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Duality, inverse problems and nonlinear problems in solid mechanics

Adhesion effects in contact interaction of solids

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Abstract

An approach to solving problems of the interaction of axisymmetric elastic bodies in the presence of adhesion is developed. The different natures of adhesion, i.e. capillary adhesion, or molecular adhesion described by the Lennard-Jones potential are examined. The effect of additional loading of the interacting bodies outside the contact zone is also investigated. The approach is based on the representation of the pressure outside the contact zone arising from adhesion by a step function. The analytical solution is obtained and is used to analyze the influence of the form of the adhesion interaction potential, of the surface energy of interacting bodies or the films covering the bodies, their shapes (parabolic, higher power exponential function), volume of liquid in the meniscus, density of contact spots, of elastic modulus and the Poisson ratio on the characteristics of the interaction of the bodies in the presence of adhesion. ***To cite this article: I. Goryacheva, Y. Makhovskaya, C. R. Mecanique 336 (2008).***

Résumé

Effet d'adhésion dans l'interaction de contact entre solides. Une approche de résolution des problèmes d'interaction de corps élastiques axisymétriques en présence d'adhésion est développée. Les différents types d'adhésion, capillaire ou moléculaire décrite par le potentiel de Lennard-Jones, sont examinés. L'effet d'un chargement additionnel des corps en interaction, exercé en dehors de la zone de contact, est également étudié. L'approche repose sur la représentation de la pression hors de la zone de contact due à l'adhésion par une fonction en échelon. Une solution analytique est obtenue et utilisée pour analyser l'influence de la forme du potentiel d'interaction décrivant l'adhésion, de l'énergie de surface des corps en interaction ou des films couvrants, de leur forme (parabolique, exponentielle d'ordre plus élevé), du volume de liquide dans le ménisque, de la densité de points de contact, et du module d'élasticité et du coefficient de Poisson, sur les caractéristiques de l'interaction des corps en présence d'adhésion. ***Pour citer cet article : I. Goryacheva, Y. Makhovskaya, C. R. Mecanique 336 (2008).***

Keywords: Contact interaction; Capillary adhesion; Molecular adhesion

Mots-clés : Interaction de contact ; Adhésion capillaire ; Adhésion moléculaire

1. Introduction

Approaches to solving the problems on adhesion of elastic bodies can be divided into two classes. The first comprises numerical methods in which integral equations of the contact problem are solved numerically for a given form of

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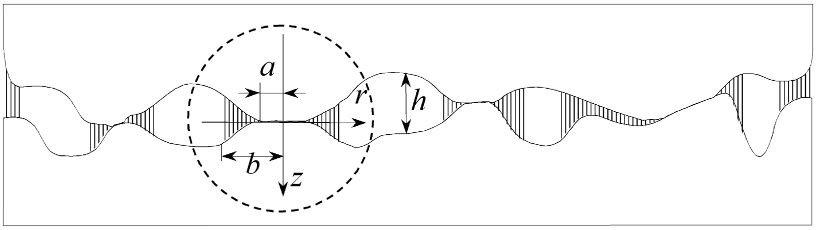

Fig. 1. Adhesion interaction of two rough elastic bodies. The cross-hatched regions correspond to the adhesion attraction of the bodies' surfaces.

the adhesion interaction potential (such as the Lennard-Jones potential [1]). The second presents approximate models (among them the classical Johnson–Kendall–Roberts theory and the classical Derjaguin–Muller–Toporov theory [2]) that yield asymptotic solutions to the adhesion problem in the case of two contacting elastic spheres. There are a number of approximate methods [3–6] developed for solving this problem in a wide range of variation of problem parameters. These methods are based on the approximation by given functions of adhesion pressure arising on the surfaces of interacting bodies.

In this study, we propose a more general approach using the representation of the adhesion pressure in the form of a piecewise-constant multistep function. This provides the possibility of considering arbitrary forms of the adhesion interaction potential (including the case of capillary adhesion), as well as of taking into account the presence of another additional load, in particular, the effect of neighboring asperities on adhesion of both rough bodies (Fig. 1) and bodies with a regular surface relief.

2. Problem formulation

We consider the interaction between two axisymmetric elastic asperities (dashed-line circle domain in Fig. 1). The separation between surfaces before loading is described by a power function $f(r) = f_1(r) + f_2(r) = Ar^{2n}$, where n is an integer. The boundary conditions at $z = 0$ have the form

$$u(r) = -f(r) - d, \quad 0 < r < a \tag{1}$$

$$p(r) = -p_a(r), \quad a \leqslant r \leqslant b \tag{2}$$

where $u(r) = u_1(r) + u_2(r)$ is the total normal displacement of the surfaces of the interacting bodies due to their deformation, $p(r)$ is the pressure on the body surfaces, and d is the variable distance between two fixed points of the interacting bodies. From condition (1), it follows that the bodies are in contact over the circular domain $0 < r < a$. If the contact is absent ($a = 0$), this condition is ignored. Condition (2) implies that the surfaces are loaded by an additional pressure $-p_a(r)$ outside the contact area. This can be either adhesion pressure or an additional load of a different nature. In what follows we assume that $p_a(r)$ at $a \leqslant r \leqslant b$ is the step function:

$$p_a(r) = \begin{cases} p_1, & b_0 \leqslant r < b_1 \\ p_2, & b_1 \leqslant r < b_2 \\ \dots\dots\dots\dots & \\ p_N, & b_{N-1} \leqslant r < b_N \end{cases} \tag{3}$$

where $b_0 = a$, $b_N = b$.

The dependence of the normal displacement $u(r)$ on pressure $p(r)$ is determined by the well-known expression for axisymmetric loading of an elastic half-space [7]:

$$u(r) = A\big[p(r), b\big], \quad 0 \leqslant r \leqslant b \tag{4}$$

Here

$$A\big[p(r), c\big] = \frac{4}{\pi E^*} \int_0^c p(r')\mathbf{K}\left(\frac{2\sqrt{rr'}}{r + r'}\right)\frac{r'\,\mathrm{d}r'}{r + r'}, \qquad \frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

where E_i and ν_i $(i = 1, 2)$ are the Young's moduli and Poisson's ratios of the interacting bodies, respectively, and $\mathbf{K}(x)$ is the complete elliptic integral of the first kind. In addition, the equilibrium condition

$$q = 2\pi \int_0^b rp(r)\,\mathrm{d}r \tag{5}$$

(where q is the normal external force acting on a single asperity) should be fulfilled.

3. The method of solution

For $0 \leqslant r \leqslant a$, the function $p(r)$ can be represented as

$$p(r) = p_*(r) - p_1 \tag{6}$$

Using conditions (2) and (3), relationship (4) can be transformed into the form

$$u(r) - \sum_{k=1}^{N} (p_{k+1} - p_k)\chi(r, b_k) = A\big[p_*(r), a\big] \tag{7}$$

where the function $\chi(r, c)$ is defined by the expression [8]

$$\chi(r, c) = A[1, c] = \frac{4}{\pi E^*} \begin{cases} c\mathbf{E}(r/c), & r \leqslant c \\ r[\mathbf{E}(c/r) - (1 - c^2/r^2)\mathbf{K}(c/r)], & r > c \end{cases}$$

Here, $\mathbf{E}(x)$ is the complete elliptic integral of the second kind. In (7) and hereinafter, it is assumed that $p_{N+1} = 0$.

In the absence of a contact between the surfaces ($a = 0$), relationship (7), in which the right-hand side is then equal to zero, is responsible for the elastic displacements of the surfaces $u(r)$ under the action of a given pressure $-p_a(r)$ (3) in the region $0 \leqslant r < b$.

When the surfaces are in contact, from (7) (with allowance for the contacting condition (1)), we find the integral equation for determining the function $p_*(r)$:

$$A\big[p_*(r), a\big] = -f_*(r) - d, \quad r \leqslant a \tag{8}$$

where

$$f_*(r) = f(r) + \frac{4}{\pi E^*} \sum_{k=1}^{N} (p_{k+1} - p_k) b_k \mathbf{E}\left(\frac{r}{b_k}\right)$$

Then, taking into account (3) and (6), equilibrium condition (5) takes the form

$$q + \pi a^2 p_1 + \pi \sum_{k=1}^{N} p_k\big(b_k^2 - b_{k-1}^2\big) = 2\pi \int_0^a r p_*(r)\,\mathrm{d}r \tag{9}$$

Since $p_*(a) = 0$ (in view of (6)), integral equation (8) is similar to that in the problem on indenting an axisymmetric stamp of a given shape $f_*(r)$ into an elastic half-space under the action of a force described by the expression on the left-hand side of condition (9). In this case, for $a < r \leqslant b$, the right-hand side of relationship (7) defines the elastic displacements $u(r)$ outside the contact area. Solving this problem with the use of the series expansion method described in [9] and [10] yields the following expressions for the contact pressure and boundary displacements of interacting bodies:

$$p(r) = \frac{AE^* a^{2n-1}}{\pi} \left[\frac{(2n)!!}{(2n-1)!!}\right]^2 \sqrt{1 - \frac{r^2}{a^2}} \sum_{k=1}^{n} \frac{(2k-3)!!}{(2k-2)!!} \left(\frac{r}{a}\right)^{2(n-k)}$$
$$- p_1 - \frac{2}{\pi} \sum_{k=1}^{N} (p_{k+1} - p_k) \arctan\sqrt{\frac{a^2 - r^2}{b_k^2 - a^2}}, \quad r \leqslant a, \tag{10}$$

$$u(r) = -\frac{2}{\pi}\left(d + Ar^{2n}\right)\arcsin\frac{a}{r} + \frac{2Ar^{2n}}{\pi}\sqrt{\frac{r^2}{a^2}-1}\sum_{i=1}^{n}\frac{(2i-2)!!}{(2i-1)!!}\left(\frac{a}{r}\right)^{2i}$$
$$-\frac{4}{\pi E^*}\sum_{j=1}^{k}(p_{j+1}-p_j)r\left\{\mathbf{E}\left(\frac{b_j}{r}\right) - \mathbf{E}\left(\arcsin\frac{a}{b_j}, \frac{b_j}{r}\right)\right.$$
$$\left.-\left(1-\frac{b_j^2}{r^2}\right)\left[\mathbf{K}\left(\frac{b_j}{r}\right) - \mathbf{F}\left(\arcsin\frac{a}{b_j}, \frac{b_j}{r}\right)\right]\right\}$$
$$+\frac{4}{\pi E^*}\sum_{j=k+1}^{N}(p_{j+1}-p_j)b_j\left[\mathbf{E}\left(\frac{r}{b_j}\right) - \mathbf{E}\left(\arcsin\frac{a}{r}, \frac{r}{b_j}\right)\right], \quad b_k \leqslant r \leqslant b_{k+1}. \tag{11}$$

The distance d between the bodies is given by the expression

$$d = -\frac{(2n)!!}{(2n-1)!!}Aa^{2n} - \frac{2}{E^*}\sum_{k=1}^{N}(p_{k+1}-p_k)b_k\sqrt{1-\frac{a^2}{b_k^2}} \tag{12}$$

and the load q has the form

$$q = \frac{(2n)!!}{(2n+1)!!}4E^*Ana^{2n+1} - \pi p_1 a^2 - \pi\sum_{k=1}^{N}p_k\left(b_k^2 - b_{k-1}^2\right)$$
$$-2\sum_{k=1}^{N}(p_{k+1}-p_k)b_k^2\left(\arcsin\frac{a}{b_k} - \frac{a}{b_k}\sqrt{1-\frac{a^2}{b_k^2}}\right) \tag{13}$$

The relationships obtained make it possible to solve the problems on the interaction of elastic bodies in the presence of adhesion of various nature.

4. Interaction of elastic bodies in the presence of molecular adhesion

An example of the application of the above approach is solving the problem on the interaction of two elastic asperities in the presence of the adhesion attraction given as a function of the distance between surfaces $p_{\mathrm{ad}}(h)$. An example of such a dependence is the Lennard-Jones function describing the molecular interaction of surfaces. To solve this problem, we set a partition b_j, $j = 1, \ldots, N$, of the region of the adhesion interaction $a \leqslant r \leqslant b$, the values of p_j being assumed to be unknown. Determining the elastic displacements $u(r)$ outside the contact area according to either relationship (7) (when the right-hand side is zero) in the absence of the contact or (11) in the presence of the contact, it is possible to express the value of the gap at the points b_j using the expression

$$h(r) = f(r) + u(r) + d \tag{14}$$

Then, the values of p_j are found from the system of equations $p_j = p_{\mathrm{ad}}(h(b_{j-1}))$, $j = 1, \ldots, N$. Additional equations for determining the coordinates a and b of the boundaries of the contact region and adhesion zone, respectively, are the continuity condition of pressure $p(a) = p_1$ at the boundary of the contact region (in the case of contact, when $a > 0$) and relationship (13) for an external load. Fig. 2 shows the calculation results for the external force q as a function of the distance d between the surfaces in the cases when the function $p_{\mathrm{ad}}(h)$ represents a step-like function and a linear function, and the shape of interacting bodies is described by a parabolic function $f(r) = r^2/(2R)$, i.e. $A = 1/(2R)$, $n = 1$. The parameters of both functions $p_{\mathrm{ad}}(h)$ under consideration are chosen in such a manner that the surface energies for both surfaces

$$\gamma = \int_0^\infty p_{\mathrm{ad}}(h)\,\mathrm{d}h$$

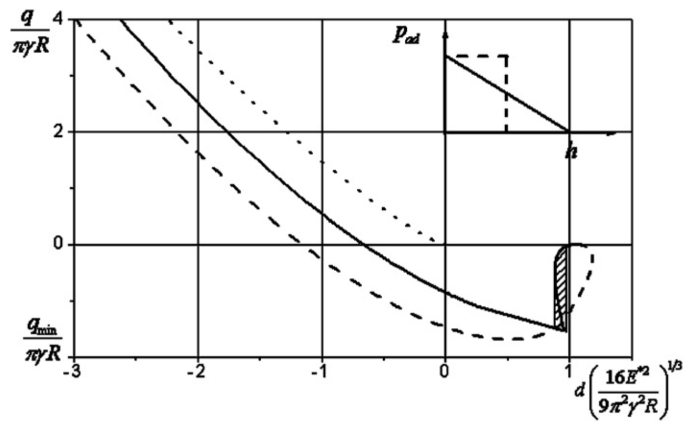

Fig. 2. Dependence of the dimensionless load on the dimensionless distance between the bodies for linear (solid line) and step (dashed line) functions $p_{\mathrm{ad}}(h)$ at $\lambda = 0.6$. The plots of the functions $p_{\mathrm{ad}}(h)$ are presented in the upper right corner. The dotted curve is obtained according to the Hertz theory in the absence of adhesion.

and the values of adhesion pressure $p_{\mathrm{ad}}(0) = p_0$ at the zero gap between the surfaces are equal to each other, respectively. We have used the parametrization proposed in [3] for which the solution depends on only the parameter

$$\lambda = p_0\left(\frac{9R}{2\pi\gamma E^{*2}}\right)^{1/3}$$

The results indicate that the shape of the function $p_{\mathrm{ad}}(h)$ most significantly affects the character of the dependences obtained for negative forces q, especially in the absence of a contact between the surfaces. It is also seen that the dependence of the force q on the distance d is non-monotonic and not uniquely defined. This implies that, if the surfaces move apart under a controlled force q, the contact break occurs in the case of a negative pull-off force corresponding to the minimum value of q on the plot. If the surfaces approach and move apart under a controlled distance d, then there is an energy loss corresponding to the cross-hatched area in Fig. 2. The calculations indicate that, in the case of the step function $p_{\mathrm{ad}}(h)$, the energy loss always differs from zero at $\lambda > 0$. At the same time, the energy loss for the linear function $p_{\mathrm{ad}}(h)$ takes place only for values of λ exceeding a certain quantity. Note that the calculation results for the dependence of the force q on the distance d for the step function of adhesion attraction $p_{\mathrm{ad}}(h)$ coincide with the solution obtained in [3]. The detailed analysis of this case, including the calculation of the pull-off force $q_{\min}$ and energy loss Δw in the approach–separation cycle of elastic bodies with different shapes (for various values of n), was performed in [11].

5. Interaction of elastic bodies in the presence of capillary adhesion

One more example illustrating the application of the above method is the case of capillary adhesion, when surface films of a liquid are merged into menisci around the interacting asperities (Fig. 1). Suppose that two elastic asperities are in contact over a region $r < a$ and a meniscus occupies a region $a \leqslant r \leqslant b$. The liquid in the meniscus exerts a constant negative pressure $-p_0$ on the surface of the asperities. This pressure is defined by the relation $p_0 \approx 2\sigma/h(b)$, which follows from the Laplace formula under the assumption that a gap is small compared to the meniscus radius (the estimations were made in [9]). Here, σ is the surface tension in the liquid. Imposing also the condition of the constant volume v of the liquid in the meniscus,

$$v = 2\pi \int_a^b r h(r)\,\mathrm{d}r$$

we arrive at the additional relation for determining the value of p_0.

The analysis of the solution to this problem presented in [9–11] showed, in particular, that an approach and separation of elastic bodies in the presence of a meniscus is accompanied by an energy loss, as is the case for adhesion defined by the function of adhesion pressure $p_{\mathrm{ad}}(h)$. For parabolic bodies ($n = 1$), the dependencies of this energy

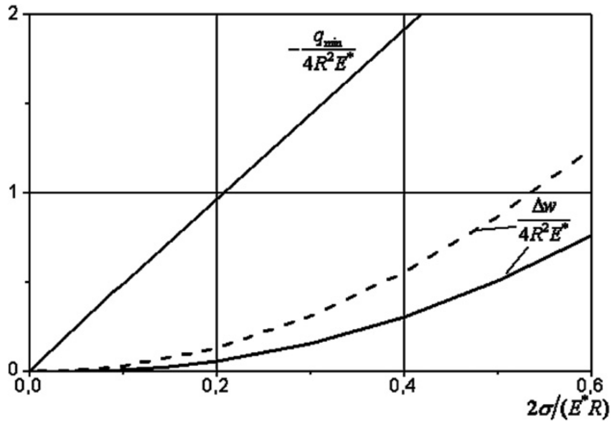

Fig. 3. Dimensionless pull-off force and dimensionless energy loss as functions of the dimensionless surface tension in the liquid in the case of capillary adhesion for $v/R^3 = 0.40$ (solid line) and 0.08 (dashed line).

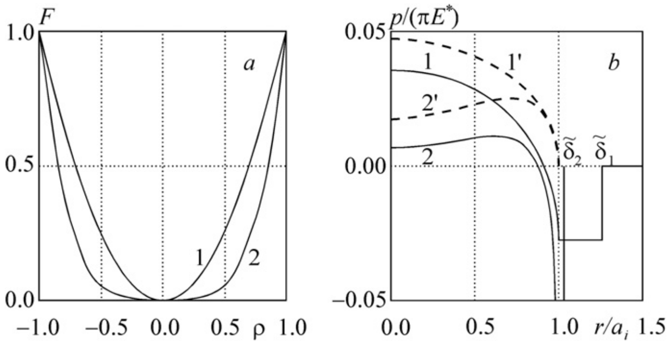

Fig. 4. The punch profiles (a) and contact pressure distribution (b) for $n = 1$ (curves 1, $1'$) and $n = 2$ (curves 2, $2'$). Curves $1'$, $2'$ correspond to the case of no capillary adhesion.

loss Δw and the pull-off force $q_{\min}$, on the surface tension in the liquid are shown in Fig. 3 (for different volumes of the liquid in the meniscus). The results indicate that the greater the surface tension σ of the liquid and the smaller the amount of the liquid v, the greater the energy loss. The pull-off force depends virtually linearly on the surface tension and weakly depends on the volume of liquid in the meniscus.

6. Influence of the shape of the interacting bodies on contact characteristics in the presence of capillary adhesion

The solution developed can be used for analysis of the adhesion effect in contact interaction of axisymmetric convex bodies of various shapes. Fig. 4(b) illustrates the dimensionless pressure in contact of the elastic half-space with the punches which shapes are described by the functions $f(r) = Ar^2$ ($n = 1$, parabolic punch, curve 1) and $f(r) = Ar^4$ ($n = 2$, curve 2). The punch profiles in dimensionless form $F(\rho) = \rho^{2n}$ ($\rho = rA^{1/(2n-1)}$) are shown in Fig. 4(a) by curves 1 ($n = 1$) and 2 ($n = 2$). The results are calculated for

$$\frac{P}{\pi E^*}A^{2/(2n-1)} = 0, \qquad \frac{\sigma}{\pi E^*}A^{1/(2n-1)} = 10^{-4}, \qquad vA^{3/(2n-1)} = 10^{-4}$$

Curves $1'$, $2'$ illustrate the pressure distribution for the case of no meniscus of liquid near the tip of asperities (no capillary adhesion) for the same contact radii a_i as the curves 1 and 2, respectively. At the plots the points $\tilde{\delta}_i = b_i/a_i$ indicate the external radius of the ring region occupied by the liquid referred to the corresponding contact radius a_i. It follows from the comparison of the curves that for the same contact area the contact pressure in the presence of the capillary adhesion is less then in the case of no meniscus near the contact tip.

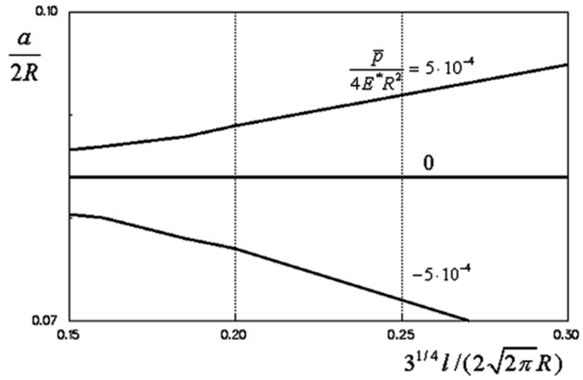

Fig. 5. Dependence of the dimensionless radius of the contact area on the dimensionless spacing between neighboring asperities in the case of the adhesion of surfaces with a regular microgeometry; $\gamma/(RE^*) = 2 \times 10^{-4}$, $p_0/E^* = 0.02$.

The shape of the punch influences on the pressure distribution and the meniscus width. The results presented in Fig. 4 show that if n increases, the absolute value of the capillary pressure p_0 increases and relative width of the meniscus ring decreases. The increase of the capillary pressure in the meniscus can be explained by the decreasing of the gap $h(r)$ for the higher n, in accordance with the Laplace formula.

7. Adhesion of bodies with regular microgeometry

The above method is also applicable for analyzing the adhesion interaction between elastic bodies with a regular surface microgeometry. Suppose that an elastic half-space interacts with a periodic system of axisymmetric identical asperities whose shape is described by the function $f(r)$. The asperities are assumed to be situated at the sites of a hexagonal lattice with a step l. The boundary conditions in the neighborhood of each of the asperities correspond to the adhesion attraction of the surfaces with a given function $p_{\mathrm{ad}}(h)$ or to the case that each asperity is surrounded by the meniscus of a liquid.

To solve this problem, we used the localization method of [12]. In the simplest variant of this method, only the interaction between the half-space and a single asperity in the presence of an additional load in the form of uniform pressure $\bar{p}$ acting in the region $L \leqslant r < +\infty$ is taken into account. The mean pressure is calculated by the formula $\bar{p} = 2q/(\sqrt{3}l^2)$, and the value of L is found from the equality condition of the mean pressure inside and outside of the region $r \leqslant L$, i.e., $\bar{p} = q/(\pi L^2)$. As a result, we have to solve the axisymmetric problem for the half-space with boundary conditions similar to those described in Section 2, in which the region of loading by an additional step pressure $-p_a(r)$ is infinitely large: $a \leqslant r < +\infty$. The results of solving this problem for the step function of the adhesion attraction $p_{\mathrm{ad}}(h)$ and $n = 1$ (parabolic asperities) are presented in [13].

Fig. 5 shows the dependences of the contact radius a on the spacing l between the neighboring asperities at a fixed mean pressure $\bar{p}$ onto the half-space. The results indicate that, for the positive mean pressure $\bar{p}$, a decrease in the spacing l reduces the size of the contact area. On the contrary, for negative pressures, provided that the surfaces are still in contact, a decrease in l leads to increasing radius a of the contact area. This implies that the character of the dependence of the actual contact area on the density of the asperities on a rough surface is determined by the sign of the external nominal pressure applied to the interacting bodies.

8. Conclusions

The approach developed makes it possible to solve the problems on interacting elastic bodies in the presence of different-type adhesion determined by the interaction potential of an arbitrary shape, including the case of capillary adhesion. In addition, it turns out to be possible to investigate the effect of adhesion on the interaction between bodies with a regular surface relief.

The method described enables one to calculate contact pressures, the actual contact area, elastic displacements of surfaces, and other characteristics of the adhesion interaction depending on diverse parameters, such as the shape of the adhesion interaction potential, its characteristics (in particular, surface energy of bodies and liquid films covering them), the amount of liquid in a meniscus, elastic constants of the bodies, the shape of interacting asperities, and the density of contact spots.

Analysis of the results obtained makes it possible to study various effects intrinsic to the adhesion interaction of elastic bodies (energy loss as a result of surface approach–separation cycle, a phenomenon of the jump separation of surfaces, increasing the real contact area with increasing density of asperities for negative values of an external force, etc.).

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