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Dual length scale two-equation modelling of the canopy turbulent kinetic energy wake budget

Christophe Sanz [a,*], Gabriel G. Katul [b]

[a] *3, rue du Mont Louvet, 91640 Fontenay lès Briis, France*
[b] *Nicholas School of the Environment and Earth Sciences, Duke University, Durham, NC 27708, USA*

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Abstract

Within vegetation canopies, the turbulent kinetic energy (k) budget is mainly modelled through source terms added to the free-air state formulation. The dependence of the modelled source term coefficients upon a dimensionless ratio (λ) between the mixing length for turbulent transport (l_m) and the relaxation length scale (l_ε) of Kolmogorov's relation is proposed. Using dimensional analysis, the order of magnitude variation of the terms involved in the newly proposed model for the coefficients of these source terms are derived. When λ is a constant, this generalized model results in a similarity constant ($C_{\varepsilon 4}$) independent of the source term model, lending support to an earlier conjecture by Seginer. ***To cite this article: C. Sanz, G.G. Katul, C. R. Mecanique 335 (2007).***

Résumé

Modélisation à 2 équations et 2 échelles de longueur de l'énergie cinétique turbulente dans la canopée. Dans la canopée, la modélisation du bilan de sillage de l'énergie cinétique turbulente (k) repose principalement sur des termes source additionnels. Nous avons déterminé la relation entre les coefficients des modèles usuels de termes source et le rapport sans dimension (λ) entre l'échelle de longueur du transport turbulent (l_m) et celle de la relation de Kolmogorov (l_ε). Nous avons généralisé les modèles de termes source par analyse dimensionnelle et nous avons déterminé l'ordre de grandeur de la variation des différents termes. Lorsque λ est une constante, le modèle de terme source généralisé présente une constante de similitude ($C_{\varepsilon 4}$) indépendante du modèle de termes source, ce qui tend à confirmer la conjecture de Seginer. ***Pour citer cet article : C. Sanz, G.G. Katul, C. R. Mecanique 335 (2007).***

Keywords: Turbulence; Dissipation; Turbid medium; Turbulence model; Vegetation canopy; Wakes

Mots-clés : Turbulence ; Dissipation ; Couvert végétal ; Milieu turbide ; Modèle de turbulence ; Sillages

* Corresponding author.
E-mail addresses: christophe.sanz@online.fr (C. Sanz), gaby@duke.edu (G.G. Katul).

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Cette Note présente deux raffinements du modèle de turbulence à deux équations pour application à la canopée. Premièrement, nous reformulons le modèle sous l'hypothèse que la longueur de mélange (l_m) diffère de l_ε, soit $\lambda(=l_m/l_\varepsilon) \neq 1$. Deuxièmement, nous généralisons les termes source et/ou puits : S_k pour l'énergie cinétique de la turbulence (k) et S_ε pour le taux de dissipation moléculaire de k (ε). L'hypothèse selon laquelle $\lambda \neq 1$ est l'hypothèse classique des modèles spécifiques de la turbulence dans un couvert végétal [1–3]. Nous avons calculé les coefficients de S_k et S_ε découlant de la contrainte que le modèle résultant reproduise la décroissance exponentielle de la vitesse moyenne (U) de l'écoulement dans le couvert, lorsque $\lambda \neq 1$. Nous en déduisons l'expression du coefficient $C_{\varepsilon 4} = \frac{S_\varepsilon/\varepsilon}{S_k/k}$. Nous avons réitéré cette procédure pour un modèle de terme source généralisé. Dans le cas usuel où λ est constant dans le couvert, l'indépendance entre $C_{\varepsilon 4}$ et le modèle de terme source atteste de la consistance physique de $C_{\varepsilon 4}$ et tend à confirmer la conjecture de Seginer sur laquelle se fondent nos résultats. Cette conjecture permet également de prédire la variation de l'ordre de grandeur des différents termes du modèle de termes source généralisé. L'analyse dimensionnelle utilisée pour généraliser les modèles de termes source est justifiée par la difficulté à déduire le bilan de sillage de k de celui de la quantité de mouvement. Certains termes du modèle généralisé sont susceptibles de rendre compte de mécanismes secondaires mais significatifs et leur prise en compte pourrait améliorer les qualités prédictives du modèle. La signification physique des termes correspondants du modèle généralisé demeure toutefois une question ouverte.

1. Introduction

The interaction between canopies and the turbulent atmosphere within them is now receiving significant attention in both vegetated and urban settings. Modelling air quality within urban areas, designing improved peri-urban environments that can maximize animal comfort and improve their water use efficiency, or explore the transfer of scalars such as CO_2 or NO from the forest floor to the free atmosphere are but a few of the applications that benefit from advancements in modelling canopy–turbulence interactions. Such modelling tools must satisfactorily introduce the aerodynamics of canopy architecture and their turbulent kinetic energy wake budget, which remains a bottleneck to progress. Modelling the turbulent kinetic energy (k) budget within plant canopies starts from a canopy-free state, which is then modified to include additional processes reflecting the effects of the canopy such as wake generation and short-circuiting of the energy cascade [4]. The common model for k production accounts for two major processes [5,6]: (i) the fine scale interaction between the air flow and the vegetation architecture, which breaks the scale of flow motion through wake turbulent production, and (ii) the overall reduction of turbulent length scales, which modifies the classical turbulence cascade and thereby enhances k destruction (often referred to as the spectral short-circuiting). Early models relied on the specification of a single intrinsic (mixing) length scale for the entire turbulence regime within the canopy [1], but this is not entirely satisfactory [6]. The two-equation modelling framework avoids prescribing a canonical mixing length but accounting for the emergent two length scales (production by shear and wakes) may provide a more general treatment of these two effects on the k budget source term (S_k). There are two major difficulties in deriving a model for S_k. First, the common [5–9] model for the form drag force ($\rho S_{\vec{u}}$) in the momentum budget involves a quadratic function in the mean flow velocity ($\vec{u}$) magnitude, which makes the formal derivation S_k from $S_{\vec{u}}$ [5,9] an open problem. Second, despite the growing amount of mathematical evidence for the form of the modelled turbulent kinetic energy dissipation rate (ε) budget equation [10], the S_ε terms lack a rigorous physical basis, which compounds the difficulties in estimating its coefficients. As early as 1974, Seginer [11] recognized the product ($\alpha \approx 0.06$) of an intrinsic mixing length (l_m) with the form drag coefficient (C_X) as being a dimensionless constant for canopy aerodynamics, which was later confirmed by experiments [1,2]. On the basis of Seginer's conjecture about a constant α, the coefficients of common models for S_k [6,8] may be determined by a dimensionless rate (β_P) of the conversion into wake k of the mean kinetic energy lost by drag. The constraint imposed on the overall model is that it must match the exponential decay of the mean velocity profile within a dense canopy for a stationary and planar homogeneous flow case in the absence of subsidence. This approach was used in Sanz [12] to determine the model constants for S_k, and we term this overall method as the [$\wedge$] method. However, the application of the [$\wedge$] method to determine the coefficients of the standard S_k model has only been carried out under the assumption that l_m matches l_ε [13], thereby missing this distinction between l_m and l_ε.

2. Dual length scales turbulence modelling

Defining $\lambda = l_m/l_\varepsilon$, it can be shown that the classical two-equation turbulence model relies on the assumption $\lambda = 1$. However, inside the canopy, l_m and l_ε are no longer identical [1,3] and λ must diverge from unity. Below, we show how the closure constants of classical two-equation models can be corrected for variations in λ. The expression for the kinematic turbulent viscosity model is given by

$$\nu_t = \lambda C_\mu \frac{k^2}{\varepsilon} \tag{1}$$

Eq. (1) derives from the one-equation closure model

$$\nu_t = C_0 l_m \sqrt{k} \tag{2}$$

and from the Kolmogorov relationship

$$l_\varepsilon = C_\varepsilon \frac{k^{3/2}}{\varepsilon} \tag{3}$$

where $C_\varepsilon = C_\mu^{3/4}$ and $C_0 = C_\mu^{1/4}$. In a stationary and planar-homogeneous flow, the problem becomes uni-dimensional and its analytical solution for

$$k = \frac{U^2}{C_g} \tag{4}$$

and

$$\varepsilon = \lambda \left(\frac{C_X}{4}\right) U^3 \tag{5}$$

derive from the closure assumptions upon identification of Eqs. (2) and (1) with the Prandtl first order closure model [12], where

$$\frac{\mathrm{d}U}{\mathrm{d}z} = \left[\frac{C_X C_g}{4\sqrt{C_\mu}}\right] U \tag{6}$$

C_X is the form drag coefficient defined such as $S_U = -\frac{C_X}{2} U^2$, $C_g = (\frac{2}{\alpha})^{2/3}\sqrt{C_\mu}$, and z is the vertical direction. The above expressions for ν_t, k and ε can now be used to derive the two coefficients $C_{\varepsilon 4}$ and β_d [12] that relate S_k and S_ε to the basic variables U, k, and ϵ solved for by two-equation models:

$$\beta_d = C_g \left(\beta_P + \frac{1-\lambda}{2}\right) + \frac{3}{\sigma_k} \tag{7}$$

and

$$C_{\varepsilon 4} = \frac{(\frac{C_\mu}{C_g^2 \sigma_\varepsilon})\frac{\mathrm{d}^2\lambda}{\mathrm{d}z^2} + 7(\frac{C_X C_\mu}{C_g \sigma_\varepsilon})\frac{\mathrm{d}\lambda}{\mathrm{d}z} + (\frac{C_X}{4})^2 f(\lambda)}{2\lambda(\frac{C_X}{4})^2[\frac{3}{\sigma_k} + C_g(\frac{1-\lambda}{2})]} \tag{8}$$

where

$$f(\lambda) = C_g[C_{\varepsilon 1} - \lambda C_{\varepsilon 2}] + \frac{12}{\sigma_\varepsilon}\lambda \tag{9}$$

and $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_k, σ_ε are the closure constants of the k–ε turbulence model [14]. The de-coupling between S_k and S_ε leading to Eq. (8) is not necessary for the model to match the exponential velocity profile decay. However, it ensures that the ε budget is independent of the β_P value [12]. The above similarity constants must be unaltered for the flow outside the canopy if the model is to reproduce the entire canopy sublayer.

Table 1
Coefficients of the generalised S_k model

Term	Scale (γ_i)	Suggested meaning	Coefficient	References
$\frac{k^2}{U}$	.01	–	$\leqslant 0$	[9]
$k^{3/2}$	.03	–	$\leqslant 0$	[9]
Uk	.1	**cascade bypass**	$-\beta_d$	[6,7,12]
$U^2k^{1/2}$	.3	return to isotropy	$\leqslant 0$	[2]
U^3	1	**wake production**	β_P	[6,7,12]

Bold types refer to the standard source terms of the k–ε model.

3. Generalizing the model for k wake budget

A generalized expression for k wake source term can be expressed as

$$S_k = \frac{C_X}{2}\sum_{i\in\mathbb{Z}}\left[\beta_i\left(\frac{U}{k^{1/2}}\right)^i\right]k^{3/2} \tag{10}$$

This form relies on dimensional analysis with additional constraints that the power series expansions in U, k and ε retain only the integer powers of U. Derivation details are provided in Appendix A. The derivation does not predict whether the individual terms in the expansion are sources or sinks. However, Table 1 provides plausible links between key physical processes pertinent to k flow within the canopy and the five terms already identified (along with corresponding references). Similarly, we derived a generalized source term model for ε

$$S_\varepsilon = \frac{C_X}{2}\sum_{i\in\mathbb{Z}}\left[C_{\varepsilon 4_i}\beta_i\left(\frac{U}{k^{1/2}}\right)^i\right]k^{1/2}\varepsilon \tag{11}$$

where

$$\forall i \in \mathbb{Z}: \quad C_{\varepsilon 4_i} \neq 0 \tag{12}$$

Interestingly, when λ is a constant independent of z, the $C_{\varepsilon 4}$ value, which derives from the [$\wedge$] method, is the same as the one predicted by Eq. (8), irrespective of $(\beta_i)_{i\in\mathbb{Z}}$. Then,

$$S_k = \frac{C_X}{2}\sum_{i\in\mathbb{Z}}\gamma_i U^3 \tag{13}$$

where $\gamma_i = C_g^{(i-3)/2}$ and $C_g \approx 10$, derives by substitution of the analytical equation (4) for k inside the canopy within Eq. (10). Eq. (13) provides a scale for the corresponding terms, as shown in Table 1. The γ_i values in Table 1 for $i = -1, 0, 1, 2$ already exhibit a 100-fold variation. This spread in γ_i values shows that modelling k wake budget beyond these four terms could lead to numerical instabilities.

4. Conclusions

For k–ε simulations of canopy flows, we determined how the coefficients of the modelled wake source terms in the turbulent kinetic energy budget vary with λ. Corresponding results for other two equation turbulence models derive from our equations, by analogy. In the constant λ case, $C_{\varepsilon 4}$ remains the same for both the standard and the generalized S_k model. This latter generalized model was derived from dimensional analysis. Our results support the argument that $C_{\varepsilon 4}$ is a constant for the viscous dissipation of the wake turbulent kinetic energy within the canopy. Furthermore, our analysis is consistent with Seginer's conjecture that α is a key dimensionless constant representing canopy wake turbulence in dense canopies. The dimensional analysis proposed here proves that the S_k model does not explicitly involve ε. However, when the first order spatial derivative operator $(\partial/\partial x_i)$ is used within the source term model, this independence does not hold. When such spatial derivative is involved within the S_k, the source term can be shown to be proportional to $1/\varepsilon$ by using a procedure similar to that presented in Appendix A. When the S_k model embeds more than two terms, its coefficients cannot be determined by the [$\wedge$] method beyond the analytical expression of the least physically based coefficient from the other coefficients and model constants. Determining more than two S_k model coefficients requires an additional method. Yet, even if our generalized model for S_k appears to encompass the main published ones, most of its terms remain unexplained. Understanding these latter terms and measuring their influence upon canopy wake budget modelling along the lines herein sketched may motivate further research and experiments.

Appendix A. Dimensional analysis of wake budget model terms

Generalized models for S_k and S_ε may be derived from dimensional analysis applied to the power series in U, k and ε

$$S_k\ [\mathrm{m}^2\,\mathrm{s}^{-3}] = \frac{C_X}{2}\sum_{i\in I}\beta_{ki}U^{a_{ki}}k^{b_{ki}}\varepsilon^{c_{ki}} \tag{A.1}$$

and

$$S_\varepsilon\ [\mathrm{m}^2\,\mathrm{s}^{-4}] = \frac{C_X}{2}\sum_{i\in I}\beta_{\varepsilon i}U^{a_{\varepsilon i}}k^{b_{\varepsilon i}}\varepsilon^{c_{\varepsilon i}} \tag{A.2}$$

where the quantities inside brackets represent dimensions and I is a fixed, though not herein defined, set of integers. More general forms for S_k and S_ε must use two distinct subsets: I_k for k and I_ε for ε. However, the Kolmogorov equation (3) implies a O(3/2) scaling between $S_\varepsilon/\varepsilon$ and S_k/k through its differential when l_ε remains almost constant [6] and this scaling ought to be verified over the whole (U, k, ε) field. This implies that $I_\varepsilon = I_k (= I)$ and thus

$$\beta_{ki} = 0 \quad \Leftrightarrow \quad \beta_{\varepsilon i} = 0 \tag{A.3}$$

Eq. (A.3) proves that there exists a set of never vanishing ($\forall i \in I\colon\ C_{\varepsilon 4_i} \neq 0$) constants such as

$$\beta_{\varepsilon i} = C_{\varepsilon 4_i}\beta_{ki} \tag{A.4}$$

for any i in I. Eq. (12) then derives when the symbols β_{ki} are replaced by the β_i. Finally, both dimensional counterparts for Eq. (A.1)

$$\frac{m^2}{s^3} = m^{-1}\sum_{i\in I}\left(\frac{m^{a_{ki}}}{s^{a_{ki}}}\frac{m^{2b_{ki}}}{s^{2b_{ki}}}\frac{m^{2c_{ki}}}{s^{3c_{ki}}}\right) \tag{A.5}$$

and (A.2)

$$\frac{m^2}{s^4} = m^{-1}\sum_{i\in I}\left(\frac{m^{a_{\varepsilon i}}}{s^{a_{\varepsilon i}}}\frac{m^{2b_{\varepsilon i}}}{s^{2b_{\varepsilon i}}}\frac{m^{2c_{\varepsilon i}}}{s^{3c_{\varepsilon i}}}\right) \tag{A.6}$$

have to be satisfied for any $I = \{i\}$ so as to ensure the dimensional consistence of Eqs. (A.1) and (A.2). This implies the generalized wake budget models (10) and (11) by retaining only the integer ($a_{ki} = a_{\varepsilon i} = i$, $i \in \mathbb{Z}$) powers of U.

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