (P0) and data completion problem (P1) with two gaps on Γ_u with $h = 0.1$.

Fig. 4. A gauche : la pression sur Γ_u du problème direct (P0) et celle du problème inverse (P1) pour le cas de deux fuites. A droite : le gradient de pression normal sur Γ_u du problème direct (P0) et celui du problème inverse (P1) pour le cas de deux fuites.

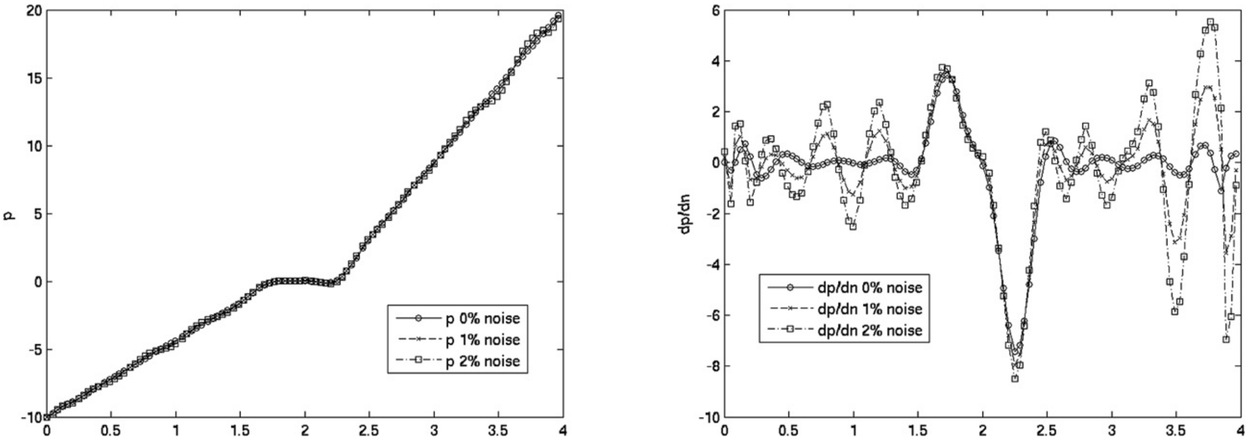

Fig. 5. Left side: pressure for data completion problem (P1) on Γ_u with medium gap: $h = 0.5$, obtained with noisy Dirichlet BC on Γ_m. Right side: normal pressure gradient for data completion problem (P1) on Γ_u with medium gap: $h = 0.5$, obtained with noisy Dirichlet BC on Γ_m.

Fig. 5. A gauche : la pression sur Γ_u du problème direct (P0) et celle du problème inverse (P1) pour des données bruitées et une largeur de fuite moyenne : $h = 0{,}5$. A droite : le gradient de pression normal sur Γ_u du problème direct (P0) et celui du problème inverse (P1) pour des données bruitées et une largeur de fuite moyenne : $h = 0{,}5$.

Fig. 4 shows the identified data for the third case where two identical leaks are located on the border Γ_u. The pressure curve exhibits a somehow large zero plateau, but the normal pressure gradient curve shows two leaks, the first outward and the second inward, clearly and separately identified. Again data completion results match very well with direct problem data.

To take into account the sensitivity of leak identification to noisy data, an uniform random white noise with null mean is applied to the Dirichlet data on Γ_m of the medium sized leak setup. Fig. 5 shows the identified data for two noise levels (1% and 2%). In spite of the noise, the leak is clearly identified on the pressure curve. However, the normal gradient pressure appears to be more sensitive to noise, hence the evaluation of the leak intensity is less accurate. Moreover, the volumic flow rate through Γ_u evaluated from the identified data is in good agreement with that obtained from the direct problem: $Q_v = -0.9837$. Table 1 summarizes the volumic flow rate up to 5% noise level.

Table 1
Computed values of volumic rate flow for data completion problem (P1) on the Γ_u

Tableau 1
Débit calculé sur Γ_u à partir du problème (P1) après convergence et pour different taux de bruit

Noise level	0%	1%	2%	5%
Rate flow (relative error)	−0.9836 (0.01 %)	−0.9831 (0.06 %)	−0.9827 (0.1 %)	−0.998 (0.74 %)

The relative error is computed against the direct problem (P0) value $Q_v = -0.9837$ (outward flow).
L'erreur relative est évaluée par rapport au débit obtenu pour le problème (P0) $Q_v = -0{,}9837$ (débit sortant).

6. Conclusion and perspectives

In this Note, leakage identification is carried out by solving a Cauchy problem derived from the Darcy equations. The Cauchy problem is solved by minimizing an energy-like functional which depends on the missing pressure and normal pressure gradient. Identification of leaks in a saturated porous media is satisfactory against different influences: leak size, position and multiplicity, in comparison to the direct problem. Analysis of results allows leak identification by means of the pressure fingerprint (size and position) and its intensity through the normal pressure gradient or volumic flow rate.

Sensitivity of the model to noisy data was addressed, as known it is as a hard issue for inverse problem. However, measurement of the leak strength from a noisy problem setup is still achievable by integration of the normal pressure gradient (volumic flow rate) even without regularization of the minimized functional.

All these numerical issues and others, such as the geometric complexity (tridimensional domain) will be addressed in further works and applied to more intricate fluid flow models like Stokes or Navier–Stokes equations.

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