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Buoyant–thermocapillary flows in a multilayer system

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Abstract

The nonlinear buoyant–thermocapillary flows in a three-layer system, filling a closed cavity and subjected to a temperature gradient directed along the interfaces, are investigated. The nonlinear simulations of convective regimes are performed by the finite-difference method. It is found that for sufficiently large values of the Grashof number, the long vortices turn into multicell structures. ***To cite this article: I.B. Simanovskii et al., C. R. Mecanique 336 (2008).***

Résumé

Écoulements provoqués simultanément par des gradients de densité et par thermocapillarité dans un système multicouche. Une étude est menée sur les écoulements induits dans un système composé de trois couches de liquides superposés, en tenant compte des effets non-linéaires. Ces liquides sont contenus dans une cavité fermée et soumise à un gradient de température parallèle aux interfaces. Les simulations numériques des régimes convectifs non-linéaires sont effectuées par la méthode des différences finies. Il est montré que pour des valeurs suffisamment grandes des nombres de Grashof, les rouleaux de convection longitudinaux se transforment en structures multicellulaires. ***Pour citer cet article : I.B. Simanovskii et al., C. R. Mecanique 336 (2008).***

Keywords: Computational fluid mechanics; Multilayer; Thermocapillarity convection; Gravity-induced convection; Flow stability; Grashof number; Marangoni number

Mots-clés : Mécanique des fluides numérique ; Multicouche ; Convection thermocapillaire ; Convection induite par densité ; Stabilité des écoulements ; Nombre de Grashof ; Nombre de Marangoni

1. Introduction

Interfacial convection in systems with interfaces is a widespread phenomenon that is of great importance in numerous branches of technology, including chemical engineering, space technologies, coating, etc. (for a review, see [1]).

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Scientific interest in such systems is due to the fact that the interfacial convection in multilayer systems is characterized by a variety of physical mechanisms and types of instability.

Numerical investigations of thermocapillary convection in multilayer systems were started in [2–4]. In these papers the linear stability of the mechanical equilibrium state and the nonlinear regimes of convection have been studied. Prakash and Koster found the analytical solution describing the velocity and the temperature fields for the parallel flow in the core region of a three-layer fluid system under the action of the temperature gradient directed along the interfaces (see [5,6]). The flow field in the end-wall region was analyzed by matching with the core region flow [6,7]. The nonlinear simulations of thermocapillary convective flows in a closed cavity filled by a symmetric three-layer system, have been performed in [8]. Thermocapillary flows in multilayer systems with periodic boundary conditions have been studied in [9]. The nonlinear simulations of buoyant–thermocapillary flows in a closed cavity filled by three immiscible viscous fluids with a temperature gradient directed along the interfaces, have been performed in [10]. Generally, *long unicell structures* have been obtained in each fluid layer. Examples of buoyant multilayer flows in a closed cavity are given in [11].

In the present Note, we investigate nonlinear *buoyant–thermocapillary convective flows* in a three-layer system, silicone oil 1 – ethylene glycol – fluorinert FC75, filling the closed cavity, subjected to a temperature gradient directed along the interfaces. It is found that at sufficiently large values of the Grashof number, the long unicell vortices turn into the multicell structures.

The Note is organized as follows. In Section 2, the mathematical formulation of the problem is presented. Nonlinear simulations of finite-amplitude convective regimes are considered in Section 3. Section 4 contains some concluding remarks.

2. General equations and boundary conditions

We consider a system of three horizontal layers of immiscible viscous fluids with different physical properties (see Fig. 1). The thicknesses of the layers are a_m, $m = 1, 2, 3$. The mth fluid has density ρ_m, kinematic viscosity ν_m, dynamic viscosity $\eta_m = \rho_m \nu_m$, thermal diffusivity χ_m, heat conductivity κ_m and thermal expansion coefficient β_m. The system is bounded from above and from below by two rigid plates, $z = a_1$ and $z = -a_2 - a_3$. A constant temperature gradient is imposed in the direction of the axis x: $T_1(x, y, a_1, t) = T_3(x, y, -a_2 - a_3, t) = -Ax + const$, $A > 0$. The surface tension coefficients on the upper and lower interfaces, σ and σ_*, are linear functions of temperature T: $\sigma = \sigma_0 - \alpha T$, $\sigma_* = \sigma_{*0} - \alpha_* T$, where $\alpha > 0$ and $\alpha_* > 0$.

Let us define

$$\rho = \frac{\rho_1}{\rho_2}, \quad \nu = \frac{\nu_1}{\nu_2}, \quad \eta = \frac{\eta_1}{\eta_2} = \rho\nu, \quad \chi = \frac{\chi_1}{\chi_2}, \quad \kappa = \frac{\kappa_1}{\kappa_2}, \quad \beta = \frac{\beta_1}{\beta_2}, \quad a = \frac{a_2}{a_1}$$

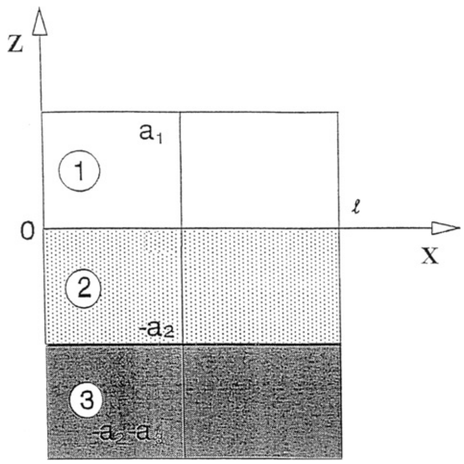

Fig. 1. Geometrical configuration of the three-layer system and coordinate axes.

Fig. 1. Coordonnées et géométrie du système trois couches.

$$\rho_* = \frac{\rho_1}{\rho_3}, \quad \nu_* = \frac{\nu_1}{\nu_3}, \quad \eta_* = \frac{\eta_1}{\eta_3} = \rho_*\nu_*, \quad \chi_* = \frac{\chi_1}{\chi_3}, \quad \kappa_* = \frac{\kappa_1}{\kappa_3}, \quad \beta_* = \frac{\beta_1}{\beta_3}, \quad a_* = \frac{a_3}{a_1}, \quad \bar{\alpha} = \frac{\alpha_*}{\alpha}$$

As the units of length, time, velocity, pressure and temperature we use a_1, a_1^2/ν_1, ν_1/a_1, $\rho_1\nu_1^2/a_1^2$ and Aa_1. The complete nonlinear equations governing convection are then written in the following dimensionless form:

$$\frac{\partial \mathbf{v}_m}{\partial t} + (\mathbf{v}_m \cdot \nabla)\mathbf{v}_m = -e_m \nabla p_m + c_m \Delta \mathbf{v}_m + b_m G T_m \mathbf{e} \tag{1}$$

$$\frac{\partial T_m}{\partial t} + \mathbf{v}_m \cdot \nabla T_m = \frac{d_m}{P} \Delta T_m \tag{2}$$

$$\nabla \mathbf{v}_m = 0, \quad m = 1, 2, 3 \tag{3}$$

Here $\mathbf{v_m} = (v_{mx}, v_{my}, v_{mz})$ is the velocity vector, T_m is the temperature and p_m is the pressure in the mth fluid; $\mathbf{e}$ is the unit vector of the axis z; $e_1 = c_1 = b_1 = d_1 = 1$, $e_2 = \rho$, $c_2 = 1/\nu$, $b_2 = 1/\beta$, $d_2 = 1/\chi$, $e_3 = \rho_*$, $c_3 = 1/\nu_*$, $b_3 = 1/\beta_*$, $d_3 = 1/\chi_*$; $\Delta = \nabla^2$, $G = g\beta_1 A a_1^4/\nu_1^2$ is the Grashof number, and $P = \nu_1/\chi_1$ is the Prandtl number determined by the parameters of the top layer.

The boundary conditions on the isothermic rigid boundaries are:

$$\mathbf{v}_1 = 0, \quad T_1 = T_0 - x \quad \text{at } z = 1 \tag{4}$$

$$\mathbf{v}_3 = 0, \quad T_3 = T_0 - x \quad \text{at } z = -a - a_* \tag{5}$$

where T_0 is constant.

The conditions on the rigid lateral boundaries, which are assumed to be thermally insulated, are

$$x = 0, L: \quad v_m = 0, \quad \frac{\partial T_m}{\partial x} = 0, \quad m = 1, 2, 3 \tag{6}$$

Let us discuss the boundary conditions at the interface between two fluids. One should be very careful by taking into account the deformation of the interface when using the Boussinesq approximation, because it is known that the interfacial deformation is a non-Boussinesq effect [12]. Indeed, the Boussinesq approximation is based on the assumption $\epsilon_\beta = \beta_1\theta \ll 1$, $G = \mathrm{O}(1)$, therefore the Galileo number $Ga = G/\epsilon_\beta = g a_1^3/\nu_1^2 \gg 1$. However, the balance of normal stresses on the interface shows that the interface deformation is proportional to $1/Ga\,\delta$, where $\delta = \rho^{-1} - 1$ (see [13]). Because $1/Ga\,\delta = \epsilon_\beta/G\delta$ is small unless $\delta \ll 1$, we come to the conclusion that in the framework of the Boussinesq approximation the interfacial deformation has to be neglected, if the densities of the fluids are not close to each other. The case of close densities is not considered in the present Note.

Thus, we assume that the interfaces between fluids are flat and situated at $z = 0$ and $z = -a$, and put the following system of boundary conditions: at $z = 0$

$$\frac{\partial v_{1x}}{\partial z} - \eta^{-1}\frac{\partial v_{2x}}{\partial z} - \frac{M}{P}\frac{\partial T_1}{\partial x} = 0, \qquad \frac{\partial v_{1y}}{\partial z} - \eta^{-1}\frac{\partial v_{2y}}{\partial z} - \frac{M}{P}\frac{\partial T_1}{\partial y} = 0 \tag{7}$$

$$v_{1x} = v_{2x}, \qquad v_{1y} = v_{2y}, \qquad v_{1z} = v_{2z} = 0 \tag{8}$$

$$T_1 = T_2 \tag{9}$$

$$\frac{\partial T_1}{\partial z} = \kappa^{-1}\frac{\partial T_2}{\partial z} \tag{10}$$

at $z = -a$

$$\eta^{-1}\frac{\partial v_{2x}}{\partial z} - \eta_*^{-1}\frac{\partial v_{3x}}{\partial z} - \frac{\bar{\alpha}M}{P}\frac{\partial T_3}{\partial x} = 0, \qquad \eta^{-1}\frac{\partial v_{2y}}{\partial z} - \eta_*^{-1}\frac{\partial v_{3y}}{\partial z} - \frac{\bar{\alpha}M}{P}\frac{\partial T_3}{\partial y} = 0 \tag{11}$$

$$v_{2x} = v_{3x}, \qquad v_{2y} = v_{3y}, \qquad v_{2z} = v_{3z} = 0 \tag{12}$$

$$T_2 = T_3 \tag{13}$$

$$\kappa^{-1}\frac{\partial T_2}{\partial z} = \kappa_*^{-1}\frac{\partial T_3}{\partial z} \tag{14}$$

Here $M = \alpha A a_1^2/\eta_1\chi_1$ is the Marangoni number.

In order to investigate the flow regimes generated by the convective instabilities, we perform nonlinear simulations of two-dimensional flows [$v_{my} = 0$ ($m = 1, 2, 3$); the fields of physical variables do not depend on y]. In this case, we can introduce the stream function ψ_m and the vorticity ϕ_m,

$$v_{m,x} = \frac{\partial \psi_m}{\partial z}, \qquad v_{m,z} = -\frac{\partial \psi_m}{\partial x}$$

$$\phi_m = \frac{\partial v_{m,z}}{\partial x} - \frac{\partial v_{m,x}}{\partial z} \quad (m = 1, 2, 3)$$

The boundary value problem was solved by the finite-difference method. Equations were solved using the explicit scheme, on a rectangular uniform mesh 168×56. The details of the numerical method can be found in the book by Simanovskii and Nepomnyashchy [13].

3. Numerical results

Below we describe results of computations of the nonlinear boundary value problem for the system silicone oil 1 – ethylene glycol – fluorinert FC75 with the following set of parameters: $\nu = 0.065$, $\nu_* = 1.251$, $\eta = 0.048$, $\eta_* = 0.580$, $\kappa = 0.390$, $\kappa_* = 1.589$, $\chi = 0.742$, $\chi_* = 2.090$, $\beta = 2.16$, $\beta_* = 0.957$, $\bar{\alpha} = 0.228$, $P = 13.9$. The ratios of the layers thicknesses have been chosen $a = a_* = 1$. Simulations have been performed for $L = 16$.

3.1. The case of buoyant flows ($G \neq 0$, $M = 0$)

For sufficiently small values of G, a steady flow takes place in the system, which contains one cell in each fluid layer (see Fig. 2). In the middle part of the cavity the flow in practice is parallel to the interfaces. With the increase of G, the intensity of the motion grows. The finite-amplitude curves corresponding to the intensities of motion in each fluid layer, are shown in Fig. 3. The presence of the rigid heat-insulated lateral walls (in comparison with periodic boundary conditions) leads to an interesting nonlinear effect—depending on G—the most intensive motion may be observed in different layers. At the larger values of G ($G \geqslant 19\,500$), the steady state becomes unstable. An additional maximum of the stream function field appears in the middle layers near the cold end, and the transient process takes place in the system (see Fig. 4). During this process, a long vortex in the middle layer breaks down and turns into the *multicell* structure. In the top layer, the maximum value of the vortices' intensity is located in the right half of the cavity. In the bottom layer, the multicell structure is located in the left half of the cavity. During the oscillatory process, a new vortex is generated near the hot end in the bottom layer. This vortex grows and shifts the neighbor vortex to the right, where the latter vortex decays. Let us note that only *long unicell structures* in each fluid layer have been obtained in [10].

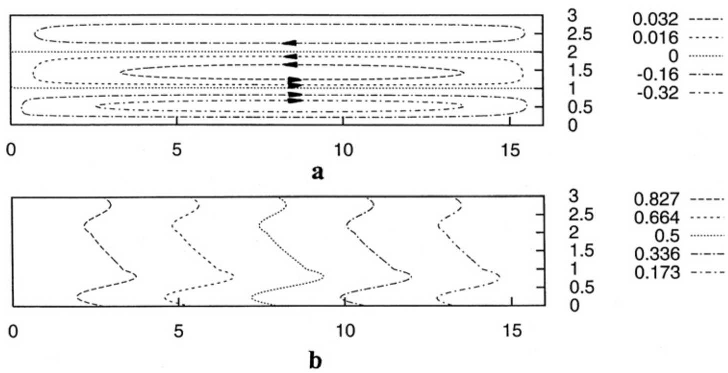

Fig. 2. Snapshot of (a) streamlines and (b) isotherms for the buoyant steady flow ($G = 1700$, $M = 0$).

Fig. 2. (a) Lignes de courant et (b) isothermes d'un écoulement provoqué par les gradients de densité ($G = 1700$, $M = 0$).

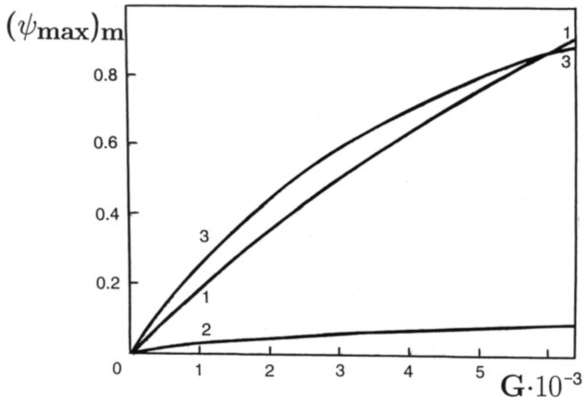

Fig. 3. Dependence of $(\psi_{\max})_m$ $(m = 1, 2, 3)$ on G for the buoyant steady flow $(M = 0)$.
Fig. 3. Variation de $(\psi_{\max})_m$ $(m = 1, 2, 3)$ en fonction de G pour un écoulement stationnaire induit uniquement par le gradient de densité $(M = 0)$.

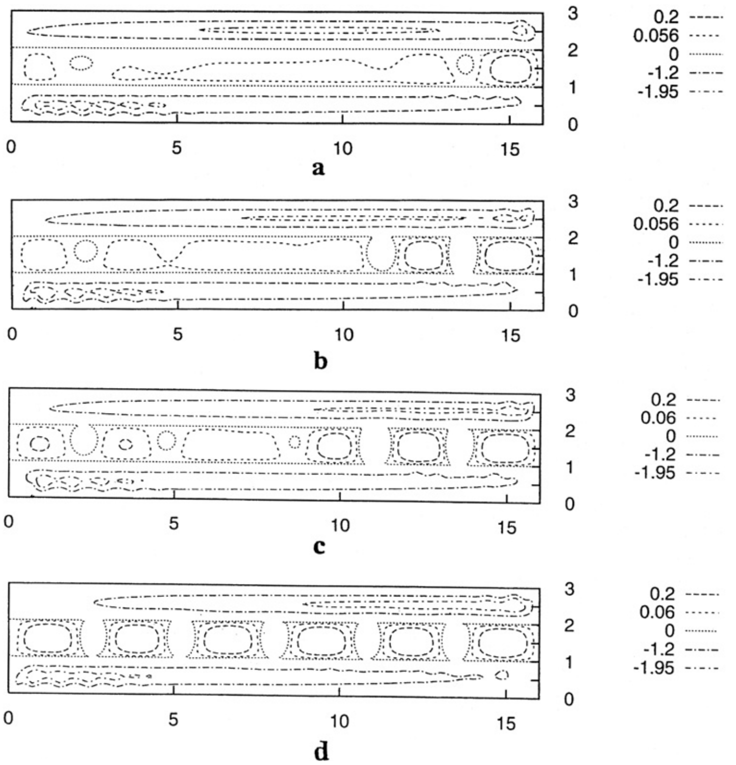

Fig. 4. Fields of (a)–(d) streamlines for the buoyant oscillatory motion $(G = 20\,400,\ M = 0)$.
Fig. 4. Lignes de courant (a)–(d) pour un écoulement oscillatoire induit par la densité $(G = 20\,400,\ M = 0)$.

3.2. *The case of buoyant–thermocapillary flows* $(G \neq 0,\ M \neq 0)$

Under the conditions of the experiment, when the geometric configuration of the system is fixed while the temperature difference θ is changed, the Marangoni number M and the Grashof number G are proportional. Define the inverse dynamic Bond number

$$K = \frac{M}{GP} = \frac{\alpha}{g\beta_1\rho_1 a_1^2}$$

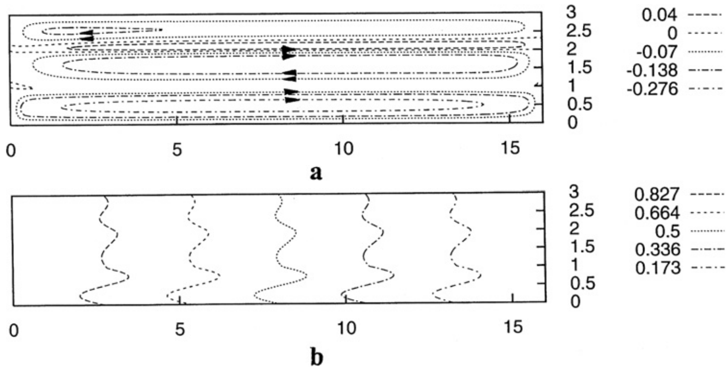

Fig. 5. Snapshot of (a) streamlines and (b) isotherms for the buoyant–thermocapillary steady flow ($G = 1700$, $M = 24\,000$).
Fig. 5. (a) Lignes de courant et (b) isothermes dans un écoulement thermocapillaire stationnaire ($G = 1700$, $M = 24\,000$).

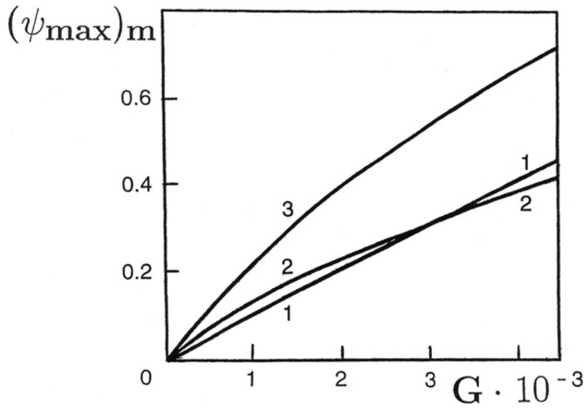

Fig. 6. Dependence of $(\psi_{\max})_m$ $(m = 1, 2, 3)$ on G for the buoyant–thermocapillary steady flow ($K = 1.02$).
Fig. 6. $(\psi_{\max})_m$ $(m = 1, 2, 3)$ en fonction de G pour un écoulement mixte (densité + thermocapillarité) stationanire ($K = 1.02$).

The simulations have been performed for $K = 1.02$.

In the case of buoyant–thermocapillary flows, the stationary flow is essentially asymmetric with respect to the reflection $x \to L - x$ (see Fig. 5). Three of the most intensive vortices are produced mainly by buoyancy (the fluids go upward near the hot wall and go downward near the cold wall). These vortices are separated by vortices rotating in the opposite direction, which are supported by the thermocapillary effect. The finite-amplitude curves for the steady flow are presented in Fig. 6. With the increase of G, the steady motion becomes unstable and one observes the oscillatory flow in the cavity (see Fig. 7). The chess-order configuration of vortices is obtained in the top layer. The vortices change their form and intensity during the oscillatory process.

4. Conclusion

The nonlinear buoyant–thermocapillary flows in a three-layer system, filling a closed cavity and subjected to a temperature gradient directed along the interfaces, are investigated. The shape and the amplitude of the convective flows are studied by the finite-difference method. It is found that depending on G, the most intensive motion may be observed in different layers. For sufficiently large values of the Grashof number, the steady state becomes unstable and the transient process of the unicell structures changing into the multicell structures takes place in the system. The specific type of nonlinear oscillations, is obtained.

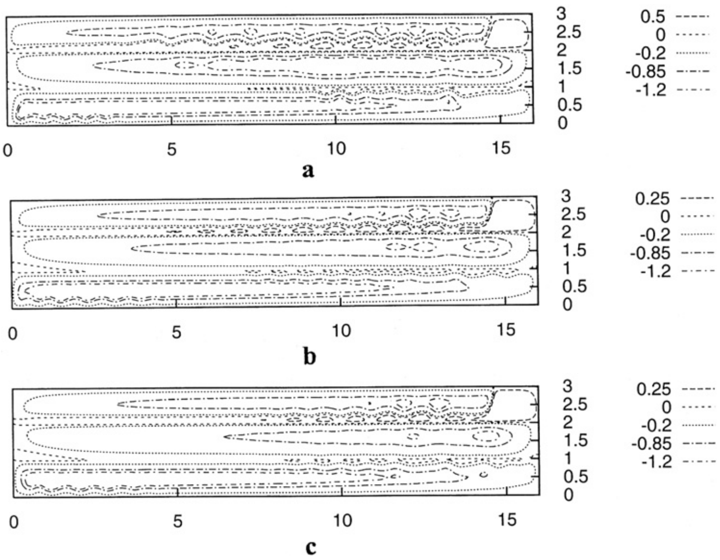

Fig. 7. Fields of (a)–(c) streamlines for the buoyant–thermocapillary oscillatory motion ($G = 15\,300$, $M = 216\,000$).

Fig. 7. (a)–(c) Lignes de courant dans un écoulement mixte oscillatoire ($G = 15\,300$, $M = 216\,000$).

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