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# Influence of high-frequency vibrations on the onset of convection in a two-layer system

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**Abstract**

This Note deals with the influence of high-frequency translational oscillations on the onset of convection in a two-layer system of weakly heterogeneous immiscible fluids with deformable interface. The averaging method is applied to the generalized Oberbeck–Boussinesq equations. Vibration-generated forces and tensions appear as the result. A transition to the Oberbeck–Boussinesq approximation is made in the averaged equations. Analysis of averaged equations leads to the following conclusions. Horizontal vibrations are obtained not influencing the onset of convection, and in the cases of other directions the influence of vibration is determined by a single parameter, depending on velocity amplitude and direction. Vibration is shown to generate effective surface tension, smoothing the interface. Critical parameters are calculated for the case of homogeneous fluids. ***To cite this article: S.M. Zenkovskaya, V.A. Novosiadliy, C. R. Mecanique 336 (2008).***


**Résumé**

**Influence des vibrations hautes fréquences sur le déclenchement de la convection dans un système à deux couches.** Le but de ce travail est d'analyser l'influence des vibrations translationnelles haute fréquence sur le dèclenchement de la convection, dans un système à deux couches de fluides non miscibles faiblement inhomogènes, oú l'interface est déformable. On applique une méthode de moyennisation sur le système des équations d'Oberbeck–Boussinesq généralisé. Il en résulte l'apparition d'une densité supplémentaire de forces extérieures et une nouvelle tension à l'interface. On étudie le seuil d'instabilité de la solution de conduction stationnaire, sur le système moyenné. Un premier résultat est que les vibrations en translation horizontale n'influencent pas le seuil d'instabilité. Un seul paramètre, fonction de l'amplitude et de la direction, intervient pour les autres directions de vibrations. On montre alors que les vibrations engendrent une tension de surface effective qui aplanit l'interface. On calcule les paramètres critiques dans le cas de fluides homogènes (cas sans pesanteur). ***Pour citer cet article : S.M. Zenkovskaya, V.A. Novosiadliy, C. R. Mecanique 336 (2008).***


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## 1. Introduction

The influence of high-frequency vertical oscillations of small amplitude on the onset of convection was first considered in [1], where averaging method was applied to Oberbeck–Boussinesq (OB) equations. Vibrations were shown to have a stabilizing effect. The case of vibration of arbitrary direction was investigated in [2]. For convection problems in a region with rigid boundaries the averaging method was implemented in [3,4]. For mechanical systems with constraints imposed, this method was developed in [5], where a unified standpoint was presented for numerous vibration effects. Vibrational Rayleigh–Marangoni convection in thin layer with deformable free boundary was researched in [6], where generalized OB equations, suggested in [7], with variable density retained also in inertia terms were used. Convection without vibrations in two-layer systems was considered in [8] for water–benzene combination, where stationary convection for heating from above was predicted theoretically, but had not been found experimentally, possibly due to interfacial contamination. Extensive theoretical and experimental research is presented in [9] with reports of experimental observation of stationary convection with heating from above, as well as oscillatory convection for heating from below for acetonitrile–$n$-hexane system. The fluid combination of Fluorinert–silicone oil with heating from below was studied numerically and theoretically in [10]. Rayleigh–Benard–Marangoni convection under influence of vibrations is the subject of present Note. Our approach, was already suggested in [6]. Vibration-generated forces and tensions are shown to appear as a result of averaging. One of the effects of vibration is the appearance of effective surface tension which can smoothen the interface. The same effect also exists in the case of an isothermal fluid [6,11]. Moreover, horizontal vibrations do not appear to influence the main terms of high-frequency asymptotics.

## 2. Problem formulation

Consider a two-layer system of infinite horizontal extension consisting of viscous incompressible immiscible fluids bounded from above and below by solid walls and separated by an interface $x_3 = \xi(x_1, x_2, t)$ with the surface tension coefficient $\sigma = \sigma_0 - \sigma_T \hat{T}^k$. Fluids are heterogeneous with densities $\hat{\rho}_k = \hat{\rho}_{0k}(1 - \beta_k \hat{T}^k)$. The $x_3$ axis is directed downwards, $\gamma = (0, 0, 1)$ is its unit vector, $x_3 = 0$ plane coincides with flat interface. Mean layer depths are $H_1$ and $H_2$ respectively (the lower layer is denoted with index 1, upper—with index 2). System as a whole is subjected to translational oscillations governed by the law $x_3 = \hat{a}/\hat{\omega} f(\hat{\omega} t)$ along the vector $\boldsymbol{s} = (\cos\varphi, 0, \sin\varphi)$, where $f$ is a $2\pi$-periodic function with zero average. Dimensionless convection equations in a moving coordinate system are written in the generalized OB approximation:

$$\rho_k\left(\frac{\partial \boldsymbol{v}^k}{\partial t} + (\boldsymbol{v}^k \cdot \nabla)\boldsymbol{v}^k\right) = -\nabla p^k + \mu_k \triangle \boldsymbol{v}^k + \rho_k \boldsymbol{g}(t), \quad \operatorname{div} \boldsymbol{v}^k = 0, \quad \frac{\partial T^k}{\partial t} + (\boldsymbol{v}^k \cdot \nabla)T^k = C_k \triangle T^k \tag{1}$$

$$x_3 = \xi(x_1, x_2, t): \quad \boldsymbol{v}^1 = \boldsymbol{v}^2, \quad \boldsymbol{v}^k \cdot \boldsymbol{\ell} = \frac{\partial \xi}{\partial t}, \quad -(p^1 - p^2)n_i + (\tau^1_{ij} - \tau^2_{ij})n_j = -2K\sigma n_i - (\nabla_\Gamma \sigma)_i \tag{2}$$

$$T^1 = T^2, \quad \varkappa_1 \frac{\partial T^1}{\partial \boldsymbol{n}} - \varkappa_2 \frac{\partial T^2}{\partial \boldsymbol{n}} = 0, \quad \boldsymbol{\ell} = (-\xi_{x_1}, -\xi_{x_2}, 1), \quad \boldsymbol{n} = \frac{\boldsymbol{\ell}}{|\boldsymbol{\ell}|}, \quad \tau^k_{ij} = \mu_k\left(\frac{\partial v^k_i}{\partial x_j} + \frac{\partial v^k_j}{\partial x_i}\right) \tag{3}$$

$$\nabla_\Gamma(\sigma)_i = \frac{\partial \sigma}{\partial x_i} - \frac{\partial \sigma}{\partial x_k} n_k n_i, \quad \sigma = C - MT^k, \quad 2K = \nabla_2 \frac{\nabla_2 \xi}{\sqrt{1 - |\nabla_2 \xi|^2}}, \quad \nabla_2 = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}\right) \tag{4}$$

$$x_3 = h_1, -h_2: \quad \boldsymbol{v}^k = 0, \quad B_{1k} \frac{\partial T^k}{\partial x_3} + B_{0k} T^k = b_k \tag{5}$$

Dimensionless quantities are defined using the following scales: $\mathcal{L}$ for length, $\mathcal{T}$ for time, $\rho$ for densities, $A\mathcal{L}$ for temperature. $\mu_k$, $C_k = \hat{\chi}_k \mathcal{T}/\mathcal{L}^2$, $\varepsilon_k = \beta_k A\mathcal{L}$, $\varkappa_k = \hat{\varkappa}_k/\varkappa$ are dynamic viscosity, thermal diffusivity, thermal expansion and thermal conductivity coefficients, $\boldsymbol{g}(t) = Q_0 \boldsymbol{\gamma} - a\omega f''(\omega t)\boldsymbol{s}$ is the variable gravity, $Q_0 = g_0 \mathcal{T}^2/\mathcal{L}$ is its mean part, $C = \sigma_0 \mathcal{T}^2/\rho \mathcal{L}^3$ is the surface tension coefficient, $M = \sigma_T A \mathcal{T}^2/\rho \mathcal{L}^2$ is the Marangoni number. The scales are left undefined up to now.

## 3. High-frequency asymptotics

We now consider the case of large vibration frequency $\omega \to \infty$ and finite velocity amplitude $a = \mathrm{O}(1)$. Under such assumptions the averaging method of Van der Pol–Krylov–Bogolubov can be applied to the system (1)–(5). Following

[1,6] the asymptotic representation of the solution is given by a sum of slow and fast components, having zero time $\tau = \omega t$ average:

$$\boldsymbol{v}^k = \bar{\boldsymbol{v}}^k(\boldsymbol{x}, t) + \tilde{\boldsymbol{v}}^k(\vec{x}, t, \tau), \quad p^k = \bar{p}^k(\boldsymbol{x}, t) + \omega \tilde{p}^k(\vec{x}, t, \tau) \tag{6}$$

$$T^k = \bar{T}^k(\boldsymbol{x}, t) + \frac{1}{\omega}\tilde{T}^k(\vec{x}, t, \tau), \quad \xi = \bar{\xi}(x_1, x_2, t) + \frac{1}{\omega}\tilde{\xi}(x_1, x_2, t, \tau) \tag{7}$$

Substituting (6), (7) into (1)–(5) and retaining the leading terms in powers of $\omega$, we reduce the problem to the fast components, from which we derive the following expressions:

$$\tilde{\boldsymbol{v}}^k = a\boldsymbol{w}^k f'(\tau), \quad \tilde{T}^k = -a(\boldsymbol{w}^k, \nabla \bar{T}^k) f(\tau), \quad \tilde{p}^k = a\rho_{0k}\Phi^k f''(\tau), \quad \tilde{\xi} = a(\boldsymbol{w}^k, \vec{\ell}) f(\tau) \tag{8}$$

$$(1 - \varepsilon_k T^k)(\boldsymbol{w}^k - \boldsymbol{s}) = -\nabla\Phi^k, \quad \operatorname{div} \boldsymbol{w}^k = 0 \tag{9}$$

$$\boldsymbol{w}_n^1|_{x_3=\xi} = \boldsymbol{w}_n^2|_{x_3=\xi}, \quad \rho_{01}\Phi^1|_{x_3=\xi} = \rho_{02}\Phi^2|_{x_3=\xi}, \quad w_3^k|_{x_3=h_1,-h_2} = 0 \tag{10}$$

Substituting (8) into expressions (6), (7), then into the problem (1)–(5), we average the equations and boundary conditions by $\tau$ and leave the terms of order of unity setting $\omega \to \infty$. In the averaged system we convert to the OB approximation, obtaining:

$$\frac{\partial \boldsymbol{v}^k}{\partial t} + (\boldsymbol{v}^k, \nabla)\boldsymbol{v}^k = -\frac{1}{\rho_{0k}}\nabla p^k + \nu_k \triangle \boldsymbol{v}^k + (1 - \varepsilon_k T^k) Q_0 \boldsymbol{\gamma} + \boldsymbol{F}_v^k, \quad \operatorname{div} \boldsymbol{v}^k = 0, \quad \nu_k = \mu_k/\rho_{0k} \tag{11}$$

$$\frac{\partial T^k}{\partial t} + (\boldsymbol{v}^k, \nabla)T^k = C_k \triangle T^k, \quad \boldsymbol{F}_v^k = Re^2(\boldsymbol{w}^k, \nabla)\nabla\Phi^k, \quad Re^2 = \frac{a^2}{2\pi}\int_0^{2\pi} f'^2(\tau)\,\mathrm{d}\tau \tag{12}$$

$$x_3 = \xi(x_1, x_2, t): \quad \boldsymbol{v}^1 = \boldsymbol{v}^2, \quad (\boldsymbol{v}^k \cdot \boldsymbol{\ell}) = \frac{\partial \xi}{\partial t}, \quad T^1 = T^2, \quad \varkappa_1 \frac{\partial T^1}{\partial \boldsymbol{n}} - \varkappa_2 \frac{\partial T^2}{\partial \boldsymbol{n}} = 0 \tag{13}$$

$$(\tau_{ij}^1 - \tau_{ij}^2)n_j - (p_1 - p_2)n_i = -(2K\sigma + \tau_v)n_i - (\nabla_\Gamma \sigma)_i, \quad \tau_v = Re^2\left(\rho_{01}\frac{\partial \Phi^1}{\partial x_3}\boldsymbol{w}^1 - \rho_{02}\frac{\partial \Phi^2}{\partial x_3}\boldsymbol{w}^2, \boldsymbol{\ell}\right) \tag{14}$$

$$x_3 = h_1, -h_2: \quad \boldsymbol{v}^k = 0, \quad B_{1k}\frac{\partial T^k}{\partial x_3} + B_{0k}T^k = b_k \tag{15}$$

Hence averaging has led to the appearance of vibration-generated forces $F_v$ in the equations of motion and vibration-generated tensions $\tau_v$ in dynamic boundary condition. Vibration-generated forces $F_v$ can be shown to be potential [12] in the case of homogeneous fluid, and if the fluid is heterogeneous they are of order $\varepsilon_k$ or higher.

## 4. Equilibrium solution. Spectral problem, dispersion relation

The problem (9)–(15) has the following solution:

$$\boldsymbol{v}^{0k} = 0, \quad T^{0k} = A_k z, \quad \xi^0 = 0, \quad \boldsymbol{w}^0 = (\cos\varphi, 0, 0), \quad \Phi^{0k} = (z - \varepsilon_k A_k z^2/2)\sin\varphi \tag{16}$$

$$p^{0k} = \rho_{0k}(z - \varepsilon_k A_k z^2/2)Q_0 + Re^2/2\rho_{0k}\cos^2\varphi \tag{17}$$

We assume that gradients $A_k$ are uniquely defined by boundary conditions (13) and (15). To study the linear stability of the solution (16)–(17) we set $\boldsymbol{v}^k = \boldsymbol{v}^{0k} + \boldsymbol{u}^k$, $\xi = \xi^0 + \bar{\eta}$, $p^k = p^{0k} + \rho_{0k}Re^2\cos\varphi\partial\bar{\Phi}^k/\partial x + q^k$, $T^k = T^{0k} + \bar{\theta}^k$, $\boldsymbol{w}^k = \boldsymbol{w}^{0k} + \bar{\boldsymbol{W}}^k$, $\Phi^k = \Phi^{0k} + \bar{\Phi}^k$ and obtain the following problem for disturbances:

$$\frac{\partial \boldsymbol{u}^k}{\partial t} = -\frac{1}{\rho_{0k}}\nabla q^k + \nu_k \triangle \boldsymbol{u}^k - \varepsilon_k(\bar{\theta}^k Q_0 + Re^2 \sin\varphi A_k \bar{W}_3^k)\boldsymbol{\gamma}, \quad \operatorname{div} \boldsymbol{u}^k = 0, \quad \operatorname{div} \bar{\boldsymbol{W}}^k = 0$$

$$\frac{\partial \bar{\theta}^k}{\partial t} + u_3^k A_k = C_k \triangle \bar{\theta}^k, \quad (1 - \varepsilon_k A_k z)\bar{\boldsymbol{W}}^k = -\nabla\bar{\Phi}^k - \sin\varphi\varepsilon_k\bar{\theta}^k\boldsymbol{\gamma}$$

$$x_3 = 0: \quad \boldsymbol{u}^1 = \boldsymbol{u}^2, \quad \bar{W}_3^1 = \bar{W}_3^2, \quad u_3^k = \frac{\partial \bar{\eta}}{\partial t}, \quad \rho_{01}(\bar{\Phi}^1 + \sin\varphi\bar{\eta}) = \rho_{02}(\bar{\Phi}^2 + \sin\varphi\bar{\eta})$$

$$\mu_1\left(\frac{\partial u_3^1}{\partial x_1} + \frac{\partial u_1^1}{\partial x_3}\right) - \mu_2\left(\frac{\partial u_3^2}{\partial x_1} + \frac{\partial u_1^2}{\partial x_3}\right) = M\left(\frac{\partial \bar{\theta}^k}{\partial x_1} + A_k\frac{\partial \bar{\eta}}{\partial x_1}\right), \quad \bar{\theta}^1 + A_1\bar{\eta} = \bar{\theta}^2 + A_2\bar{\eta}$$

$$\mu_1\left(\frac{\partial u_2^1}{\partial x_3}+\frac{\partial u_3^1}{\partial x_2}\right)-\mu_2\left(\frac{\partial u_2^2}{\partial x_3}+\frac{\partial u_3^2}{\partial x_2}\right)=M\left(\frac{\partial\bar\theta^k}{\partial x_2}+A_k\frac{\partial\bar\eta}{\partial x_2}\right),\quad \varkappa_1\frac{\partial\bar\theta^1}{\partial x_3}=\varkappa_2\frac{\partial\bar\theta^2}{\partial x_3}$$

$$2\left(\mu_1\frac{\partial u_3^1}{\partial x_3}-\mu_2\frac{\partial u_3^2}{\partial x_3}\right)-\left(q^1-q^2+(\rho_{01}-\rho_{02})Q_0\bar\eta-Re^2(\rho_{01}-\rho_{02})\sin\varphi\bar W_3^1\right)=-C\triangle_2\bar\eta$$

$$x_3=h_1,-h_2:\quad \boldsymbol{u}^k=0,\quad \bar W_3^k=0,\quad B_{1k}\frac{\partial\bar\theta^k}{\partial x_3}+B_{0k}\bar\theta^k=0$$

The vibrational terms contain only one $\sin\varphi$ multiplier, hence *high-frequency horizontal vibrations have no effect on the linear stability of solution* (16), (17). Consequently, we can substitute $\bar{\boldsymbol{W}}^k=\sin\varphi\tilde W^k$, $\bar\Phi^k=\sin\varphi\tilde\Phi^k$. Assuming that the disturbances are flat, introducing flow functions $\bar\psi^k$ and $\bar\zeta^k$, so that $u_1^k=\partial\bar\psi^k/\partial z$, $u_3^k=-\partial\bar\psi^k/\partial x$, $\tilde W_1^k=\partial\bar\zeta^k/\partial z$, $\tilde W_3^k=-\partial\bar\zeta^k/\partial x$, excluding pressures $q^k$ and functions $\tilde\Phi^k$, separating time $t$ and variable $x_1$ by substitution $(\bar\psi^k(x,z,t),\bar\zeta^k(x,z,t),\bar\theta^k(x,z,t),\bar\eta(x,t))=\mathrm{e}^{\lambda t+\mathrm{i}\alpha x}(\psi^k(z),\zeta^k(z),\mathrm{i}\alpha\theta^k(z),\mathrm{i}\alpha\eta)$ we obtain the following spectral problem for normal disturbances:

$$\lambda L\psi^k=\nu_kL^2\psi^k-\varepsilon_k\alpha^2(\theta^kQ_0-\mu_vA_k\zeta^k),\quad \lambda\theta^k-A_k\psi^k=C_kL\theta^k$$

$$L\zeta^k=\varepsilon_k\left(-\alpha^2\theta^k+A_k(zL\zeta^k+D\zeta^k)\right),\quad L=(D^2-\alpha^2),\quad D=\frac{\partial}{\partial z},\quad \mu_v=Re^2\sin^2\varphi$$

$$z=0:\quad \psi^1=\psi^2,\quad D\psi^1=D\psi^2,\quad \zeta^1=\zeta^2,\quad \rho_{01}D\zeta^1-\rho_{02}D\zeta^2=-\alpha^2(\rho_{01}-\rho_{02})\eta$$

$$\psi^k=-\lambda\eta,\quad \mu_1D^2\psi^1-\mu_2D^2\psi^2+\alpha^2(\mu_1-\mu_2)\psi^1=-\alpha^2M(\theta^k+A_k\eta),\quad \varkappa_1D\theta^1=\varkappa_2D\theta^2$$

$$\left(3\alpha^2(\mu_1-\mu_2)+\lambda(\rho_{01}-\rho_{02})\right)D\psi^1-(\mu_1D^3\psi^1-\mu_2D^3\psi^2)$$
$$+\alpha^2\left(\left(Q_0(\rho_{01}-\rho_{02})+C\alpha^2\right)\eta+\mu_v(\rho_{01}-\rho_{02})\zeta^1\right)=0,\quad \theta^1+A_1\eta=\theta^2+A_2\eta$$

$$z=h_1,-h_2:\quad \zeta^k=0,\quad \psi^k=0,\quad D\psi^k=0,\quad B_{1k}D\theta^k+B_{0k}\theta^k=0$$

It can be easily seen that the influence of vibration is described by a single vibrational parameter $\mu_v$. When vibrations are vertical, the same result can be also obtained when initial equations are taken in OB approximation [12]. Under the assumption of weakly heterogeneous fluids, we retain only the main $\varepsilon_k$ terms by setting $\zeta^k=\zeta_0^k+\varepsilon_k\zeta_1^k+\varepsilon_k^2\zeta_2^k+\cdots$. For $\zeta_0^k$ we obtain:

$$\zeta_0^1=\alpha\eta S(\operatorname{ch}\alpha z-\operatorname{cth}\alpha h_1\operatorname{sh}\alpha z),\quad \zeta_0^2=\alpha\eta S(\operatorname{ch}\alpha z+\operatorname{cth}\alpha h_2\operatorname{sh}\alpha z),\quad S=\frac{\rho_{01}-\rho_{02}}{\rho_{01}\operatorname{cth}\alpha h_1+\rho_{02}\operatorname{cth}\alpha h_2}$$

With this solution a the spectral problem for unknown disturbances is received:

$$\lambda L\psi^k=\nu_kL^2\psi^k-\alpha^2\left(\theta^kGr_k-A_k(Gv_{1k}\zeta_0^k+Gv_{2k}\zeta_1^k)\right)\tag{18}$$

$$\lambda\theta^k-A_k\psi^k=C_kL\theta^k,\quad L\zeta_1^k=-\alpha^2\theta^k+A_kD\zeta_0^k,\quad Gr_k=\varepsilon_kQ_0,\quad Gv_{1k}=\varepsilon_k\mu_v,\quad Gv_{2k}=\varepsilon_k^2\mu_v\tag{19}$$

$$z=0:\quad \psi^1=\psi^2,\quad D\psi^1=D\psi^2,\quad \zeta_1^1=\varepsilon\zeta_1^2,\quad \psi^k=-\lambda\eta,\quad \rho_{01}D\zeta_1^1=\rho_{02}\varepsilon D\zeta_1^2,\quad \varepsilon=\varepsilon_2/\varepsilon_1\tag{20}$$

$$\mu_1D^2\psi^1-\mu_2D^2\psi^2+\alpha^2(\mu_1-\mu_2)\psi^1=-\alpha^2M(\theta^k+A_k\eta),\qquad \theta^1+A_1\eta=\theta^2+A_2\eta\tag{21}$$

$$\left(3\alpha^2(\mu_1-\mu_2)+\lambda(\rho_{01}-\rho_{02})\right)D\psi^1-(\mu_1D^3\psi^1-\mu_2D^3\psi^2)+\alpha^2\left(\left(Q_0(\rho_{01}-\rho_{02})+\alpha C_v\right)\eta\right.$$
$$\left.+Gv_{11}(\rho_{01}-\rho_{02})\zeta_1^1\right)=0,\quad \varkappa_1D\theta^1=\varkappa_2D\theta^2,\quad C_v=C\alpha+\mu_v\frac{(\rho_{01}-\rho_{02})^2}{\rho_{01}\operatorname{cth}\alpha h_1+\rho_{02}\operatorname{cth}\alpha h_2}\tag{22}$$

$$z=h_1,-h_2:\quad \zeta_1^k=0,\quad \psi_k=0,\quad D\psi_k=0,\quad B_{1k}D\theta^k+B_{0k}\theta^k=0\tag{23}$$

Here $Gr_k$ are Grashof numbers, $Gv_{1k}$ and $Gv_{2k}$ are vibrational Grashof numbers. Second vibrational Grashof number $Gv_{2k}$ is included because the first vibrational Grashof number $Gv_{1k}$ is multiplied by a function depending on the interface deformation, which can become small, for example for the case of large surface tension. $C_v$ is the effective surface tension, thus, vibration increases the surface tension, smoothing the interface. Dispersion relations are obtained from (18)–(23) in explicit form $M=\Gamma(\alpha,\lambda,\mu_v,Gr_k,Gv_{1k},Gv_{2k})$ in the case of homogeneous fluids, and $\tilde\Gamma(\alpha,\lambda,M,\mu_v,Gr_k,Gv_{1k},Gv_{2k})=0$ in the case of heterogeneous fluids. Functions $\Gamma$, $\tilde\Gamma$ also depend on all other parameters of the system. The dispersion relations themselves are rather cumbersome, for detailed derivation see [12]. The obtained relations can be used for calculating the spectral parameter $\lambda$, or the critical parameter values with $Re\,\lambda=0$.

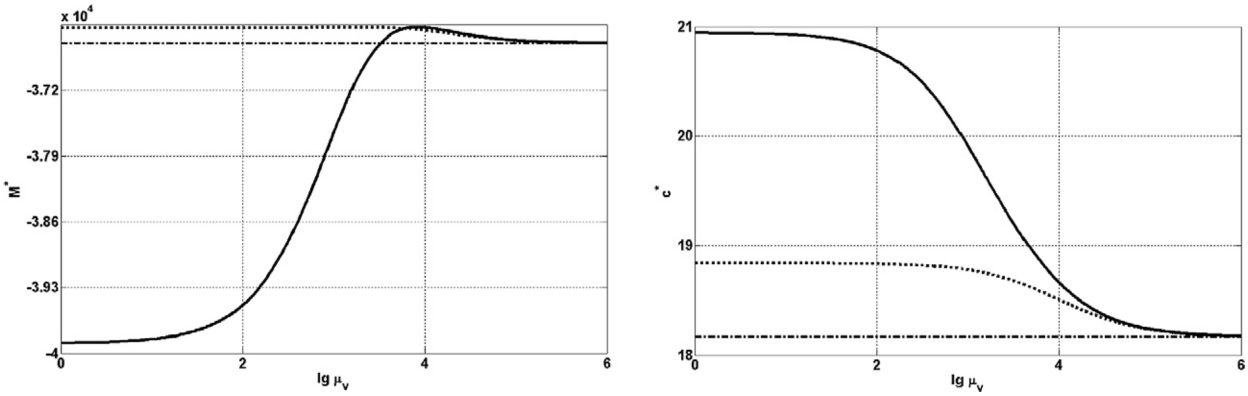


Fig. 1. Fluorinert FC70–silicone oil 10 cSt, $h_1 = 0.6$, $h_2 = 0.4$. Solid line corresponds to deformable interface with $g_0 = 0$; dashed line—deformable interface with $g_0 = 9.8$; dot-dashed line—undeformable in average interface.

Fig. 1. L'huile Fluorinert FC70–silicone 10 cSt, $h_1 = 0{,}6$, $h_2 = 0{,}4$. La ligne continue correspond à l'interface déformable avec $g_0 = 0$ ; en pointille—à l'interface déformable avec $g_0 = 9{,}8$ ; petits traits—l'interface en moyenne non déformable.

## 5. Numerical results

For the purpose of numerical calculations, we define the scales as follows: $\mathcal{T} = \rho\mathcal{L}^2/(\hat{\mu}_1 + \hat{\mu}_2)$, $\rho = \hat{\rho}_{01} + \hat{\rho}_{02}$, $\mathcal{L} = H_1 + H_2$, $A = \hat{A}_1 + \hat{A}_2$, $\varkappa = \hat{\varkappa}_1 + \hat{\varkappa}_2$. The results are provided for the case of oscillatory instability ($\lambda = \mathrm{i}c$) of homogeneous fluids. Quantitative agreements have been reached for the cases of fluid combinations taken from [8,9] for heating from above where thermocapillary forces are leading in destabilizing the equilibrium. Qualitative agreement for heating from below has also been reached, for example, oscillatory convection exists for system of acetonitril–$n$-hexane in homogeneous approximation. We take the system fluorinert FC70–silicone oil 10 cSt (fluorinert fills the lower layer) for investigations. The parameters of this system are given in [10]. The total depth of the system is 5 mm. Fig. 1 shows the calculated critical Marangoni number $M^*$ (absolute value minimized by wave number $\alpha$) and corresponding critical oscillation frequency $c^*$ for gravity values $g_0 = 0$ and $g_0 = 9.8$. When vibration reaches sufficient force ($\mu_v \approx 1.6 \times 10^5$), both $M^*$ and $c^*$ for deformable interface come within 2% of the values of undeformable in average interface. Critical wave number also slightly lowers from 3.96 to 3.58, always staying in thermocapillary range. Thus calculations support theoretical conclusions, given in Section 4.

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