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Influence of wall properties on peristaltic transport with heat transfer

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Abstract

The effect of elasticity of the flexible walls on peristaltic transport of an incompressible viscous fluid, with heat transfer, in a two dimensional uniform channel has been investigated under long wave length approximation. The perturbation solution has been obtained in terms of wall slope parameter and closed form expressions have been derived for velocity, temperature and heat transfer. The effects of elastic tension, damping and mass characterizing parameters on temperature and heat transfer have been studied. It is found that heat transfer increases with elastic tension and mass characterizing parameters. ***To cite this article: G. Radhakrishnamacharya, Ch. Srinivasulu, C. R. Mecanique 335 (2007).***

Résumé

L'influence des propriétés de la paroi sur transport péristaltique avec transfert de chaleur. Nous avons étudié le rôle de l'élasticité des parois flexibles sur le transport péristaltique d'un fluide visqueux incompressible, avec échange de chaleur, dans une conduite uniforme bi-dimensionnelle dans l'approximation de grande longueur d'ondes. Une solution perturbative a été obtenue, dépendant de la pente de la paroi comme paramètre, ainsi que les expressions explicites de la vitesse, température et le flux de chaleur. On a étudié également les effets des paramètres caractérisant la tension élastique, mouillage et la masse sur la température et l'échange thermique. On a trouvé que le transfert de chaleur augmente avec la tension élastique et la masse. ***Pour citer cet article : G. Radhakrishnamacharya, Ch. Srinivasulu, C. R. Mecanique 335 (2007).***

Keywords: Heat transfer; Peristaltic transport; Elastic walls; Viscous fluids; Biomedical instruments

Mots-clés : Transferts thermiques ; Transport péristaltique ; Parois élastiques ; Fluides visqueux ; Appareils biomédicaux

1. Introduction

The term peristalsis is used for the mechanism by which a fluid can be transported through a distensible tube when progressive waves of area contraction and expansion propagate along its length. This is known to be a major mechanism for transport of fluids in physiological systems. Some biomedical instruments, like the blood pumps in dialysis and the heart lung machine, use this principle. Peristaltic transport of a toxic liquid is used in nuclear industry

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to avoid contamination of the outside environment. The industrial use of this pumping mechanism in roller/finger pumps to pump slurries and corrosive fluids is well known.

Shapiro et al. [1], Srinivasacharya et al. [2] and Muthu et al. [3] have studied peristalsis in both mechanical and physiological situations under different conditions. However, the interaction of peristalsis and heat transfer has not received much attention. This may become highly relevant and significant in several industrial processes. Also thermodynamical aspects of blood may become significant in processes like oxygenation and hemodialysis when blood is drawn out of the body. Victor and Shah [4] studied heat transfer to blood using the Casson model. Radhakrishnamacharya and Radhakrishna Murty [5] investigated peristalsis with heat transfer in a non-uniform channel. Mittra and Prasad [6] considered peristaltic transport in a two-dimensional channel considering the elasticity of the walls.

Hence, in this Note, the interaction of peristalsis with heat transfer has been investigated for the motion of a viscous incompressible Newtonian fluid in a two dimensional uniform channel with wall effects. Following the analysis of Mittra and Prasad [6], the wall is modeled as a stretched membrane and this together with the equation of motion of flexible wall and continuity of stresses result in a more realistic boundary condition. Perturbation forms of solutions have been obtained for velocity, temperature and heat transfer. The effects of pertinent parameters on temperature and heat transfer have been studied.

2. Formulation of the problem

Consider the flow of a Newtonian viscous fluid through a two dimensional channel of uniform thickness. The walls of the channel are assumed to be flexible and are taken as a stretched membrane, on which traveling sinusoidal waves of moderate amplitude are imposed.

The geometry of the channel wall is given by

$$y = \eta(x,t) = d + a\sin\frac{2\pi}{\lambda}(x - ct) \tag{1}$$

and the equations governing the motion for the present problem are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \upsilon\nabla^2 u \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y} + \upsilon\nabla^2 v \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$C'\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{k}{\rho}\nabla^2 T + \upsilon\phi \tag{5}$$

where u, v are the components of velocity along x and y directions respectively, d is the mean half width of the channel, a is the amplitude, λ is the wavelength and c is the phase speed of the wave, ρ is the density of fluid, υ is the kinematic coefficient of viscosity, p is the pressure, T is the temperature, C' is the specific heat and k is the thermal conductivity,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{and} \quad \phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 \tag{6}$$

Following the analysis of Srinivasulu and Radhakrishnamacharya [7], the boundary conditions are

$$u = 0 \quad \text{at } y = \pm\eta = \pm\left[d + a\sin\frac{2\pi}{\lambda}(x - ct)\right] \tag{7}$$

$$\frac{\partial}{\partial x}L(\eta) = \frac{\partial p}{\partial x} = \rho\upsilon\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] \quad \text{at } y = \pm\eta \tag{8}$$

$$\left.\begin{aligned} T &= T_0 \quad \text{on } y = -\eta \\ T &= T_1 \quad \text{on } Y = \eta \end{aligned}\right\} \tag{9}$$

where $L = -\tau \frac{\partial^2}{\partial x^2} + m \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t}$. Here τ is the elastic tension in the membrane, m is the mass per unit area and C is the coefficient of viscous damping.

Introducing ψ such that $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ and the following non-dimensional quantities

$$x' = \frac{x}{\lambda}; \quad y' = \frac{y}{d}; \quad \psi' = \frac{\psi}{cd}; \quad t' = \frac{ct}{\lambda}; \quad \theta = \frac{T - T_0}{T_1 - T_0}$$

in Eqs. (1)–(5), (7)–(9) and eliminating p, we finally get (after dropping primes)

$$\delta R\left[\left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_t + \psi_y\left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_x - \psi_x\left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_y\right] = \left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \psi \tag{10}$$

$$R\delta\left(\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}\right) = \frac{1}{P}\left(\frac{\partial^2}{\partial y^2} + \delta^2 \frac{\partial^2}{\partial x^2}\right)\theta + E\left(4\delta^2\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2}\right)^2\right) \tag{11}$$

$$\frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm\eta = \pm\left[1 + \varepsilon \sin 2\pi(x - t)\right] \tag{12}$$

$$\delta^2(\psi_{xxy} + \psi_{yyy}) - R\delta(\psi_{yt} + \psi_y \psi_{yx} - \psi_x \psi_{yy}) = \left[E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t}\right](\eta) \tag{13}$$

$$\left.\begin{aligned} \theta &= 0 \quad \text{on } y = -\eta \\ \theta &= 1 \quad \text{on } y = \eta \end{aligned}\right\} \tag{14}$$

where ε $(= a/d)$, δ $(= d/\lambda)$ are geometric parameters, R $(= cd/\upsilon)$ is the Reynolds number, E_1 $(= -\tau d^3/\lambda^3 \rho \upsilon c)$, E_2 $(= mcd^3/\lambda^2 \rho \upsilon)$, E_3 $(= Cd^3/\upsilon \lambda^2 \rho)$ are the non-dimensional elasticity parameters, $P(-\rho \upsilon C'/K)$ is the Prandtl number and $E = c^2/[C'(T_1 - T_0)]$ is the Eckert number.

3. Method of solution

We seek perturbation solution in terms of small parameter δ as follows:

$$F = F_0 + \delta F_1 + \delta^2 F_2 + \cdots \tag{15}$$

where F represents any flow variable.

Substituting (15) in Eqs. (10) to (14) and collecting the coefficients of various powers of δ and solving the resultant equations under the relevant boundary conditions, we finally get

$$\psi = \psi_0 + \delta \psi_1 + \cdots \tag{16}$$

$$\theta = \theta_0 + \delta \theta_1 + \delta^2 \theta_2 \cdots \tag{17}$$

$$\psi_0 = A_1\left(y^3 - 3\eta^2 y\right), \qquad \psi_1 = R\left[B_1 y^7 + B_2 y^5 + K_3 y^3 + k_1 y\right]$$

$$\theta_0 = -3PEA_1^2 y^4 + C_1 y + C_2$$

$$\theta_1 = PR\left[(D_1/56)y^8 + (D_2/30)y^6 + (D_3/20)y^5 + (D_4/12)y^4 + (D_5/6)y^3 + (D_6/2)y^2\right] + D_7 y + D_8$$

$$A_1 = -\left(4\varepsilon\pi^3/3\right)\left[(E_1 + E_2)\cos 2\pi(x - t) - E_3 \sin 2\pi(x - t)\right], \qquad B_1 = A_1 A_{1x}, \qquad C_1 = 1/(2\eta)$$

$$B_2 = \left(6A_1^2 \eta \eta_x - A_{1x}\right)/20$$

$$K_1 = -\left[A_1 A_{1x}\eta^6/10 + 9A_1 A_{1x}\eta^5/2 + 21A_1^2 \eta_x \eta^5/2 + 5A_{1x}\eta^4/4 + 3A_1 \eta_x \eta^3\right]$$

$$K_3 = \eta\left(3\eta^3 B_1 + 6\eta^2 \eta_x A_1^2 + \eta A_{1x} + 2A_1 \eta_x\right)/2, \qquad C_2 = \left(1 + 6PEA_1^2 \eta^4\right)/2$$

$$D_1 = -(6P + 504)EA_1 B_1, \qquad D_2 = 6PEB_1 - 18PE\eta^2 A_1 B_1 - 72PEA_1^2 \eta \eta_x - 240EA_1 B_2$$

$$D_3 = 3A_1 C_{1x} - A_{1x} C_1 - 72EA_1 K_3, \qquad D_4 = 3A_1 C_{2x}, \qquad D_6 = C_{2t} - 3A_1 C_{2x}\eta^2$$

$$D_5 = C_{1t} - 3A_1 C_{1x}\eta^2 + 3C_1\left(A_{1x}\eta^2 + 2A_1 \eta \eta_x\right), \qquad D_7 = -(D_3/20)\eta^4 + (D_5/6)\eta^2$$

$$D_8 = -(D_1/56)\eta^8 + (D_2/30)\eta^6 + (D_4/12)\eta^4 + (D_6/2)\eta^2$$

The coefficient of heat transfer at the wall is given by

$$Z = \frac{\partial \eta}{\partial x}\frac{\partial \theta_0}{\partial y} + \delta\left[\frac{\partial \theta_0}{\partial x} + \frac{\partial \eta}{\partial x}\frac{\partial \theta_1}{\partial y}\right] \tag{18}$$

4. Results and discussion

The temperature profiles are plotted in Fig. 1 for various values of E_1, E_2, E_3 and fixed values of P $(=1)$, E $(=1)$, R $(=1)$, ε $(=0.2)$, δ $(=0.2)$ and $x = 0.7$. It can be seen that the temperature remains negative for all the values of various parameters considered. The absolute value of the temperature increases with the elastic tension i.e., the rigidity of the wall (E_1) and also the mass characterizing parameter i.e., the stiffness of the wall (E_2). This increase is more

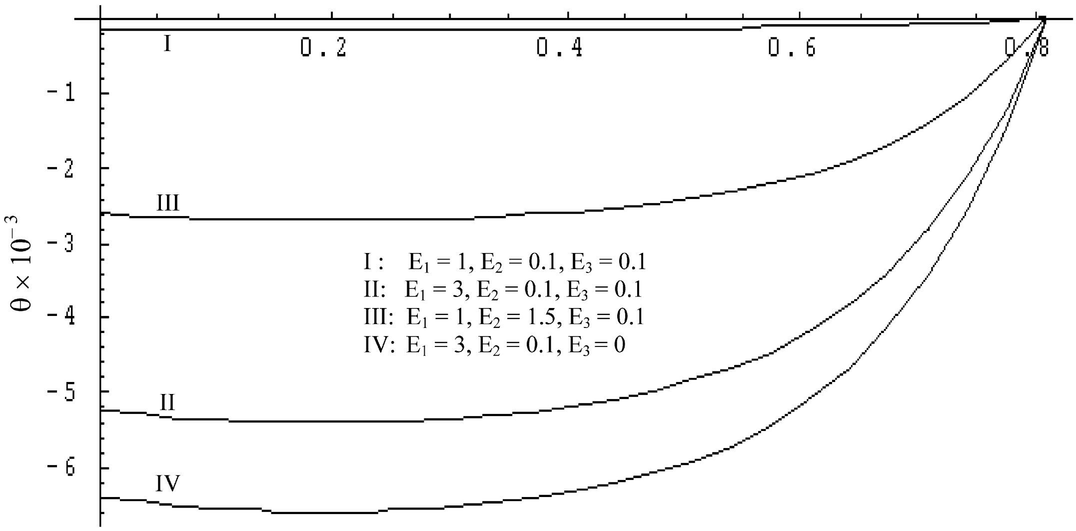

Fig. 1. Effect of E_1, E_2, E_3 on temperature.

Fig. 1. Effet des paramètres E_1, E_2, E_3 sur la température.

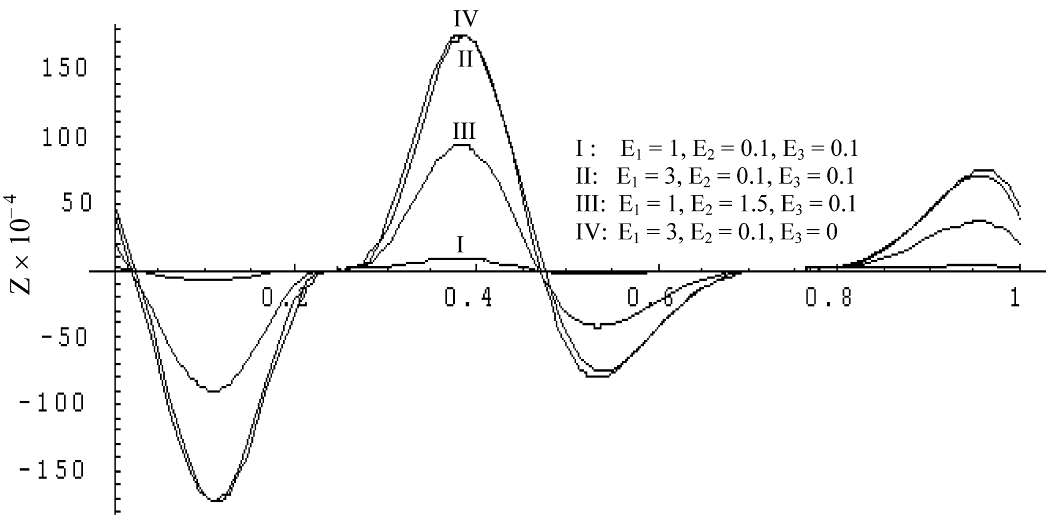

Fig. 2. Effect of E_1, E_2, E_3 on heat transfer.

Fig. 2. Effet des paramètres E_1, E_2, E_3 sur le transfert de chaleur.

significant in the case of rigidity of the wall. Further, it can be seen that the influence of the damping nature of the wall (E_3) on the temperature is not very significant.

The effects of elastic parameters on heat transfer at the wall are graphically shown in Fig. 2. These figures show typical oscillatory behavior of heat transfer which may be due to peristalsis. The absolute value of heat transfer increases significantly as rigidity of the wall (E_1) increases. Also, it increases with stiffness (E_2) but this increase is not as predominant as in the case of rigidity. The damping nature of the wall (E_3) has very insignificant influence on the heat transfer.

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