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Use of the Boussinesq solution in geotechnical and road engineering: influence of plasticity

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Abstract

The Boussinesq solution for the distribution of stresses in a half-space resulting from surface loads is largely used in geotechnical and road engineering. It is based on the assumption of a linear–elastic homogeneous isotropic half-space for the soil media. Since the soil exhibits nonlinear and irreversible behavior, it is of major interest to study the validity of this solution for elastoplastic soils. This paper includes an investigation of this issue using finite element modeling. The study is conducted by comparing the elastic stress distribution to that obtained using elastoplastic finite element analyses. Results show that the plasticity reduces the attenuation of the vertical stresses in the soil mass, which means that the Boussinesq solution underestimates the stresses in an area which contributes to the soil settlement. ***To cite this article: M. Sadek, I. Shahrour, C. R. Mecanique 335 (2007).***

Résumé

Utilisation de la solution de Boussinesq en géotechnique et dans le domaine des chaussées : influence de la plasticité. La solution de Boussinesq pour la distribution des contraintes dans un espace semi infini est utilisée en géotechnique et dans le domaine des chaussées. Cette solution est fondée sur l'hypothèse d'un milieu isotrope, homogène et linéaire–élastique. Puisque le sol a un comportement non-linéaire et irréversible, il est intéressant d'étudier la validité de cette solution pour les sols élastoplastiques. Cet article comporte une analyse de cette question en utilisant une modélisation par éléments finis. L'analyse est réalisée en comparant la solution élastique à celle obtenue par une modélisation élastoplastique par éléments finis. Les résultats montrent que la plasticité réduit l'atténuation des contraintes verticales dans le sol, ce qui signifie que la solution de Boussinesq sous-estime les contraintes dans un domaine qui contribue aux tassements. ***Pour citer cet article : M. Sadek, I. Shahrour, C. R. Mecanique 335 (2007).***

Keywords: Soils; Boussinesq; Plasticity; Finite element; Half-space; Settlement; Stress

Mots-clés : Sols ; Boussinesq ; Plasticité ; Éléments finis ; Demi-espace ; Élastique ; Tassement ; Contraintes

1. Introduction

The Boussinesq solution (Boussinesq [1]) for the distribution of stresses in a half-space resulting from surface loads is largely used in geotechnical and road engineering. It is based on the assumption of linear–elastic homogeneous

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isotropic half-space for the soil media. In geotechnical engineering, this solution is generally used together with semi-empirical constitutive relations for the determination of the soil settlement due to the application of surface loads such as foundations and embankments. Conventional approaches in road pavement design are also based on linear elasticity. They use the Boussinesq solution, which was extended by Burminster [2,3] for two-layered and three-layered systems, then by Schiffman [4] for multilayered systems. Several pavement tests were carried out to evaluate the pertinence of the linear elasticity in pavements design. While some authors reported acceptable agreement between experimental results and the elastic theory (Siddharthan et al. [5], Nilsson et al. [6], Ullidtz [7]), others observed significant differences between the elastic theory and field measurements, which attained in some cases 200% (Ullidtz et al. [8], Zhang et al. [9]). The disparity between the experimental results and the elastic theory could be attributed to several factors such as the nonlinear behavior of the soil and the efficiency of the experimental system. Vrettos [10] examined the influence of the soil non-homogeneity on the stress distribution in an elastic soil. The non-homogeneity was modelled using depth-dependent soil stiffness. Analyses showed that the variation of the soil stiffness with depth did not significantly affect the distribution of the vertical stress, consequently the Boussinesq solution could be used for soils with depth-dependent stiffness.

It is well known that the soil material exhibits a nonlinear and irreversible behavior, even at low deformations. This behavior is generally considered using elastoplastic constitutive relations. Since this behavior largely influences the soil deformations and displacement resulting from the application of surface loads such as those due to foundations, traffic and embankments, it is of major interest to analyse its influence on the stress distribution. This analysis will provide information about the possibility of the use of the Boussinesq solution for the determination of stresses in elastoplastic soils.

2. Methodology

The Boussinesq solution for a soil mass subjected to a uniformly distributed load is compared to elastoplastic finite element analyses conducted for both cohesive and frictional soils. Finite element analyses were carried out using the finite element program ABAQUS. The soil behavior is described using an elastoplastic constitutive relation based on the non-associated Mohr–Coulomb criterion. Computations were carried out up to an advanced plasticity state. The first part of the paper is devoted to a purely cohesive soil, while the second concerns frictional soil. Numerical simulations were conducted under plane strain conditions with a loading width $B = 0.4$ m.

3. Purely cohesive soil

Analyses on a purely cohesive soil were conducted with the following soil characteristics: cohesion $C = 50$ kPa, a Young's modulus $E = 20$ MPa, and a Poisson's ratio $\nu = 0.3$. The ultimate bearing capacity in this case is equal to $q_{\max} = 257$ kPa ($q_{\max} = 5.14$ C).

Results of the finite element analysis are depicted in Fig. 1. It shows that plasticity induces a strong nonlinearity in the variation of the soil settlement at the loading centre (S) with the applied load (q). Figs. 1(b) and (c) illustrate a comparison of the vertical stress distribution between the elastic and elastoplastic solutions at the loading level $q = 240$ kPa which corresponds to an important development of the plasticity in the soil mass ($q/q_{\max} = 0.93$). It can be observed that plasticity leads to an important change in the vertical stress distribution under the loading area, characterized by a better transmission of the vertical stress in this area. This result is illustrated in Fig. 1(d) which shows the influence of plasticity on the variation of the vertical stress along the vertical axis. It can be observed that the increase in the loading level, leads to a significant diminution in the stress attenuation under the loading up to depth $Z/B = 2$. In this area ($Z/B \leqslant 2$), the stress ratio σ_v/q exceeds 0.2 which means that this area largely contributes to the soil settlement.

Fig. 1(e) depicts the influence of plasticity on the distribution of the lateral stress. It shows a progressive diminution of the normalized horizontal stress σ_H/q at the surface with the increase in the applied pressure. For example, the normalized lateral pressure σ_H/q at the surface is about 60% of that obtained with the elastic theory for $q = 240$ kPa. On the other hand, the profile of normalized lateral stress shows that plasticity leads to an important reduction of the attenuation in the lateral stress in the upper part of the soil mass, up to a depth $Z/B = 1.5$.

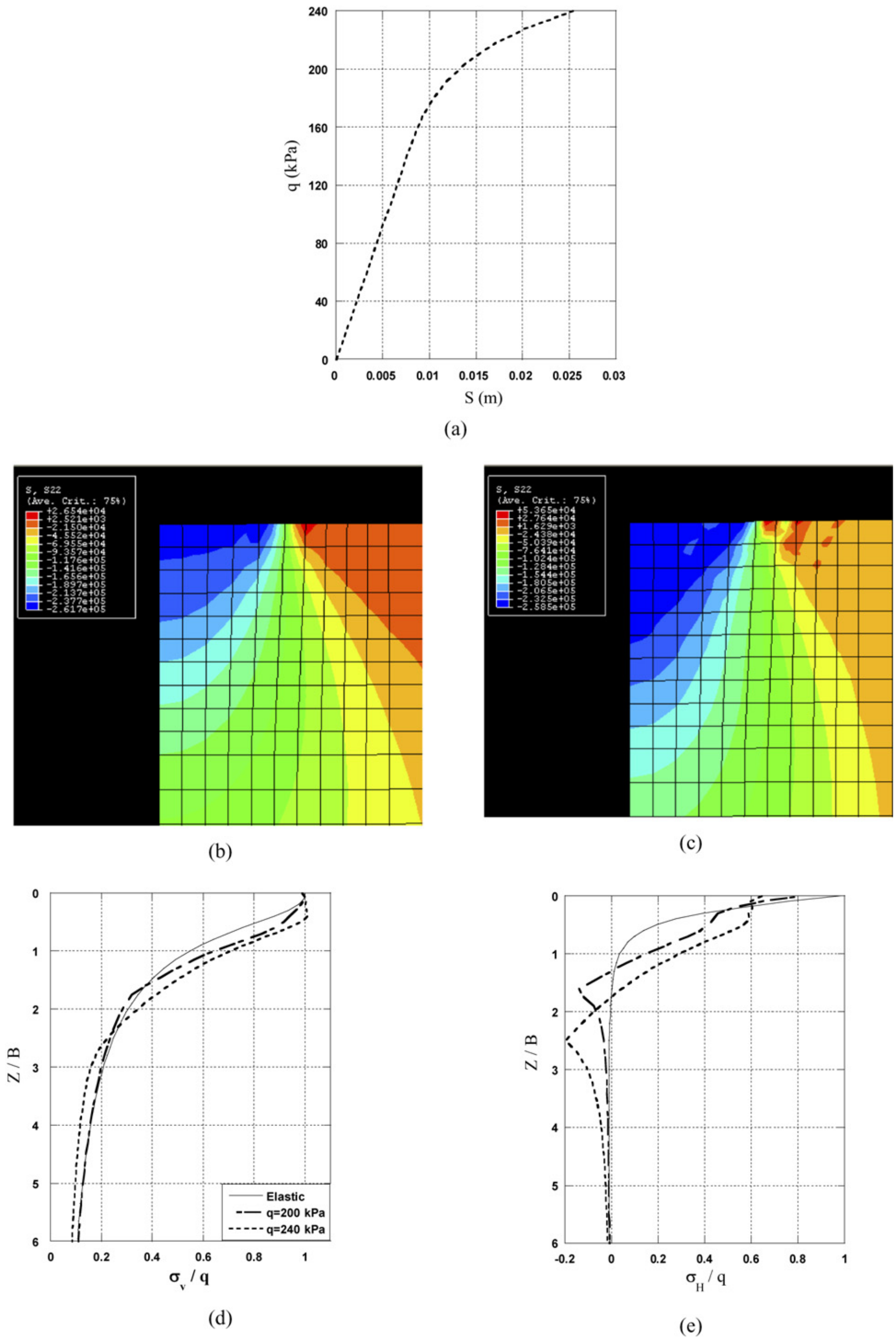

Fig. 1. Influence of plasticity on the stress distribution in a purely cohesive soil subjected to a uniformly distributed load ($C = 50$ kPa). (a) Settlement versus applied load; (b) vertical stress distribution—elastic theory, $q = 240$ kPa ($q/q_{\max} = 0.93$); (c) vertical stress distribution—elastoplastic theory, $q = 240$ kPa; (d) variation of the vertical stress along the vertical axis; (e) Variation of the horizontal stress along the vertical axis.

Fig. 1. Influence de la plasticité sur la distribution des contraintes dans un sol purement cohérent soumis à une charge uniformément repartie ($C = 50$ kPa). (a) Tassement ; (b) distribution de la contrainte verticale—élasticité ; (c) distribution de la contrainte verticale—élastoplasticité ; (d) variation de la contrainte verticale le long de l'axe vertical ; (e) variation de la contrainte horizontale le long de l'axe vertical.

4. Frictional weightless soil

Analyses were also conducted on a frictional weightless soil characterized by: a Young's modulus $E = 20$ MPa, a Poisson's ration $\nu = 0.3$, a cohesion $C = 2$ kPa, a friction angle $\varphi = 30°$ and a dilatancy angle $\psi = 10°$. The ultimate bearing capacity (Prandtl [11]) is equal to $q_{\max} = 60$ kPa ($q_{\max} = CN_c$, with $N_c = 30$).

Finite element results are depicted in Fig. 2. It can be observed that plasticity induces nonlinearity in the variation of the soil settlement at the loading centre (S) with the applied load (q). Fig. 2(b) shows a comparison of the vertical

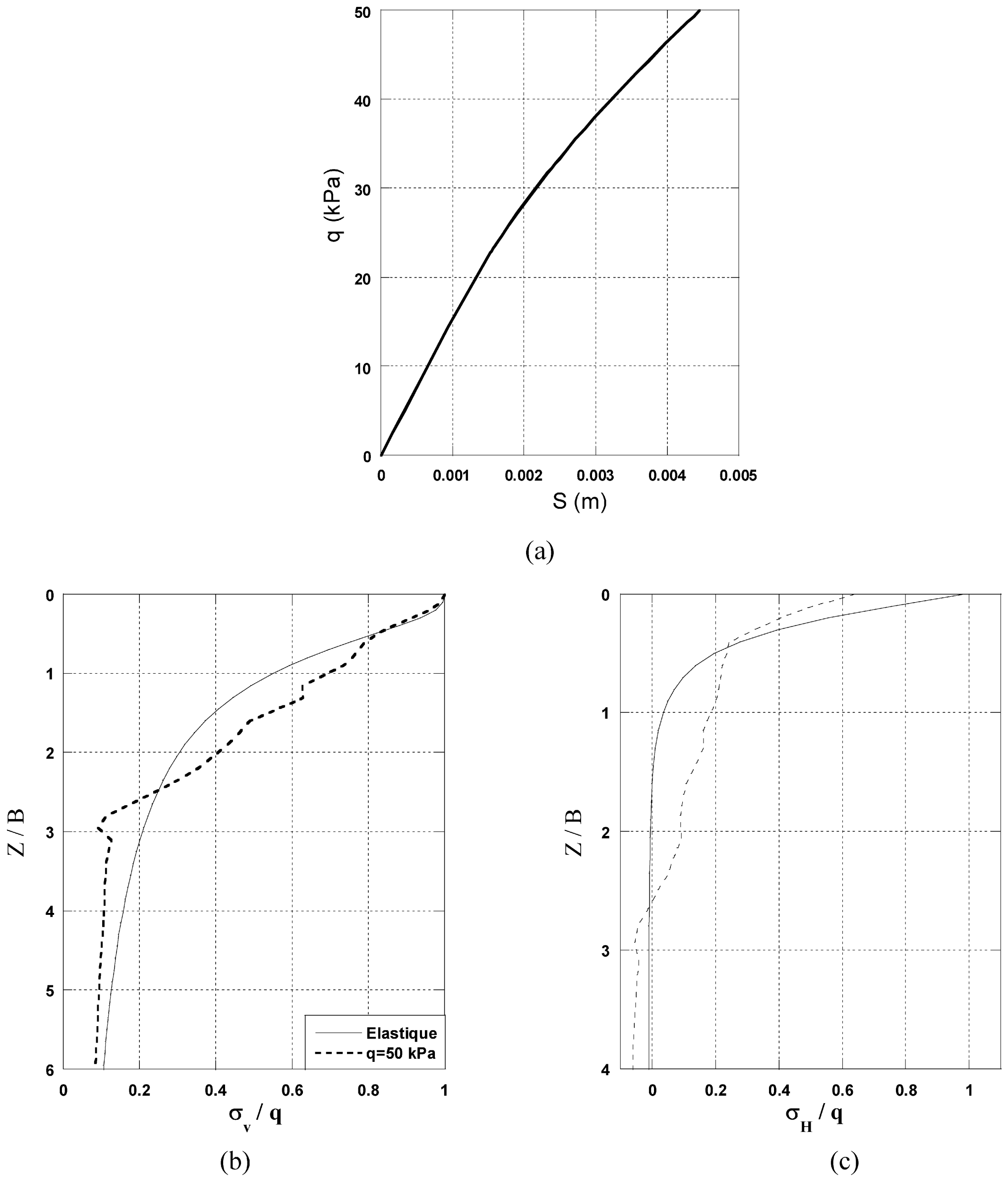

Fig. 2. Influence of plasticity on the stress distribution in a weightless frictional soil subjected to a uniformly distributed load ($\varphi = 30°$, $C = 2$ kPa). (a) Settlement versus applied load; (b) variation of the vertical stress along the vertical axis; (c) variation of the horizontal stress along the vertical axis.

Fig. 2. Influence de la plasticité sur la distribution des contraintes dans un sol frottant non pesant soumis à une charge uniformément repartie ($\varphi = 30°$, $C = 2$ kPa). (a) Tassement ; (b) variation de la contrainte verticale le long de l'axe vertical ; (c) variation de la contrainte horizontale le long de l'axe vertical.

stress distribution between the elastic and the elastoplastic solutions at the loading level $q = 50$ kPa ($q/q_{\max} = 0.83$) which corresponds to an important development in the plasticity in the soil mass. It can be noted that plasticity leads to a reduction in the stress attenuation up to depth $Z/B = 2$. Fig. 2(c) depicts the influence of plasticity on the distribution of the lateral stress. It confirms results obtained for the purely cohesive soil concerning the influence of plasticity in the diminution of the attenuation of the lateral stress in the upper part of the soil mass, up to a depth $Z/B = 2$. It is worth noting that numerical results agree well with the results of experimental tests conducted by Schmertmann [12]. These tests show that the ratio $\Delta\sigma_v/q$ at a depth $Z/B = 1.75$ increases from 0.52 to 0.72 when the applied pressure increases from $q = 27.2$ kPa to 136 kPa. It means that the increase in the load, which induces an amplification of the plasticity in the soil mass results in an augmentation of the vertical stress transfer below the loading.

5. Conclusion

This article has included a study of the validity of the Boussinesq solution for the distribution of stresses in an elastoplastic soils. The study was conducted by comparing the elastic stress distribution to that obtained by elastoplastic finite element analyses for uniformly distributed loads. Results show that plasticity results in a diminution of the attenuation of the vertical stress in the soil mass under the loading, which means that the Boussinesq solution underestimates the stresses in a region that has an important effect on the soil settlement. The influence of plasticity on the distribution of the lateral stress is also significant; the increase in plasticity leads to a significant decrease in the attenuation of the lateral stress in the vicinity of the load.

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