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Duality, inverse problems and nonlinear problems in solid mechanics

Eigenspectra and orders of stress singularity at a mode I crack tip for a power-law medium

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Abstract

In this Note eigenspectra and orders of singularity of the stress field near a mode I crack tip in a power-law material are discussed. The perturbation theory technique is employed to pose the required asymptotic solution. The whole set of eigenvalues is obtained. It is shown that the eigenvalues of the nonlinear problem are fully determined by the corresponding eigenvalues of the linear problem and by the hardening exponent. ***To cite this article: L. Stepanova, C. R. Mecanique 336 (2008).***

Résumé

Spectre et ordre de la singularité de contrainte à la pointe d'une fissure chargée en mode I dans un milieu dont le comportement suit une loi exponentielle. Dans cette Note, on détermine le spectre de valeurs propres du champ de contrainte asymptotique au voisinage de l'extrémité d'une fissure, dans le cas d'un matériau à comportement non linéaire. Toutes les valeurs propres sont obtenues par une méthode de perturbation. L'analyse indique que la valeur propre du problème non linéaire est complètement déterminée par la valeur propre du problème linéaire et le coefficient de consolidation plastique. ***Pour citer cet article : L. Stepanova, C. R. Mecanique 336 (2008).***

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Mots-clés : Rupture ; Analyse asymptotique ; Problème non linéaire aux valeurs propres

1. Introduction

Solutions for crack-tip fields are very important in understanding the mechanisms of crack initiation and propagation in elastic–plastic and creeping materials. The stress field in the vicinity of the crack tip in power-law materials (power-law hardening materials, power-law creeping materials) is widely discussed in literature. The stress singularity for a crack in a homogeneous power-hardening material with hardening exponent n was first studied by Hutchinson [1], Rice and Rosengren [2]. They obtained variable separable solutions for the leading 'HRR'-term of the asymptotic series for the crack tip stress field in power-law materials. In [1] the problem of plastic stress singularity is reduced to a nonlinear eigenvalue problem and the shooting method is used to solve the homogeneous differential equation obtained in the analysis.

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It should be noted that for some time multi-term asymptotic solutions with the well-known HRR-field as the leading order term of the asymptotic expansion aroused considerable interest of many researchers [3–5]. Thus, in [3] analytical solutions of higher order fields at a crack under antiplane shear in a fully plastic power-law hardening material are presented. By the use of hodograph transformation and asymptotic analysis the stress and strain exponents, angular distributions of shear stresses and strains are analytically determined. An algorithm and a computer program for the three-term asymptotic expansion of elastic–plastic crack tip stress and displacement fields are proposed in [4]. Here the leading order term is the classical HRR stress distribution. Noting growing appreciation of the role of higher-order terms in asymptotic stress and deformation fields near cracks in nonlinear materials, Nguyen et al. [5] focused on the higher-order near-tip fields in a steadily creeping power-law material.

Nowadays the whole eigenspectrum and orders of stress singularity at the crack tip for a power-law medium are of prevailing interest.

Thus, in [6] it is noted that most of works have not paid any attention to the higher order nonsingular and singular terms like $r^{3/2}$, r, $r^{1/2}$, $r^{-3/2}$, $r^{-5/2}, \ldots$ in the complete Williams expansion for elastic–plastic problems. If there exists a plastic zone around the crack tip, the complete solution in elastically deformed material outside the plastic zone should include the higher order singular terms as pointed out by Hui and Ruina [7]. They questioned the validity of some customary arguments related to the elastic–plastic crack problem under small scale yielding. They concluded that the higher order singularities should not be disregarded. The central subject of [7] is not whether or not singular terms and non-singular terms always exist in the elastic field outside the nonlinear zone. The authors conclude that the higher order terms cannot be neglected. These studies thus lead to the following questions [8]:

(1) How many singular terms, for a specific nonlinearly inelastic medium, exist mathematically at the crack tip and how they, if any, can be determined completely?
(2) Under what conditions a singular term can be regarded as the physically preferred one and plays a dominant role in practice?
(3) Does any other form of higher or lower order asymptotic solution exist and, if yes, how can one find them?

Namely, if the dominant term is not the classical HRR-field with the theoretically known eigenvalue $s = -1/(n+1)$ [1,2] how one can determine the function $s = s(n)$?

In [8] some additional eigenvalues for the stress field at a static mode I crack under plane stress condition are numerically obtained for some values of the exponent n via the Runge–Kutta method in conjunction with the shooting method. However, in this case the shooting method is multi-parametric since it is necessary to select two parameters and, consequently, the results obtained still require further investigations.

The present article offers a technique developed in the perturbation theory for study of nonlinear eigenvalue problems arising from fracture mechanics analysis.

2. Mode I crack—basic equations

A static mode I crack problem under the plane strain condition is considered. The equilibrium and compatibility equations in the polar coordinate system can, respectively, be written as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \qquad \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + 2\frac{\sigma_{r\theta}}{r} = 0 \tag{1}$$

$$2\frac{\partial}{\partial r}\left(r\frac{\partial \varepsilon_{r\theta}}{\partial \theta}\right) = \frac{\partial^2 \varepsilon_{rr}}{\partial \theta^2} - r\frac{\partial \varepsilon_{rr}}{\partial r} + r\frac{\partial^2 (r\varepsilon_{\theta\theta})}{\partial r^2} \tag{2}$$

The power-law constitutive relations for the plane strain condition are described by

$$\varepsilon_{rr} = -\varepsilon_{\theta\theta} = 3B\sigma_e^{n-1}(\sigma_{rr} - \sigma_{\theta\theta})/4, \qquad \varepsilon_{r\theta} = 3B\sigma_e^{n-1}\sigma_{r\theta}/2 \tag{3}$$

where the Mises equivalent stress is expressed by $\sigma_e^2 = 3(\sigma_{rr} - \sigma_{\theta\theta})^2/4 + 3\sigma_{r\theta}^2$.

The problem is completely defined by Eqs. (1), (2) and (3) and by the traction free boundary conditions on the crack faces:

$$\sigma_{\theta\theta}(r, \theta = \pm\pi) = 0, \qquad \sigma_{r\theta}(r, \theta = \pm\pi) = 0$$

The Airy stress potential $F(r,\theta)$ can be used to obtain

$$\sigma_{\theta\theta} = \frac{\partial^2 F}{\partial r^2}, \qquad \sigma_{rr} = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2}, \qquad \sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \theta}\right) \tag{4}$$

In analyzing the asymptotic behaviour of the stress field near the crack tip the Airy stress potential can be presented in the following form

$$F(r,\theta) = r^{\lambda+1} f(\theta) \tag{5}$$

Substitution (5) into (4) immediately yields

$$\sigma_{rr}(r,\theta) = r^{\lambda-1}\big[(\lambda+1)f(\theta) + f''(\theta)\big], \qquad \sigma_{\theta\theta}(r,\theta) = r^{\lambda-1}\lambda(\lambda+1)f(\theta), \qquad \sigma_{r\theta}(r,\theta) = -r^{\lambda-1}\lambda f'(\theta)$$

Using the constitutive equations (3) and the compatibility equation (2) one finds

$$\begin{aligned}
&f_e^2 f^{\mathrm{IV}}\big\{(n-1)\big[\big(1-\lambda^2\big)f + f''\big]^2 + f_e^2\big\} + (n-1)(n-3) \\
&\quad\times\big\{\big[\big(1-\lambda^2\big)f + f''\big]\big[\big(1-\lambda^2\big)f' + f'''\big] + 4\lambda^2 f'f''\big\}^2\big[\big(1-\lambda^2\big)f + f''\big] \\
&\quad+ (n-1)f_e^2\big\{\big[\big(1-\lambda^2\big)f' + f'''\big]^2 + \big[\big(1-\lambda^2\big)f + f''\big]\big(1-\lambda^2\big)f'' \\
&\quad+ 4\lambda^2\big(f''^2 + f'f'''\big)\big\}\big[\big(1-\lambda^2\big)f + f''\big] + 2(n-1)f_e^2 \\
&\quad\times\big\{\big[\big(1-\lambda^2\big)f + f''\big]\big[\big(1-\lambda^2\big)f' + f'''\big] + 4\lambda^2 f'f''\big\}\big[\big(1-\lambda^2\big)f' + f'''\big] \\
&\quad+ C_1(n-1)f_e^2\big\{\big[\big(1-\lambda^2\big)f + f''\big]\big[\big(1-\lambda^2\big)f' + f'''\big] + 4\lambda^2 f'f''\big\}f' \\
&\quad+ C_1 f_e^4 f'' - C_2 f_e^4\big[\big(1-\lambda^2\big)f + f''\big] + f_e^4\big(1-\lambda^2\big)f'' = 0
\end{aligned} \tag{6}$$

where

$$f_e^2 = \big[\big(1-\lambda^2\big)f + f''\big]^2 + 4\lambda^2 f'^2, \qquad C_1 = 4\lambda\big[(\lambda-1)n + 1\big], \qquad C_2 = (\lambda-1)n\big[(\lambda-1)n + 2\big]$$

The fourth order nonlinear ordinary differential equation (6) with the boundary conditions

$$f(\theta = \pm\pi) = 0, \qquad f'(\theta = \pm\pi) = 0 \tag{7}$$

defines a nonlinear eigenvalue problem in which the constant λ is the eigenvalue and $f(\theta)$ is the corresponding eigenfunction. The direct integration of the differential equation (6) is generally realized by the Runge–Kutta method in conjunction with the shooting method. Obviously, the eigenvalue λ and the initial value $f''(\theta = -\pi)$ are coupled with each other in general, and they have to be searched simultaneously. Only in some special cases one can assign a certain λ a prior through additional physical presumptions. Now the whole eigenspectrum and orders of stress singularity at the crack tip are of interest. The whole eigenspectrum stipulates the possible stress distributions in the neighbourhood of the crack tip. Therefore, the shooting procedure becomes multi-parametric here and the numerical results obtained need to be proved additionally. To overcome this difficulty in the problem the perturbation theory approach can be applied. A further reason to consider this problem is in the need for a formula expressing eigenvalues for the nonlinear problem through eigenvalues of the linear problem and the hardening exponent. In [9] a closed form solution for the eigenvalues, determining the asymptotic behaviour of the field at a crack tip under longitudinal shear is analytically derived by applying the perturbation method. It is shown [9] that the eigenvalues of the nonlinear problem solely depend on the eigenvalues of the corresponding linear problem and on the hardening exponent.

The purpose of this study is to obtain the whole eigenspectrum for the stress field near a mode I crack in a power-law material.

3. The perturbation theory approach

The underlying idea of the method is to consider the expansion representing the eigenvalue λ of the nonlinear eigenvalue problem (6), (7) for an arbitrary exponent n to be a sum of the eigenvalue λ_0 corresponding to the 'undisturbed' linear problem ($n = 1$) and a small parameter ε which quantitatively describes the nearness of the eigenvalues:

$$\lambda = \lambda_0 + \varepsilon \tag{8}$$

The exponent n and the stress function $f(\theta)$ can be presented as formal series with respect to ε:

$$n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \cdots, \qquad f(\theta) = f_0(\theta) + \varepsilon f_1(\theta) + \varepsilon^2 f_2(\theta) + \cdots \tag{9}$$

where $f_0(\theta)$ denotes the solution of the linear problem ($n = 1$).

Introducing (8), (9) into (6) and collecting terms of equal power in ε, the set of linear differential equations is obtained.

The first equation describing the linear problem

$$f_0^{\mathrm{IV}} + 2\left(\lambda_0^2 + 1\right) f_0'' + \left(\lambda_0^2 - 1\right)^2 f_0 = 0 \tag{10}$$

has the following solution

$$f_0(\theta) = B_1 \cos\left[(\lambda_0 - 1)\theta\right] + B_2 \sin\left[(\lambda_0 - 1)\theta\right] + B_3 \cos\left[(\lambda_0 + 1)\theta\right] + B_4 \sin\left[(\lambda_0 + 1)\theta\right] \tag{11}$$

The boundary conditions

$$f_0(\theta = \pm\pi) = 0, \qquad f_0'(\theta = \pm\pi) = 0 \tag{12}$$

lead to the characteristic equation $\sin 2\pi\lambda_0 = 0$. As is expected, the eigenspectrum of (10), (12) distributes discretely and has infinite number of eigenvalues

$$\lambda_0 = m/2, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Using the eigenvalues λ_0 obtained one can find the relations among B_j:

$$B_{3m} = -\frac{m-2}{m+2} B_{1m}, \quad B_{4m} = -B_{2m}, \quad m = \pm 1, \pm 3, \pm 5, \ldots$$
$$B_{3m} = -B_{1m}, \quad B_{4m} = -\frac{m-2}{m+2} B_{2m}, \quad m = 0, 2, \pm 4, \pm 6, \ldots$$

Considering odd integers m here one can represent (11) in the form

$$f_0(\theta) = \beta \cos(\alpha\theta) - \alpha \cos(\beta\theta), \quad \text{where } \alpha = \lambda_0 - 1,\ \beta = \lambda_0 + 1$$

The dimensionless angular function $f_1(\theta)$ must satisfy the fourth order linear ordinary differential equation

$$f_1^{\mathrm{IV}} + 2\left(\lambda_0^2 + 1\right) f_1'' + \left(\lambda_0^2 - 1\right)^2 f_1 = -n_1 \frac{x_0(f_0^{\mathrm{IV}} x_0 + \omega_0)}{g_0} + 2\lambda_0 f_0'' - C_1^1 f_0'' + C_2^1 x_0 + 2\lambda_0 a_0 f_0 \tag{13}$$

where, for brevity's sake, the following notations are adopted:

$$a_0 = 1 - \lambda_0^2, \qquad x_0 = a_0 f_0 + f_0'', \qquad g_0 = x_0^2 + 4\lambda_0^2 (f_0')^2$$
$$\omega_0 = (x_0')^2 + a_0 x_0 f_0'' + 4\lambda_0^2 (f_0'')^2 + 4\lambda_0^2 f_0' f_0'''$$
$$C_1^1 = 4\lambda_0\left[2 + n_1(\lambda_0 - 1)\right], \qquad C_2^1 = 2\lambda_0\left[1 + n_1(\lambda_0 - 1)\right]$$

Boundary conditions follow from the traction free conditions on the crack faces:

$$f_1(\theta = \pm\pi) = 0, \qquad f_1'(\theta = \pm\pi) = 0 \tag{14}$$

Thus, the boundary value problem (13), (14) for the nonhomogeneous fourth order linear differential equation is formulated. It is known [10] that if the boundary value problem for the homogeneous differential equation has a nontrivial solution then there can exist no solution of the corresponding nonhomogeneous differential equation unless the solvability condition is realized.

The solvability condition can be formulated by using a solution of the self-adjoint problem [10]:

$$\int_{-\pi}^{\pi} u g(\theta)\, \mathrm{d}\theta = 0, \quad u = f_0(\theta) = \beta \cos(\alpha\theta) - \alpha \cos(\beta\theta) \tag{15}$$

where u is the solution of the self-adjoint problem corresponding to (13), (14); $g(\theta)$ is the right hand side of (13).

The solvability condition (15) enables to obtain the first perturbation of n:

$$n_1 = -\frac{2}{\lambda_0 - 1} \tag{16}$$

and, consequently, the two-term asymptotic expansion for the exponent n has the following form

$$n = 1 - \frac{2\varepsilon}{\lambda_0 - 1} + \mathrm{O}(\varepsilon^2) \tag{17}$$

The nonhomogeneous linear differential equation for the function $f_2(\theta)$ can be presented as

$$\begin{aligned}
&g_0^2\big[f_2^{\mathrm{IV}} + 2(\lambda_0^2+1)f_2'' + (\lambda_0^2-1)^2 f_2\big] \\
&\quad + g_0^2\big(-x_0 + C_1^2 f_0'' - C_2^2 x_0 + 2\lambda_0 C_2^1 f_0\big) + n_1\big\{-x_0\big(f_0^{\mathrm{IV}}x_0 + \omega_0\big)\big[-4\lambda_0 f_0 x_0 + 8\lambda_0 (f_0')^2\big] \\
&\quad + g_0 x_0\big[-4\lambda x_0' f_0' - 2\lambda_0 a_0 f_0 f_0'' - 2\lambda_0 x_0 f_0'' + 8\lambda_0 (f_0'')^2 + 8\lambda_0 f_0' f_0'''\big] \\
&\quad + 2h_0 x_0'\big[-4\lambda_0 f_0 x_0 + 8\lambda_0 (f_0')^2\big] - 2h_0 x_0[-2\lambda_0 x_0 f_0' - 2\lambda_0 f_0 x_0' + 8\lambda_0 f_0' f_0''] \\
&\quad + 4\lambda_0^2 h_0 f_0'\big[-4\lambda_0 f_0 x_0 + 8\lambda_0 (f_0')^2\big] - 2\lambda_0 g_0 f_0\big(f_0^{\mathrm{IV}}x_0 + \omega_0\big) - 2\lambda_0 g_0 f_0 f_0^{\mathrm{IV}} x_0 + n_1 h_0^2 x_0 \\
&\quad + 4\lambda_0 h_0^2 f_0 - 4\lambda_0 g_0 h_0 f_0' - x_0\big(f_0^{\mathrm{IV}}x_0 + \omega_0\big)\big[2x_0 x_1 + 8\lambda_0^2 f_0' f_1'\big] + g_0 x_0^2 f_1^{\mathrm{IV}} \\
&\quad + g_0 x_0\big[2x_0' x_1' + a_0 f_0'' x_1 + a_0 x_0 f_1'' + 8\lambda_0^2 f_0'' f_1'' + 4\lambda_0^2 f_0''' f_1' + 4\lambda_0^2 f_0' f_1'''\big] + 2h_0 x_0'\big[2x_0 x_1 + 8\lambda_0^2 f_0' f_1'\big] \\
&\quad - 2h_0 x_0\big[x_0 x_1' + x_0' x_1 + 4\lambda_0^2 f_0' f_1'' + 4\lambda_0^2 f_0'' f_1'\big] + 4\lambda_0^2 h_0 f_0'\big[2x_0 x_1 + 8\lambda_0^2 f_0' f_1'\big] \\
&\quad + g_0\big(f_0^{\mathrm{IV}}x_0 + \omega_0\big)x_1 + 2h_0 g_0 x_1' - 2h_0^2 x_1 + 4\lambda_0^2 h_0 g_0 f_1' + f_0^{\mathrm{IV}} g_0 x_0 x_1\big\} = 0
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
&h_0 = x_0 x_0' + 4\lambda_0^2 f_0' f_0'', \qquad x_1 = a_0 f_1 + f_1'' \\
&C_1^2 = 4\big\{\lambda_0\big[n_1 + n_2(\lambda_0-1)\big] + 1 + n_1(\lambda_0-1)\big\}, \qquad C_2^2 = 2\lambda_0\big[n_1 + n_2(\lambda_0-1)\big] + \big[1 + n_1(\lambda_0-1)\big]^2 \\
&f_1(\theta) = -n_1(\beta\cos\alpha\theta - \alpha\cos\beta\theta)\ln\cos(\theta/2) \\
&\qquad + n_1\beta\begin{bmatrix} \displaystyle\sum_{k=1}^{(2\lambda_0-1)/2} \frac{\cos(2k-\lambda_0)\theta}{2k-1}, & 2\lambda_0 - 1 \geqslant 0 \\ \displaystyle\sum_{k=1}^{(1-2\lambda_0)/2} \frac{\cos(2k+\lambda_0-2)\theta}{2k-1}, & 2\lambda_0 - 1 < 0 \end{bmatrix} \\
&\qquad - n_1\alpha\begin{bmatrix} \displaystyle\sum_{k=1}^{(2\lambda_0+1)/2} \frac{\cos(2k-\lambda_0-2)\theta}{2k-1}, & 2\lambda_0 + 1 \geqslant 0 \\ \displaystyle\sum_{k=1}^{-(1+2\lambda_0)/2} \frac{\cos(2k+\lambda_0)\theta}{2k-1}, & 2\lambda_0 + 1 < 0 \end{bmatrix}
\end{aligned}$$

The boundary conditions for the function $f_2(\theta)$ are given by

$$f_2(\theta = \pm\pi) = 0, \qquad f_2'(\theta = \pm\pi) = 0 \tag{19}$$

Analysis of the solvability condition for the boundary value problem (18), (19) results in the three-term asymptotic expansions of the exponent n:

$$\begin{aligned}
&\lambda_0 = -5/2, \quad n = 1 + \frac{4}{7}\varepsilon - \frac{79}{2401}\varepsilon^2 + \mathrm{O}(\varepsilon^3) \\
&\lambda_0 = -3/2, \quad n = 1 + \frac{4}{5}\varepsilon + \frac{669}{625}\varepsilon^2 + \mathrm{O}(\varepsilon^3) \\
&\lambda_0 = -1/2, \quad n = 1 + \frac{4}{3}\varepsilon + \frac{92}{81}\varepsilon^2 + \mathrm{O}(\varepsilon^3)
\end{aligned}$$

$$\lambda_0 = 1/2, \quad n = 1 + 4\varepsilon + 8\varepsilon^2 + O(\varepsilon^3)$$
$$\lambda_0 = 3/2, \quad n = 1 - 4\varepsilon + \frac{53}{5}\varepsilon^2 + O(\varepsilon^3)$$
$$\lambda_0 = 5/2, \quad n = 1 - \frac{4}{3}\varepsilon + \frac{683}{567}\varepsilon^2 + O(\varepsilon^3) \tag{20}$$

For the eigenvalue $\lambda_0 = 1/2$ corresponding to the classical HRR-problem the following closed form solution

$$n_k = -\frac{(-1)^k}{(\lambda_0 - 1)^{k+1}}, \quad n = 1 - \frac{1}{\lambda_0 - 1}\sum_{k=1}^{\infty}\left(-\frac{\varepsilon}{\lambda_0 - 1}\right)^k = -\frac{\lambda}{\lambda - 1}, \quad \lambda = \frac{n}{n+1} \tag{21}$$

is found.

Hence, the well-known formula (21) connecting the hardening exponent n and the eigenvalue λ for the HRR-problem is derived.

Generalizing (20), one can find the second perturbation of n for $\lambda_0 \leqslant -3/2$ and for $\lambda_0 \geqslant 3/2$

$$n_2 = -\frac{\lambda_0^5 - 2\lambda_0^4 - 7\lambda_0^3 + 11\lambda_0^2 + 4\lambda_0 - 5 - (\lambda_0^2 - 1)\,\mathrm{sgn}(\lambda_0)}{(\lambda_0 + 1)(\lambda_0 - 1)^4} \tag{22}$$

4. Summary

Using the perturbation method the whole set of eigenvalues for a mode I crack tip in a power-law material is determined. The three-term asymptotic expansions for the exponent n (20) allowing to find the eigenvalue via (8) for the nonlinear problem (6), (7) are obtained.

The relative error of the three-term asymptotic expansion (20) for a crack in the power-law material with $n = 2$ to the exact HRR-solution is 2%.

The results obtained for $\lambda_0 = -1/2$ were compared with those found for the same problem by the Runge–Kutta method in conjunction with the shooting method [11]. The comparison of the eigenvalues for $n = 2$ calculated by the three-term asymptotic expansion (20) and by the numerical scheme $\lambda = -0.9801$ and $\lambda = -1.000$ shows the good agreement. The eigenvalues for $n = 3$ given by the four-term asymptotic expansion for $\lambda_0 = -1/2$ and by the Runge–Kutta method are $\lambda = -0.7716$ and $\lambda = -0.7755$. Consequently, a quite satisfactory solution is obtained by taking the asymptotic expansion achieved.

Thus, estimates (8), (9), (16) and (22) effectively constitute an asymptotic approximation of the solution to the nonlinear eigenvalue problem (6), (7).

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