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Duality, inverse problems and nonlinear problems in solid mechanics

Two non-linear finite element models developed for the assessment of failure of masonry arches

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Abstract

In this article a comparison between two non-linear finite element approaches for the numerical estimation of the ultimate failure load of masonry arches is presented. According to the first model, the geometry of the arch is divided into a number of unilateral contact interfaces which simulate potential cracks. Opening or sliding for some of the interfaces indicates crack initiation. The second model uses two-dimensional finite elements for the simulation of the arch. When tensile stresses appear, upon an adaptive stepwise procedure, the corresponding elements are replaced by unilateral contact elements which represent cracks. In both models the fill over the arch, that could strongly affect the collapse behaviour increasing the bridge load carrying capacity, is taken into account. Moreover, the ultimate load and the collapse mechanism have been calculated by using a path-following (load incrementation) technique. Both models are developed and applied on a real scale masonry arch; results are comparable with both the experimental collapse mechanism and the ultimate load failure. ***To cite this article: M. Betti et al., C. R. Mecanique 336 (2008).***

Résumé

Deux modèles éléments finis non-linéaires développés pour le calcul de la ruine d'arches en maçonnerie. Dans cet article, on présente une comparaison entre deux approches par éléments finis non-linéaires permettant d'estimer numériquement la charge ultime de ruine d'arches en maçonnerie. Dans le premier modèle, la géométrie de l'arche est divisée en un certain nombre d'interfaces de contact unilatéral qui simulent des fissures potentielles. L'ouverture ou le glissement sur certains de ces interfaces indiquent une initiation de fissure. Le second modèle utilise des éléments finis bidimensionnels pour la simulation de l'arche. Quand des contraintes de traction apparaissent, les éléments correspondants sont remplacés, suivant une procédure pas-à-pas adaptative, par des éléments de contact unilatéral qui représentent des fissures. Dans les deux modèles, on prend en compte le remplissage sur l'arche, qui peut affecter considérablement le comportement à la rupture en augmentant la capacité portante du pont. De plus, la charge limite et le mécanisme de ruine ont été calculés en utilisant une technique pas-à-pas tenant compte du trajet de chargement. Les deux modèles sont développés et appliqués à une arche en maçonnerie en vraie grandeur ; les résultats reproduisent bien tant le mécanisme de ruine expérimental et la charge limite de rupture. ***Pour citer cet article : M. Betti et al., C. R. Mecanique 336 (2008).***

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Mots-clés : Solides et structures ; Arche en maçonnerie ; Contact unilatéral ; Interaction arche–remplissage ; Analyse-limite ; Charge ultime de ruine

1. Introduction

A large number of arch bridges built in Europe during the 19th century according to codes and design criteria developed for the 19th century loads [1] are still in service. Most of these bridges are nowadays subjected to heavier loads, and sometimes show signs of deterioration. Due to their importance for the transportation systems, especially for the European railway network, as well as due to their cultural value, tools for a reliable estimation of their actual load carrying capacity are needed.

The most widely distributed and well-know method for the assessing of masonry arch bridges is, perhaps, the Military Engineering Experimental Establishment (MEXE) [2] based on Pippard's papers (e.g. [3,4]) and developed with the aim to determinate the ability of masonry arches to survive the crossing of a military tank. Substantially the MEXE method is a family of semi-empirical methods, including several reduction factors based on the condition noted in visual inspection of the bridge. In fact, these methods consider a representative single-arch and the load carrying capacity of the whole bridge is deduced from the single arch analysis, after introduction of some reducing factor. These methods, which has been incorporated (with some modifications) into the UK standard for the assessment of bridges, are crude, and the rational basis of the reduction factor has never been well established [5]. Despite their very conservative features, they are widely used because of their simplicity.

The aforementioned procedures are questionable because it is known that the geometry of the bridge, the actual position of the loads and the compliance of the arch and of the pier greatly affect the response. Masonry arch bridges are complex three-dimensional systems, whose structural behaviour is affected by the interaction between their components. Both full scale tests [6,7], and model scale experiments, show the significant contribution of the overlying fill as well as the spandrel walls to the structural stiffness. The variation in the mechanical properties of the bridge's materials and components (barrel vaults, piers, spandrel walls, fill, etc.) makes the study of them quite demanding and has led to the development of a number of theories in order to represent, as accurately as possible, the actual mechanical behaviour of the masonry bridge. Experimental results obtained from collapse tests on full scale and model scale bridges have shown the complexity of the collapse behaviour and highlight the strong influence of fill and spandrels on the collapse mechanism and the load carrying capacity.

In order to understand, and to apprise, the mechanics of these structures within a reasonable degree of confidence a classical method proposed in the literature is the mechanism method established by Heyman [8] who considered the actual way in which arches failed (by the formation of hinges, therefore forming a mechanism). The hypotheses on the masonry behaviour are: no tensile strength; infinite compressive strength and absence of sliding at failure. Under these hypotheses, the masonry bridge at collapse can be approximated by an assemblage of rigid parts, held up by mutual pressure, and the collapse of the structural elements is characterised by the development of non-dissipative hinges transforming the structure into a mechanism. The term mechanism indicates that a displacement distribution in the structure is produced by inelastic deformations (the formation of hinges) which occur in a finite number of sections due to disconnections and cracking. It is based on the assumption that an arch fails by the development of a collapse mechanism with four hinges. In fact, the classical collapse mechanism method of Heyman is based on the estimation of the thrust line (a funicular polygon which defines the resultant force in a cross-section of the arch). When the thrust line in a cross-section is adjacent to the ring of the arch, the eccentricity of the normal force, from the center line of the arch at the given cross-section, becomes maximum. As a result, a bending moment is developed around the center line of the arch and a hinge opens, on the assumption that the arch does not develop any tensile strength. Since a three-pin arch is a statically determinate structure, opening of a fourth hinge converts the structure into a mechanism and collapse occurs. Therefore, the four hinges collapse mechanism is the collapse mode for an arch loaded with a vertical concentrated force in the quarter-span of the arch [9,10]. In addition, no compressive failure for the masonry is usually expected. Collapse load predictions from this method depend on an assumed distribution of lateral soil pressures on the extrados regardless of the arch's deformed shape. A mechanism analysis is accurate only when all the forces and their positions are known and the assumptions of the method are not violated. For example, friction and sliding failure are not permitted. On the other hand, when transverse collapse mechanisms may be ignored, as discussed by Fanning et al. [11], two-dimensional models that include the strengthening effects of the fill can be useful for the longitudinal bridge assessment and provide direct and simple information on the structural response. In this case, a reduced number of constitutive parameters is required.

In the sequel several methods have been proposed for the assessment of masonry arches. Part of them are related to the limit analysis of block structures with a frictional contact interface. Melbourne and Gilbert [12] studied the limit analysis problem of stone arches using the upper bound theorem [13] and a linear programming formulation. Orduna and Lourenço [14,15] developed two- and three-dimensional models of discrete structures (like stone arches) and they took into account torsion failure mode as well as reinforcement elements.

Other methods are based on the development of a finite element model and incremental analysis. Crisfield [16,17] proposed a model in which the arch is simulated with beam elements. He took into account the fill over the arch as well as the active and passive soil pressure induced by the fill, by using non-linear one-dimensional elements. Lofti and Shing [18] developed a discrete finite element model for the description of the mortar joints of masonry structures. They simulated mortar with interface elements that obey to a non-linear constitutive law. Molins and Roca [19] used a three-dimensional finite element model for the investigation of the behaviour of stone arches. They applied the Mohr–Coulomb criterion for the shear failure of the masonry, and the perfect-plastic constitutive law for the simulation of the tensile failure mode.

Cavicchi and Gambarotta [20] simulated arches and piers with beam elements having zero tensile strength. For the fill they used two-dimensional plane strain finite elements with the Mohr–Coulomb failure criterion. For the arch–fill interaction they applied interface elements. They presented applications in a single span as well as in a multi-span masonry bridge. From another point of view, Ng and Fairfield [5] proposed a modification of the collapse mechanism method by considering the interaction between the arch deflections and the backfill pressures.

A relatively recent technique, in order to assess the backfill interaction, is the discontinuous deformation analysis. The method, originally developed in the field of rock mechanics, is adopted by Thavalingam et al. [21].

Due to the great variety of the approaches proposed in the inherent literature for the assessment of masonry arch bridge, in this study a comparison between two approaches developed for the investigation of the mechanical behaviour is presented.

According to the first technique, the ultimate failure load is found by the use of a discrete-type model. In particular, the geometry of the structure is divided into a number of interfaces, perpendicular to the center line of the ring. Those interfaces are uniformly distributed along the arch. Unilateral contact law governs the behaviour in the normal direction of an interface, indicating that no tension forces can be transmitted in this direction. The behaviour in the tangential direction takes into account that sliding may or may not occur, by the use of the Coulomb friction law. The backfill can be included in this model and it is simulated with two-dimensional finite elements. The interaction between arch and fill is also simulated by a unilateral contact–friction interface. The either/or decisions incorporated in the unilateral contact and friction mechanisms make the whole mechanical model highly non-linear. Due to the presence of non-differentiable functions within these models, they are characterized as non-smooth or inequality mechanics models [22–24]. In fact, contact analysis has been used in several works for the non-linear study of masonry structure, including collapse, see e.g. [25]. Furthermore the limit analysis problem has been recently studied within the framework of optimization with complementarity constraints, i.e. a problem known as mathematical program with equilibrium constraints (MPEC); see, among others, [26,13,14] and the references given therein. For practical applications carefully tuned path-following iterative techniques are used for the numerical solution. Furthermore, the limit analysis problem is related to the solvability of the underlying mechanical problem using analogous theoretical results concerning the solvability of variational inequalities and complementarity problems [22,23,27].

According to the second approach a non-linear incremental adaptive method is proposed in order to assess the load carrying capacity. The method is based on a simplified two-dimensional finite element discretization of the masonry bridge arch barrel. Masonry in assumed to be a no-tension material. Therefore some unilateral contact elements are added iteratively in order to respect this assumption. The lateral response of the fill is modelled by one-dimensional horizontal elements having a non linear equation with different responses at active and passive fill states.

Results of the comparison between the two models will prove to be quite interesting, as they will demonstrate how a more complex finite element model interacts with a simplified model, both useful for the assessment of stone arches.

2. The FEM model with unilateral contact–friction interfaces

The behaviour in the normal direction of an interface is described by the unilateral contact model. In particular, let us consider the boundary of an elastic body which comes in contact with a rigid wall. Let u be the single degree of freedom of the system, g be the initial opening and t^n be the corresponding contact pressure in case contact occurs. The basic unilateral contact law is described by the set of inequalities (1a), (1b) and by the complementarity relation (1c) [22–24].

$$h = u - g \leqslant 0 \Rightarrow h \leqslant 0, \qquad -t^n \geqslant 0, \qquad t^n(u-g) = 0 \tag{1}$$

Inequality (1a) represents the non penetration relation, relation (1c) implements the requirement that only compressive stresses (contact pressures) are allowed and Eq. (1c) is the complementarity relation according to which either separation with zero contact stress occurs or contact is realized with possibly non-zero contact stress.

The behaviour in the tangential direction is defined by a static version of the Coulomb friction model. Two contacting surfaces start sliding when the shear stress at the interface reaches a maximum critical value equal to

$$t^t = \tau_{\mathrm{cr}} = \pm\mu\left|t^n\right| \tag{2}$$

where t^t, t^n are the shear stress and the contact pressure at a given point of the contacting surfaces respectively and μ is the friction coefficient. There are two possible directions of sliding along an interface, therefore t^t can be positive or negative depending on that direction. Furthermore, there is no sliding if $|t^t| < \mu \mid t^n|$ (stick conditions). The stick–slip relations of the frictional mechanism can be mathematically described with two sets of inequalities and complementarity relations, similar to (1), (2), by using appropriate slack variables [24].

Due to the nonpenetration assumption and the stick–slip inequalities of friction the variational formulation of the problem takes the form of a variational inequality [22,23].

Lagrange multipliers are used in the Principle of the Virtual Work to enforce the no-penetration and sticking conditions. The arising set of the non-linear equations is solved by the Newton–Raphson incremental iterative procedure, or by specialized algorithms (for linear or non-linear complementarity or nonsmooth optimization problems).

For example, the frictionless unilateral contact problem takes the following matrix form:

$$\mathbf{Ku} + \mathbf{N}^{\mathbf{T}}\mathbf{r} = \mathbf{P}_0 + \lambda\mathbf{P} \tag{3}$$

$$\mathbf{Nu} - \mathbf{g} \leqslant 0, \qquad \mathbf{r} \geqslant 0, \qquad (\mathbf{Nu} - \mathbf{g})^T \cdot \mathbf{r} = 0 \tag{4}$$

Eq. (3) expresses the equilibrium equations of the unilateral contact problem, where for simplicity frictional terms are omitted. **K** is the stiffness matrix and **u** is the displacement vector. $\mathbf{P}_0$ denotes the self-weight of the structure and **P** represents the concentrated live load. **N** is an appropriate geometric transformation matrix and vector **g** contains the initial gaps for the description of the unilateral contact joints. Relations (4a), (4b), (4c) represent the constraints of the unilateral contact problem for the whole discretized structure and are based on the local description given by relations (1).

The enforcement of the constraints is achieved by using Lagrange multipliers. Thus, **r** is the vector of Lagrange multipliers corresponding to the inequality constraints and is equal to the corresponding contact pressure ($-\mathbf{t}^n$).

The problem described above is a nonsmooth parametric linear complementarity problem (LCP) [24,27,28] parametrized by the one-dimensional load parameter λ. All required quantities can be calculated by using finite element techniques. Using path-following the solution of the problem can be calculated in the interval $0 \leqslant \lambda \leqslant \lambda_{\mathrm{failure}}$, where $\lambda_{\mathrm{failure}}$ is the value of the loading factor for which the unilateral contact problem does not have a solution. This is the limit analysis load. The analysis reported here has been completed within the ABAQUS code (see [29] for further details).

3. A FEM model with adaptive crack elements

A relatively recent development in the analysis of masonry arch bridges has been the use of simplified finite element techniques. Towler and Sawko [30] showed the potential of this general approach by computing load deflection curves and collapse loads for an arch modelled with one-dimensional beam elements to represent the arch barrel. The presence of fill was considered only as a dead load and no further soil-structure interaction effects were modelled.

Crisfield [17] later introduced non-linear spring elements in an attempt to model, in a simplified way, the lateral resistance from the soil. He was the first to introduce non-linear spring elements for the modelling of the fill material and for the modelling of the lateral resistance of the soil. The spring stiffness has been termed the sub-grade modulus, and was initially pre-compressed to the equivalent pressure at rest. The maximum horizontal pressure was limited by the active or passive fill state, depending upon the type of movement of the arch. Other researchers, [31,32], presented more accurate iterative schemes and more detailed two- or three-dimensional models [33] where the tools of the modern incremental analysis are applied. If one-dimensional models are efficient in assessment and design procedures for single bridges, two- and three-dimensional models may give detailed information on local phenomena (e.g. the fill stress and strain state) at the expense of high computational costs.

Here a simplified numerical procedure for the limit analysis of masonry arch bridge is developed in order to take into account, and to evaluate, the arch–fill interaction. A non-linear incremental approach, assuming masonry as no tensile material, is developed. The procedure is based on a simplified two-dimensional finite element discretization of the masonry bridge. The arch is modelled by means of four-node plane stress elastic elements and the fill material is modelled by means of non-linear links (see Fig. 4) whose properties are evaluated according to the active and passive fill states.

The passive and active pressures of the fill are estimated by the following force, taking into account the geotechnical fill properties:

$$H_f(d_s) = 0.5 \cdot \rho \cdot K_{a,p} \cdot \left(d_R^2 - d_S^2\right) \tag{5}$$

where d_R and d_S represent, respectively, the depth of the fill with respect to the upper surface of the bridge and the depth of the fill at the point of interest; ρ is the fill density; $K_a(K_p)$ is the passive (active) resistance factor of the fill material that could be obtained from the angle of internal friction φ by

$$K_a = \frac{1-\sin\varphi}{1+\sin\varphi}; \qquad K_p = \frac{1+\sin\varphi}{1-\sin\varphi} \tag{6}$$

The proposed simplified, and approximate procedure, starts from the elastic analysis of the structure (stress state on the arch elements) taking into account only the self-weight. Next, with the increasing external collapse load, unilateral contact elements are applied in order to take into account the non-linear behaviour of the arch barrel. At each load step the stress state on the arch ring elements is checked at every joint, and if the stress state is not admissible for the modelled masonry material (that is, tensile stresses arise) the corresponding joints that connect the adjacent elements are substituted by unilateral contact interfaces (see Fig. 4). The procedure proposed here is based on an original idea of Castigliano.

Here, this stepwise process is implemented inside the commercial code ANSYS. By means of this adaptive procedure the development of the cracked area follow the load application, i.e. the area where the crack appears is not imposed a priori. At the beginning, in the first step of the procedure, when only dead loads are present, each plane element is connected to the others directly by joints; by applying the external collapse load these connections are modified by following the internal stress state in order to respect the no-tensile behaviour of the masonry material. The actual behaviour of the arch, when the load is increasing from zero to the final load, is followed. More technical details can be found in [35].

It is interesting to point out that the calibration of this model does not require expensive experimental in situ investigation, since only the knowledge of a reduced number of parameters is required: the Young modulus, the Poisson coefficient, the self weight and the geotechnical fill properties.

4. Application to a real arch

As a case study the two proposed approaches are applied for the assessment of the load carrying capacity of Prestwood bridge, a single-span bridge tested to collapse within the experimental research on masonry bridges supported by the Transport Research Laboratory [7].

This bridge has a span length equal to 6.55 m, with upper rise 1.42 m. The vault thickness is 0.22 m and comprises a single ring of bricks laid as headers; the fill depth at the crown is 0.165 m and the total bridge width is 3.80 m (see Fig. 1). Therefore the slenderness (thickness/span) is equal to 1/30 and the flatness (rise/span) is equal to 1/6–1/7. The load is applied on a strip of the road surface along the full width of the bridge between the parapets at a point

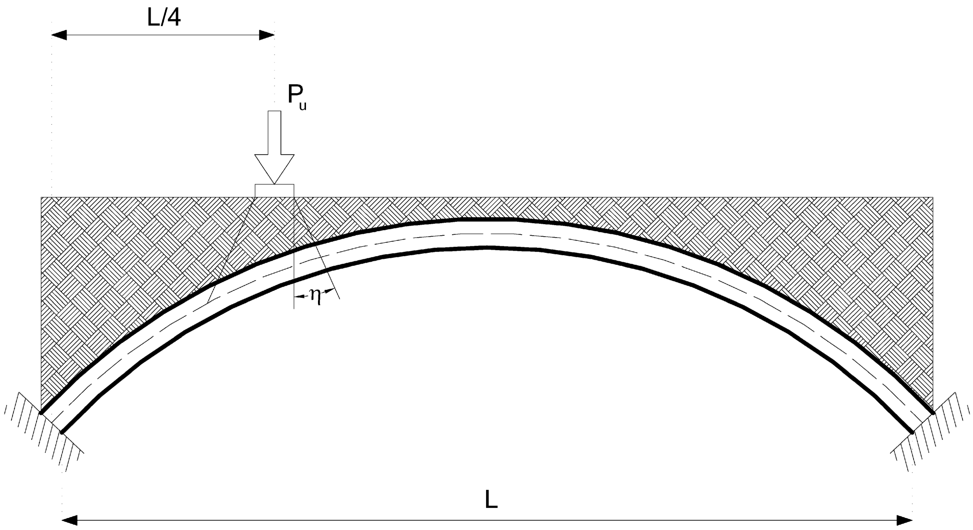

Fig. 1. Geometry of the Prestwood bridge.

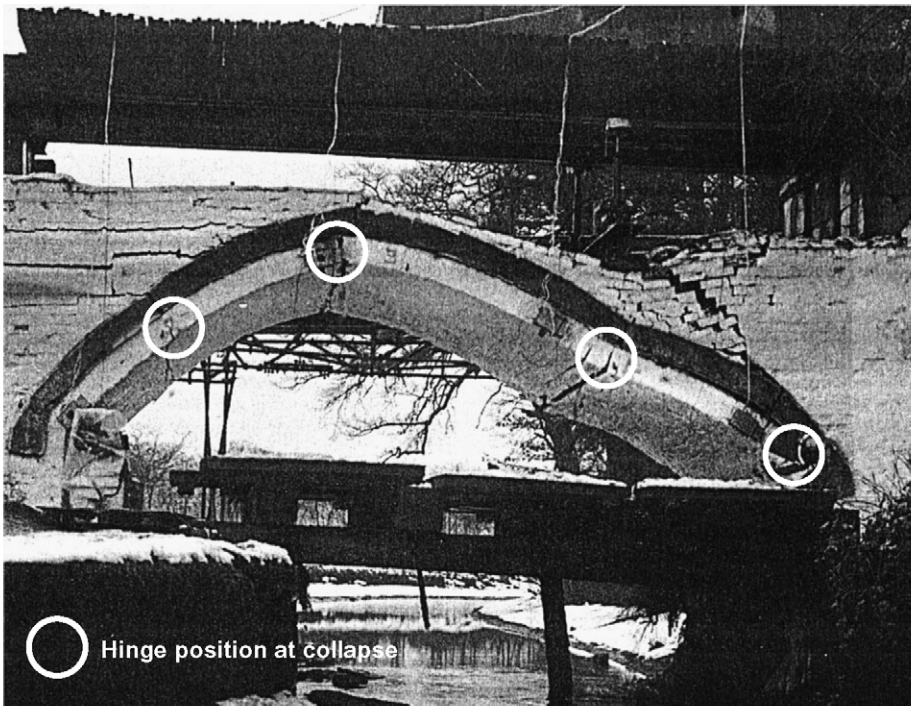

Fig. 2. Collapse mechanism of the Prestwood bridge after the destructive test (DT).

where the lower load value leading to failure was expected. The latter has been evaluated by assuming that the arch would fail as a four hinged mechanism and was, then, applied at the quarter-span of the bridge. The strip was 0.30 m wide in order to distribute the load and to avoid a premature failure of the fill. The load has been applied by means of hydraulic jacks, while the required reaction for the load was provided by the weight of concrete blocks on a steel frame above the bridge (see Fig. 2). The fill density c_{fill} is 20 kN/m^3 and the masonry density is 20 kN/m^3. Furthermore, the fill cohesion is equal to 10 kPa, the fill angle of internal friction φ is 37° and the masonry compressive strength σ_m is equal to 7.7 N/mm^2. The experimental collapse mechanism of the bridge is shown in Fig. 2 [6,7]. The vault collapse mechanism exhibits four hinges that are clearly visible in the picture; the mechanism developed with

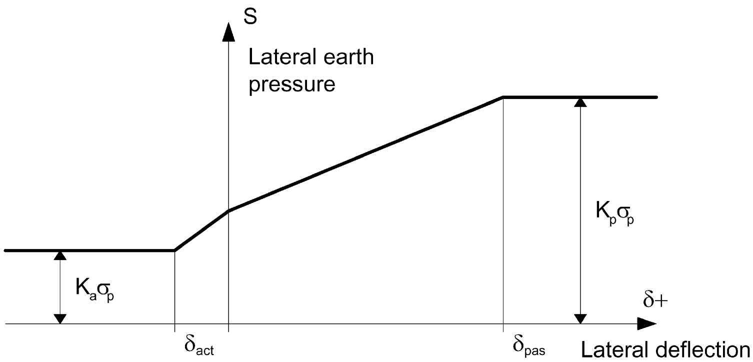

Fig. 3. Non-linear fill pressure model.

negligible material crushing. The arch mechanism constrains the fill region under the applied load to move downward and the fill at the other side of the bridge to move upward. The first visible sign of damage occurred at $P_f = 173$ kN and the experimental collapse load was $P_{\text{exp}} = 228$ kN.

4.1. Analysis without fill

In a preliminary step an analysis where the fill is assumed to be heavy but not resistant is made. Loading conditions in both models include the self-weight (which is applied first) and a concentrated load at the quarter span of the bridge (see Fig. 1). In the contact–friction model the Young's modulus of the arch is 15 GPa, the Poisson's ratio is 0.3 and the density is equal to 2000 kg/m^3. The contact model consists of quadrilateral, four-node, bilinear, plane strain elements with two-translational degrees of freedom per node. The exact number of contact–friction interfaces along the length of the bridge tends to be meaningless in the case where many interfaces are used (more details can be found in [28]). In this study, forty interfaces are used for both the arch without and with the fill. The arch is considered to be fixed to the ground. The out of plane width of the bridge is taken equal to 3.8 m. The friction coefficient of the interfaces is equal to 0.6.

In the adaptive model only arch barrel elements are modelled by four node linear plain stress elements. According to the stepwise procedure, unilateral contact elements are added iteratively along the ring thickness at the position where tensile stresses arise. In order to evaluate the influence of the material properties of the arch barrel some parametric analyses have been performed by varying the elastic modulus of the elastic elements. The analyses have shown that (neglecting second order effects connected with geometric non-linearities) the material properties are not significant for the evaluation of the ultimate load [34].

In accordance with the adaptive model in the numerical collapse mechanism, four groups of hinges are developed (see Figs. 5 and 6). The loaded side of the arch moves downward, and the arch mechanism pushes the left side. This behaviour agrees with the experimental result only with respect to the collapse mechanism; the numerical critical load is 79.8 kN, which is approximately equal to the 35% of the experimental collapse load $P_{\text{exp}} = 228$ kN. It is interesting to observe that the value reached by the tensional state on the arch at the collapse is admissible for the masonry material, confirming that collapse is due to a development of a mechanism (see Fig. 6). Fig. 5 shows, immediately before the collapse, the principal stress in vector form and this could be a representation of the thrust line. Accumulation of principal stresses near the ring of the arch denotes that a hinge is opened in that point. The relation between displacements and loads is almost linear until a load level approximately equal to 58% of the numerical ultimate load when the first crack appears on the arch barrel. With increasing load it is possible to observe a crack propagation on the arch ring, up to the development of a hinge.

4.2. Analysis with fill

The presence of the fill above the arch significantly affects its behaviour. The self-weight of the fill increases the stability, either directly by inducing additional compression in the arch, or indirectly by allowing a smoother distribution of the concentrated forces over the bridge. Comparison of the magnitude of the failure load obtained by

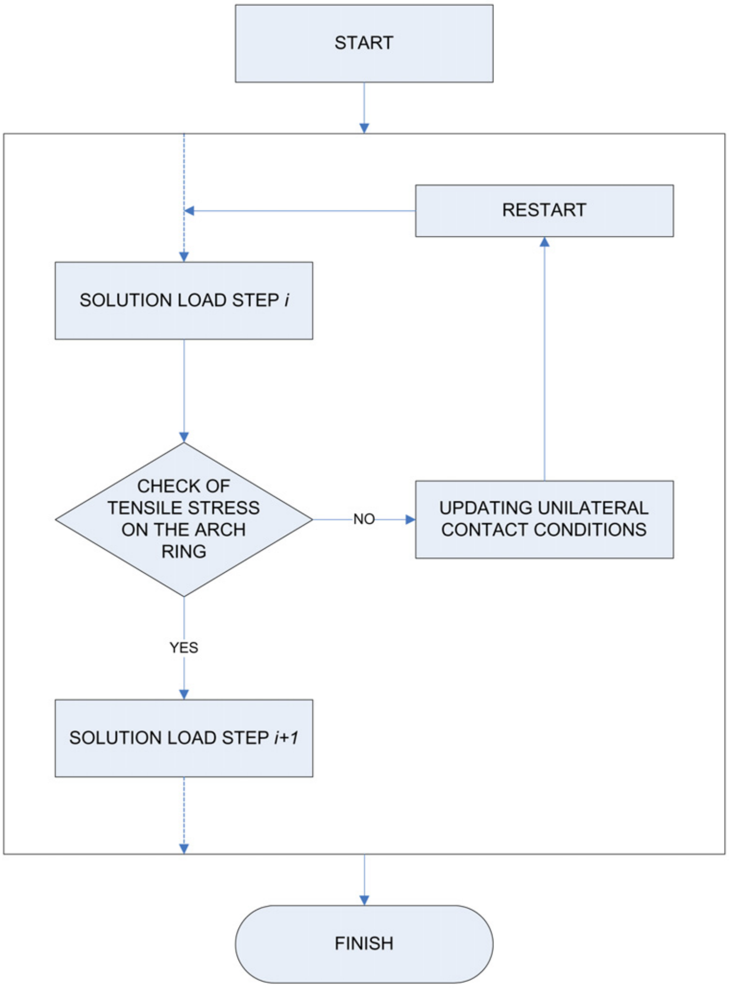

Fig. 4. Flow chart of the adaptive procedure.

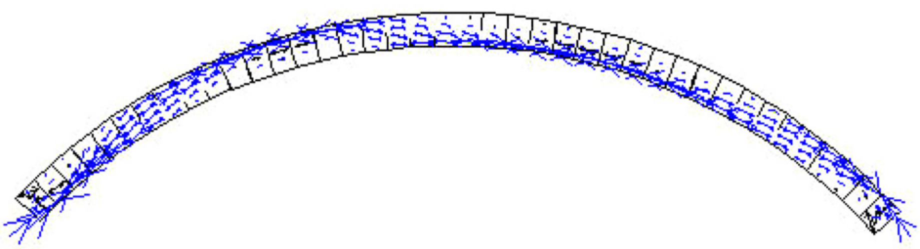

Fig. 5. Principal stress (vector representation, trust line) before collapse.

each model, with the experimentally obtained one, confirms this experimental result. A concentrated load is applied on a rigid basis (0.3 m wide) upon the fill, in the quarter span of the bridge.

4.2.1. Results from the unilateral contact–friction model

In the framework of the contact model, the fill is simulated with two-dimensional plane strain finite elements. The arch–fill interaction is taken into account, as well by a unilateral contact–friction interface. Failure of the fill material is representing by the Mohr–Coulomb failure criterion. Concerning the boundary conditions of the contact model,

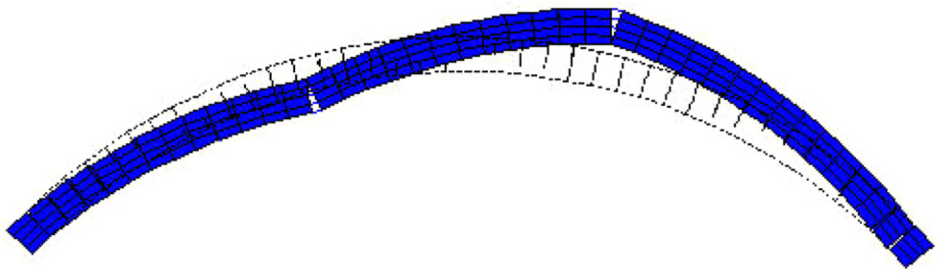

Fig. 6. Deformation before collapse.

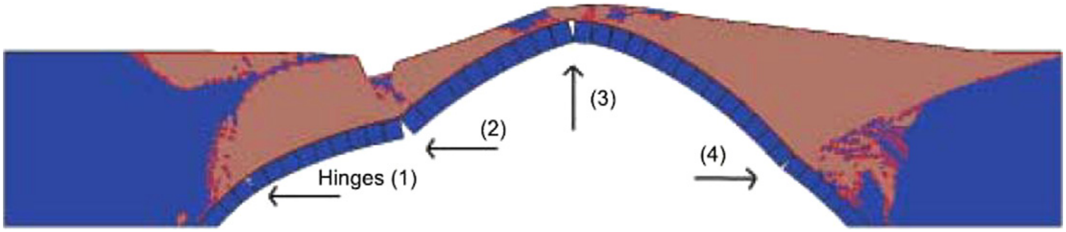

Fig. 7. Four hinges collapse mechanism from the unilateral contact–friction.

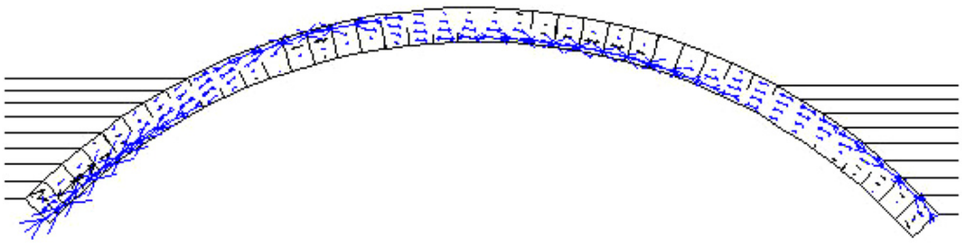

Fig. 8. Principal stress (vector representation, trust line) before collapse.

both the arch and the fill are initially considered to be fixed to the ground. In particular, the horizontal as well as the vertical boundaries of the fill are fixed.

For the mechanical properties mentioned here the collapse load which is obtained by the contact model is $P_{u,c} = 225.4$ kN which is comparable with the experimental value (228 kN). A four hinges mechanism arises in the arch, which is the same with the experimental one. In Fig. 7 the collapse mechanism obtained by the contact model is shown, suitably scaled to make deformation visible. The brown colour in the fill indicates the region where yield occurs. Both the arch and the fill move downwards in the left-hand side of the structure (e.g. the loaded side). As a consequence, the side of the bridge opposite to the loading moves upwards (the four hinges collapse mechanism of the arch pushes the fill upwards in this side). The hinge number (4) of Fig. 7 does not appear very close to the right springing but is moved toward the left side. The presence of the fill is responsible for the offset of the hinge's position.

4.2.2. Results from the adaptive unilateral element model

The arch barrel structural elements are modelled again with four node linear plain stress elements. Unilateral contact elements are added along the arch barrel in order to set zero tensile stresses with the adaptive stepwise procedure: at each load step the internal unilateral contact conditions are upgraded. In this second analysis the interaction between the fill material and the arch barrel has been taken into account by means of non-linear springs. Properties of this non-linear spring are evaluated assuming an angle of internal friction equal to 37° corresponding to the geotechnical fill material properties (reddish-brown sand).

Fig. 9 shows that the loaded side of the arch moves downwards, and the arch mechanism pushes the left side; as a consequence, the fill over the left side is moved upwards. The first hinge on the left side of the arch does not develop

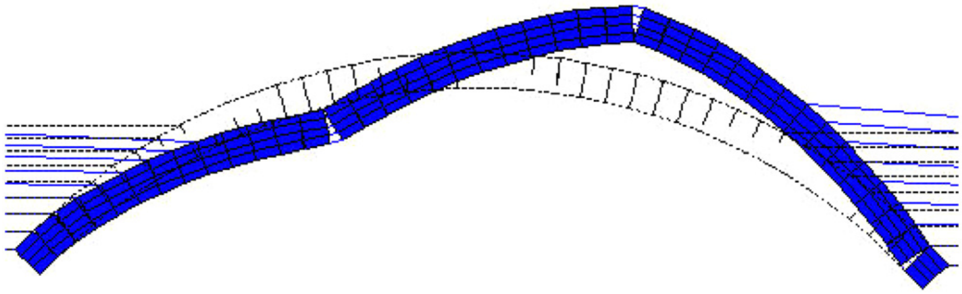

Fig. 9. Deformation before collapse.

at the springing, as it happens in the arch where the fill is heavy but not resistant; the presence of the fill constrains the hinge to move upward. This behaviour agrees with the experimental result shown in Fig. 2. The numerical load carrying capacity is $P_{u,\text{ad}} = 225.7$ kN, a value that agree with the collapse load found with the contact model ($P_{u,c} = 225.4$ kN). Fig. 8 shows principal stresses; close to the area where hinges arise there is a local crushing of masonry material but, again, the value reached by the tensional state on the arch at the collapse denotes that collapse is due to a development of a mechanism rather than a global crushing. Fig. 8 shows the trust line, which is adjacent to the ring of the arch on four sections denoting that hinges appear in that points. The section of the arch that first reaches the compressive elastic limit is located under the area where the load is applied. This result agrees with the experimental behaviour of the bridge.

4.2.3. Comparison between the two methods

Both models with the fill provide results that are very close to the experimental collapse load $P_{\text{exp}} = 228$ kN.

The strengthening effects of the fill correspond to a higher exploitation of the strength resources of the arch; in particular, the development of the collapse mechanism predicted by the limit analysis requires the development of plastic compressive strains higher than those predicted by simplified models without fill resistance that could be no longer consistent with the masonry behaviour.

A significant difference between the two models is introduced by the fact that in the model with the unilateral contact elements the development of the cracked area is not imposed a priori, but follows the actual mechanical behaviour of the arch when the loading increases from zero to the final load level. Of course the position of these elements is determined by the mesh size adopted for the plane elements used on the model of the arch ring. In contrast, the contact model uses fixed positions for the interfaces of the arch indicating that the positions of the potential cracks are imposed a priori. In practice the two models converge, provided that a large number of interfaces is used in the contact model.

Furthermore, the fill is simulated in two different ways. In the model with the unilateral contact elements, non-linear springs are used. This simplified procedure is particular useful for a first quick assessment of the behaviour of the stone arch. This is attributed to the fact that springs contribute to a relatively simple and fast numerical solution. On the other hand, the contact–friction model uses two dimensional finite elements with the Mohr–Coulomb failure criterion. This is a more complex way of simulating the fill over the arch. It gives information about the failure of the fill and a more realistic representation of it, but it demands more computational time and effort.

Moreover, an interesting comment can be made by observing the collapse mechanism obtained by the two models. In particular, it seems that the position of the hinge number (4) in the right side of the contact model (see Fig. 7), does not coincide with the corresponding hinge in the model with the unilateral elements (first hinge in the right side of the model, see Fig. 9). The latter is developed very near to the springing, while the hinge of the contact model arises at some distance from it. This is attributed to the fact that in the contact model both the vertical and the horizontal boundaries of the fill are fixed. In contrast, in the model with unilateral elements the fill is simulated with horizontal springs which permit a movement in the horizontal direction. This corresponds to free vertical boundaries, for the contact model. Indeed, if vertical boundaries of the contact model become free, the collapse mechanism will be slightly modified and the particular hinge will be in the same position for both models, e.g. very close to the springing. The limit load in this case is equal to 223.4 kN. As the boundary conditions have been changed, fill parameters have to be changed too, in order to obtain the above mentioned limit load. Consequently, the angle of internal friction has been considered to be equal to 37°, the cohesion equal to 10 kPa and the dilation angle equal to 33°.

5. Conclusions

Two procedures for the analysis of the load carrying capacity of masonry arch bridges have been discussed. Both the value of the collapse load and the corresponding collapse mechanisms, taking into account arch–fill interaction, can be predicted with high accuracy. Comparison with experimental results shows that both methods are able to access with good confidence the ultimate load for masonry arch structures. With the contact model, where the fill is modelled with plane elements, it is possible, in addition, to obtain information about the arising stresses and strains of the fill material. The unilateral model cannot supply detailed information about the fill because of the simplified assumption of the springs. Anyway, the method has proved to be user-friendly since it can be developed by making use of the programming facilities of commercial FE codes and, in spite of some approximations, it has proved to give good precision and it could be an interesting alternative to the standard non-linear facilities of commercial FEM codes, which are generally related to concrete-like materials. Finally, it should be emphasized that unilateral phenomena arise in a larger number of heritage and monumental structures, like domes and vaults. The techniques proposed here can, in principle, be extended to cover more general structures as well.

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