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Turbulent transport of a passive contaminant in an initially anisotropic turbulence subjected to rapid rotation: an analytical study using linear theory

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Abstract

The linear effect of rapid rotation is studied on the transport by homogeneous turbulence of a passive scalar with vertical mean scalar gradient. Connection with one-particle diffusion studied by Cambon et al. [C. Cambon, F.S. Godeferd, F. Nicolleau, J.C. Vassilicos, Turbulent diffusion in rapidly rotating turbulence with and without stable stratification, J. Fluid Mech. 499 (2004) 231–255] is discussed. The input of the initial anisotropy of the velocity field is then investigated in the axisymmetric case, using a general and systematic way to construct axisymmetric initial data: a classical expansion in terms of scalar spherical harmonics for the 3D spectral density of kinetic energy and a modified expansion for the polarization anisotropy. The scalar variance exhibits a quadratic evolution ($\propto t^2$) for short times and a linear one ($\propto t$) for larger times. The long-time behaviour looks similar to the classical 'Brownian' evolution but it has a very different origin: a linear impact of dispersive inertial waves via phase-mixing instead of a nonlinearly-induced random walk. It is shown that this trend is not altered by the polarization anisotropy. The vertical scalar flux varies linearly with time for short times and tends to a plateau for larger times. ***To cite this article: A. El Bach et al., C. R. Mecanique 336 (2008).***

Résumé

Transport turbulent d'un contaminant passif dans une turbulence initialement anisotrope, assujetti à une rotation rapide : une étude analytique utilisant la théorie linéaire. Le transport d'un scalaire passif est étudié dans un écoulement turbulent en rotation rapide. Ce problème est analogue à celui de la diffusion à une particule, le carré du déplacement moyen d'un élément de fluide dans la direction verticale (le long de l'axe de la rotation d'ensemble) étant régi par les mêmes équations que la variance de concentration de scalaire en présence d'un gradient vertical moyen de scalaire. La solution linéaire conduit à des résultats purement analytiques à partir des distributions spectrales détaillées. L'introduction de conditions initiales à symmétrie de révolution, via une décomposition de type harmoniques sphériques, permet de généraliser l'étude en analysant les trois contributions « isotrope », de « directivité » et de « polarisation » du champ de vitesse turbulent. Une loi d'évolution quadratique ($\sim t^2$), de type « ballistique », est retrouvée à temps court pour les trois contributions à la variance du scalaire, tandis qu'une loi linéaire ($\sim t$) apparait à la fois sur la composante isotrope et la composante de directivité à partir d'un temps critique un peu inférieur à une période de révolution, la

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contribution de la polarisation tendant, comme le flux vertical de scalaire, vers un plateau. Comme dans Cambon et al. [C. Cambon, F.S. Godeferd, F. Nicolleau, J.C. Vassilicos, Turbulent diffusion in rapidly rotating turbulence with and without stable stratification, J. Fluid Mech. 499 (2004) 231–255], la loi linéaire ($\sim t$) résulte du mélange de phase des ondes d'inertie en rotation rapide, avec donc une physique très différente du comportement non-linéaire classique de marche aléatoire en turbulence isotrope. ***Pour citer cet article : A. El Bach et al., C. R. Mecanique 336 (2008).***

Keywords: Turbulence; Rapid rotation; Linear theory

Mots-clés : Turbulence ; Rotation rapide ; Théorie linéaire

Transport of passive scalar in rotating flows is found in many geophysical (see, e.g. [1]) or industrial applications. One-particle turbulent diffusion in a rapidly rotating fluid was studied by Cambon et al. [2] by means of linear theory (or RDT, Rapid Distortion Theory), KS (Kinematic Simulation) and DNS (Direct Numerical Simulation). As initially shown by [3], application of RDT is relevant if a simplified Corrsin Hypothesis is assumed, which amounts to identify two-time second order Lagrangian velocity correlations with their Eulerian counterparts. The use of linear dynamics for velocity is justified for rapid rotation, at small Rossby number, not only because of the very definition and meaning of the Rossby number (as classically advocated in RDT), but also because nonlinearity is dramatically reduced by inertial wave phase-mixing, as shown in wave turbulence theory.

The linear study is generalized here to the case of anisotropic initial turbulence, replacing the vertical diffusivity by the variance of the scalar concentration. On the one hand, it is possible to study the Lagrangian displacement, using the trajectory equation $\dot{\chi}_i = u_i$ (in which, u_i is the velocity field, χ_i the position of a fluid particle, the 'overdot' denoting a substantial derivative along turbulent trajectories), and to derive the mean square displacement, denoted $\Delta_{ij} \sim \langle \chi_i \chi_j \rangle$ in [2] from it. On the other hand, the concentration of a purely advected scalar, ϑ, is governed by a similar equation, in the presence of a mean scalar gradient $\boldsymbol{C}$, or $\dot{\vartheta} + C_i u_i = 0$. This analogy, pointed out by Batchelor, among others, allows us to recover the Δ_{ii}, $i = 1, 2, 3$, not summed, from the variance of the scalar concentration $\langle \vartheta^2 \rangle$ ($\langle \cdot \rangle$ denotes ensemble averaging) choosing sequentially the mean scalar gradient in the three directions, or $C_i = \delta_{i1}$, $C_i = \delta_{i2}$, $C_i = \delta_{i3}$. A similar calculation was performed, for instance, by Brethouwer [4], in the case of rotating shear, using DNS with and without nonlinear advection term (the latter corresponds to *numerical* RDT solutions). For the sake of brevity, only the vertical $\boldsymbol{C} = C\boldsymbol{n}$ gradient case is studied here, corresponding to vertical one-particle diffusivity, where $\boldsymbol{n}$ denotes the vertical unit vector which also bears system vorticity $f\boldsymbol{n}$. The new emphasis on passive scalar concentration gives also access to the scalar fluxes $\langle \vartheta u_i \rangle$.

The main motivation of this new study is introducing initial anisotropy for the turbulent velocity field. The effect of rapid (linear 'RDT' limit) rotation on the structure of turbulence is very different, regarding the level of statistical correlations. Single-point second-order velocity correlations are only affected if the initial data are anisotropic: for instance the structure of an initially axisymmetric Reynolds stress tensor is dramatically changed by rapid rotation, passing from a 'pancake' (two equal transverse components larger than the axial one) to a 'cigar' type (two equal transverse components smaller than the axial one). This effect was explained via the following three-fold splitting of the Reynolds stress tensor

$$\langle u_i u_j \rangle = 2\mathcal{K}\left(\frac{\delta_{ij}}{3} + b_{ij}^{(e)} + b_{ij}^{(z)} \right) \tag{1}$$

in which the first term is the pure isotropic contribution, involving only the turbulent kinetic energy $\mathcal{K}(t)$, whereas the deviatoric tensor b_{ij} is decomposed into a 'directional' (superscript 'e') and a 'polarization' (superscript 'z') contribution from spectral anisotropy (the reader is referred to [5] for the spectral 'e–Z–h' (energy/polarization/helicity) decomposition which underlies Eq. (1) and to [6] for a recent application. The effect of rapid rotation, entirely missed in any single-point RSM (Reynolds-Stress Model), results in a selective rapid damping of $b_{ij}^{(z)}$ by angular phase-mixing, all the other terms being preserved. On the other hand, initial isotropy can be broken for two-time second order correlations [2], with direct application to passive scalar transport (as discussed in the very beginning) and for third order statistics with many different applications (nonlinear energy transfer [5], triple vorticity statistics [7]).

The linearized inviscid equations of the fluctuating velocity $\boldsymbol{u}$ and fluctuating passive scalar concentration ϑ are

$$\dot{\boldsymbol{u}} + f\boldsymbol{n} \times \boldsymbol{u} = -\nabla p, \qquad \nabla \cdot \boldsymbol{u} = 0, \qquad \dot{\vartheta} + C_3 u_3 = 0 \tag{2}$$

where f is the Coriolis parameter (local system vorticity), p is a modified pressure term (divided by the fluid density) only needed to ensure the solenoidal property ($\nabla \cdot \boldsymbol{u} = 0$), and $\dot{\boldsymbol{u}}$ is only $\partial \boldsymbol{u}/\partial t$ in the linear limit. Discarding laminar diffusive terms is justified because high Reynolds number, small Rossby number, and small Ekman number, are consistent limits. High Smith (or Prandtl) number allows us to discard the scalar dissipation in similar conditions. Linearization of the velocity equation is justified at small Rossby number, or $Ro = u'/(2\Omega L) \ll 1$ (u' and L being velocity and length scales, respectively) and not too large elapsed time. As a classical result in wave turbulence theory, nonlinearity is dramatically damped by phase-mixing and becomes relevant only at very long time $\Omega t \sim Ro^{-2}$ at small Rossby number. Linearization of the scalar equation, or $\dot{\theta} \sim \partial\theta/\partial t$, could be justified if the mean velocity gradient is large enough. Such an argument can be valid, as shown by [4]; nevertheless, the order of magnitude of C_3 is not arbitrary if one wishes to use the analogy of the third equation (2) with the trajectory equation $\dot{\chi}_3 = u_3$. In a more realistic way, one can say that linearization of the scalar equation yields the same final results as using the Simplified Corrsin Hypothesis, looking either at scalar variance with vertical mean scalar gradient or at mean square vertical displacement. On the one hand, the Simplified Corrsin Hypothesis is probably a weaker assumption than linearizing from the very beginning. On the other hand, the present study is purely Eulerian and there is no need to calculate two-time correlation using the scalar equation, although – single-time – scalar variance and scalar flux are implicitly related to two-time velocity correlations. As a final remark on our basic equations (2), suggested by one of the Referees , the case of a scalar with horizontal (in the plane of mean rotation) mean scalar gradient, or similarly the case of one-particle diffusion in the horizontal direction, can be addressed in the same way and is justified under the same assumptions; this case is not treated here for the sake of brevity, and because we did not succeed in founding eventual analytical laws, given a more complicated algebra, when the velocity field is initially anisotropic.

The solutions of linearized equations (2) are most easily obtained via Fourier synthesis, $[\boldsymbol{u}(\boldsymbol{x}, t), \vartheta(\boldsymbol{x}, t)] = \sum_k [\hat{\boldsymbol{u}}(\boldsymbol{k}, t), \hat{\vartheta}(\boldsymbol{k}, t)] \exp(\imath \boldsymbol{k} \cdot \boldsymbol{x})$, where $\boldsymbol{k}$ is the wave vector, the discrete summation being possibly replaced by an integral. Spectral formalism allows us to take into account the solenoidal condition, which amounts to the orthogonality of $\hat{\boldsymbol{u}}$ with $\boldsymbol{k}$. The latter geometrical condition then yields two components only, $u^{(1)}$ (toroidal-type) and $u^{(2)}$ (poloidal-type), by projecting $\boldsymbol{u}$ in the Craya–Herring [8] frame of reference

$$\boldsymbol{e}^{(1)} = \boldsymbol{k} \times \boldsymbol{n}/|\boldsymbol{k} \times \boldsymbol{n}|, \qquad \boldsymbol{e}^{(2)} = \boldsymbol{k} \times \boldsymbol{e}^{(1)}/k$$

in which $\boldsymbol{n}$ is the unit vector along the angular velocity vector. For convenience, the third variable $\hat{\vartheta}$ is scaled to have the same dimension as the velocity, or $u^{(3)} = -(f/C_3)\hat{\vartheta}$. The general problem in five components in physical space $(u_1, u_2, u_3, p, \vartheta)$ therefore reduces to a three-component $(u^{(1)}, u^{(2)}, u^{(3)})$ problem, and Eq. (2) becomes

$$\dot{u}^{(1)} - \sigma_r u^{(2)} = 0, \qquad \dot{u}^{(2)} + \sigma_r u^{(1)} = 0, \qquad \dot{u}^{(3)} + \sigma_c u^{(2)} = 0 \tag{3}$$

where $\sigma_r = f \boldsymbol{k} \cdot \boldsymbol{n}/k = f \cos\theta$ is the dispersion frequency of inertial waves, and $\sigma_c = f \sin\theta$. It clearly appears that the anisotropic distribution is characterized by the polar angle $\theta = \widehat{(\boldsymbol{k}, \boldsymbol{n})}$. The solution of the latter equation in (3) is easily derived as

$$u^{(3)}(t) = \left[(1 - \cos\sigma_r t)\tan\theta\right]u_0^{(1)} - \left[(\sin\sigma_r t)\, \tan\theta\right]u_0^{(2)} + u_0^{(3)} \tag{4}$$

where the subscript 0 denotes the initial values.

In the present study, attention is paid to the evolution of scalar variance $\langle\vartheta\vartheta\rangle$ and scalar fluxes $\langle u_i \vartheta\rangle$, obtained by integrating their spectra. All these spectra can be derived from their initial values. The second order spectral tensor, underlying Eq. (1), is built in anisotropic homogeneous turbulence using the spherically averaged energy spectrum $E(k, t)$, the scalar $\mathcal{E}(\boldsymbol{k}, t)$ which generates directional anisotropy, and the complex-valued scalar $Z(\boldsymbol{k}, t)$ which generates polarization anisotropy [5]. Half the trace of the spectral tensor is given by $E/(4\pi k^2) + \mathcal{E}$, which is also the total (poloidal + toroidal) spectral density of energy, whereas Z quantifies the unbalance between toroidal and poloidal energy components [5,9]. Initial anisotropy, here axial symmetry with mirror symmetry, consistent with the basic configuration (e.g. created by an axisymmetric convergent duct), is generated by means of a modified decomposition in terms of Legendre polynomials $P_m^0(x)$, with $x = \cos\theta$, so that the above-mentioned spectral scalar $\mathcal{E}$ (directional anisotropy) and Z (polarization anisotropy) reduce to

$$\mathcal{E}(\boldsymbol{k}) = \frac{E(k)}{4\pi k^2} \sum_{n=1}^{N_0} e_{2n}(k) P_{2n}^0(x), \qquad Z(\boldsymbol{k}) = (1 - x^2)\frac{E(k)}{4\pi k^2} \sum_{n=0}^{N_0} z_{2n}(k) P_{2n}^0(x) \tag{5}$$

for arbitrary degree (integer N_0). The reader is referred to [5] and [9] for the most general definition of $e = E/(4\pi k^2) + \mathcal{E}$ and Z for arbitrary anisotropic, homogeneous turbulence. Here, the use of zeroth-degree Legendre polynomials, and not more general angular harmonics, is allowed by axisymmetry. The weighting factor $\sin^2\theta = (1 - x^2)$ allows a correct convergence of the polarization Z towards 0 when the wavevector becomes aligned with the axial (vertical here) direction (see [9]). Eqs. (5) are completely consistent with those of [10] anyway.

Looking at the velocity dynamics only, linear solutions yield

$$E(k,0) = E(k,t), \qquad \mathcal{E}(\boldsymbol{k},t) = \mathcal{E}(\boldsymbol{k},0), \qquad Z(\boldsymbol{k},t) = e^{2\imath f t\cos\theta} Z(\boldsymbol{k},0)$$

for arbitrary initial anisotropy [5], meaning that the distribution of energy, with its directional anisotropy, is not affected, whereas axes of polarization are rotated from an angle $2\sigma_r t$, which involves twice the dispersion frequency $\sigma_r = f t\cos\theta$ of inertial waves.

Using the solution (4) and expressions of spectral components

$$\langle u^{(1)}u^{(1)*}\rangle = E/(4\pi k^2) + \mathcal{E} - \Re Z, \qquad \langle u^{(2)}u^{(2)*}\rangle = E/(4\pi k^2) + \mathcal{E} + \Re Z$$

consistently with (3), one finds the following three-fold decomposition for the spectrum of the scalar variance

$$\frac{1}{2}\langle u^{(3)}u^{(3)*}\rangle = \frac{1}{2}\langle u^{(3)}u^{(3)*}\rangle^{(iso)} + \frac{1}{2}\langle u^{(3)}u^{(3)*}\rangle^{(e)} + \frac{1}{2}\langle u^{(3)}u^{(3)*}\rangle^{(z)}$$

with

$$\frac{1}{2}\langle u^{(3)}u^{(3)*}\rangle^{(iso)} = \frac{1}{2}\langle u_0^{(3)}u_0^{(3)*}\rangle + \frac{E(k)}{4\pi k^2}\frac{1-x^2}{x^2}\bigl(1-\cos(fxt)\bigr) \tag{6}$$

$$\frac{1}{2}\langle u^{(3)}u^{(3)*}\rangle^{(e)} = \frac{E(k)}{4\pi k^2}\sum_{n=1}^{N_0} e_{2n}(k)\frac{1-x^2}{x^2}\bigl(1-\cos(fxt)\bigr)P_{2n}^0(x) \tag{7}$$

$$\frac{1}{2}\langle u^{(3)}u^{(3)*}\rangle^{(z)} = -\frac{E(k)}{8\pi k^2}\sum_{n=0}^{N_0} z_{2n}(k)\frac{(1-x^2)^2}{x^2}\bigl[1-2\cos(fxt)+\cos(2fxt)\bigr]P_{2n}^0(x) \tag{8}$$

in which the 'star' denotes complex conjugate, and e_{2n}, z_{2n} are the coefficients of the expansion in terms of angular harmonics, defined in (5). The spectrum of initial scalar variance $\langle\theta_0\theta_0\rangle$ is chosen as purely isotropic, but Eq. (4) changes its isotropic contribution and generates a directional dependence. It is important to point out that the notations $\langle u^{(3)}u^{(3)*}\rangle$ with the different superscripts (iso), (e) and (z) do not denote the same splitting as in Eq. (1): they distinguish the part of the scalar spectrum which is separately affected by the initial parts in (5) of the velocity spectral tensor via linear dynamics, but not an intrinsic decomposition of the scalar spectrum itself (for instance, polarization anisotropy has no meaning for a scalar spectrum, it is only relevant for a spectral tensor).

Furthermore, an explicit form of the Legendre polynomials used in (5), in terms of x [11]

$$P_{2n}^0(x) = \sum_{\ell=0}^{n} a_{n-\ell,n}x^{2\ell}, \qquad a_{\ell,n} = \frac{1}{2^{2n}}\frac{(-1)^{n-\ell}(2n+2\ell)!}{(n-\ell)!(n+\ell)!(2\ell)!}$$

allows us to integrate analytically equations (6)–(8) over wave space, with $\mathrm{d}^3\boldsymbol{k} = -2\pi k^2\,\mathrm{d}k\,\mathrm{d}x$ using axisymmetry. The contribution from $(1/2)\langle u^{(3)}u^{(3)*}\rangle^{(iso)}$ in (8) – i.e. generated by the pure isotropic part of the velocity field –

$$\frac{(f/C_3)^2}{2\mathcal{K}(0)}\bigl[\langle\vartheta\vartheta\rangle^{(iso)} - \langle\vartheta_0\vartheta_0\rangle\bigr] = \left(-2 + \tau\,\mathrm{Si}(\tau) + \cos\tau + \frac{\sin\tau}{\tau}\right) \tag{9}$$

with $\tau = ft$, is the same as the one for vertical diffusion square-length in [2], in which $\mathrm{Si}(x) = \int_0^x [\sin s/s]\,\mathrm{d}s = \mathrm{si}(x) + \pi/2$, where $\mathrm{si}(x)$ is the sine integral [11]. By using the series representation of the sine integral [11], we easily deduce that the time-evolution of $\langle\vartheta\vartheta\rangle^{(iso)}$ exhibits a quadratic law (i.e., $\langle\vartheta\vartheta\rangle^{(iso)} \propto \tau^2$) for short times, and a linear law (i.e., $\langle\vartheta\vartheta\rangle^{(iso)} \propto \tau$) at larger times. As stressed before, the quadratic law reflects a ballistic behaviour, but the linear one has nothing to do with the classical Brownian law found in isotropic turbulence, which is mediated by nonlinear interactions generating a random walk.

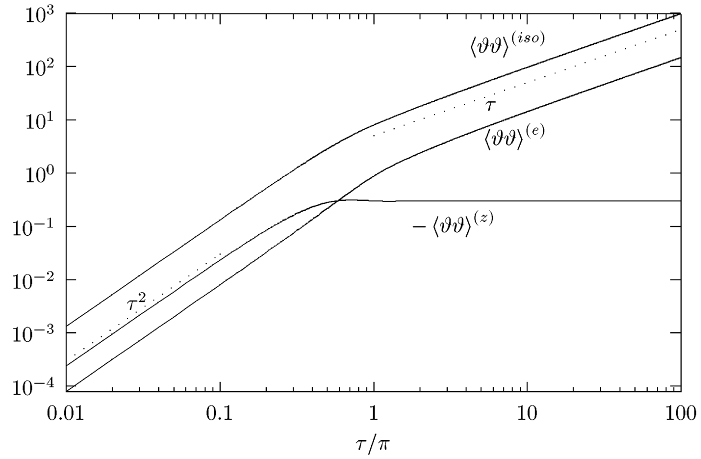

Fig. 1. Time evolution of the three contributions $\langle\vartheta\vartheta\rangle^{(iso)}$, $\langle\vartheta\vartheta\rangle^{(e)}$ and $-\langle\vartheta\vartheta\rangle^{(z)}$ to the scalar variance (zero initially), normalized by $2\mathcal{K}(0)/[(f/C_3)^2]$.

Similarly, the contribution from the initial directional anisotropy of the velocity field is found as

$$(f/C_3)^2\frac{\langle\vartheta\vartheta\rangle^{(e)}}{2\mathcal{K}(0)} = 15\sum_{n=1}^{N_0}\beta_{2n}^{(e)}a_{n,n}\left(2-\tau\,\mathrm{Si}(\tau)-\cos\tau-\frac{\sin\tau}{\tau}\right) - 15\sum_{n=1}^{N_0}\beta_{2n}^{(e)}\sum_{p=1}^{n}a_{n-p,n}\left(\frac{2}{4p^2-1}+\mathrm{I}_{2p}(\tau)-\mathrm{I}_{2p-2}(\tau)\right) \tag{10}$$

where the parameters $\beta_{2n}^{(e)} = -[\int_0^\infty E(k)e_{2n}(k)\,\mathrm{d}k]/[15\mathcal{K}(0)]$ quantify the initial contribution from e_{2n} in (5). The coefficient $\mathrm{I}_{2p}(\tau)$ is easily calculated using recurrence formulas

$$\mathrm{I}_{2p}(\tau) = \mathrm{I}_0(\tau) + \frac{2p}{\tau^2}\cos\tau - \frac{2p(2p-1)}{\tau^2}\mathrm{I}_{2p-2}(\tau), \quad \mathrm{I}_0(\tau) = \frac{\sin\tau}{\tau}$$

with $p = 1,\ldots,n$. It clearly appears that the behaviour of the part $\langle\vartheta\vartheta\rangle^{(e)}$, induced by initial directional anisotropy, for short and long times is similar to that of the part $\langle\vartheta\vartheta\rangle^{(iso)}$.

The last contribution to the scalar variance, induced by the initial polarization of the velocity field, is found as

$$\begin{aligned}(f/C_3)^2\frac{\langle\vartheta\vartheta\rangle^{(z)}}{2\mathcal{K}(0)} = &-\frac{15}{4}\sum_{n=0}^{N_0}\beta_{2n}^{(z)}a_{n,n}\left[-\frac{4}{3}+\tau\big(\mathrm{Si}(\tau)-\mathrm{Si}(2\tau)\big)+\left(\cos\tau-\frac{1}{2}\cos 2\tau\right)\right.\\ &\left.+\left(\frac{\sin\tau}{\tau}-\frac{\sin 2\tau}{4\tau}\right)-\left(2\frac{\cos\tau}{\tau^2}-\frac{\cos 2\tau}{4\tau^2}\right)+\left(2\frac{\sin\tau}{\tau^3}-\frac{\sin 2\tau}{8\tau^3}\right)\right]\\ &-\frac{15}{4}\sum_{n=1}^{N_0}\beta_{2n}^{(z)}\sum_{p=1}^{n}a_{n-p,n}\left(\frac{4}{(4p^2-1)(2p+3)}+2\mathrm{I}_{2p}(\tau)-\mathrm{I}_{2p-2}(\tau)-\mathrm{I}_{2p+2}(\tau)\right)\\ &-\frac{15}{8}\sum_{n=1}^{N_0}\beta_{2n}^{(z)}\sum_{p=1}^{n}a_{n-p,n}\big(\mathrm{I}_{2p+2}(2\tau)+\mathrm{I}_{2p-2}(2\tau)-2\mathrm{I}_{2p}(2\tau)\big)\end{aligned} \tag{11}$$

A 'ballistic' behaviour is shown again at short times, but followed by a plateau at larger times, easily expressed in terms of $\beta_{2n}^{(z)} = [4\int_0^\infty E(k)z_{2n}(k)\,\mathrm{d}k]/[15\mathcal{K}(0)]$ from (5). Finally, the vertical scalar flux, $\langle u_3\vartheta\rangle$, derived from the previous equations by $\mathrm{d}\langle\vartheta\vartheta\rangle/\mathrm{d}t = -2C_3\langle u_3\vartheta\rangle$, varies linearly with time for short times, and tends to a plateau at larger times (horizontal scalar fluxes are zero here).

Typical results are plotted in Fig. 1 for normalized $\langle\vartheta\vartheta\rangle^{(iso)}$, $\langle\vartheta\vartheta\rangle^{(e)}$ and $\langle\vartheta\vartheta\rangle^{(z)}$, using only the first spherical harmonics $e_2(k)$ and $z_0(k)$ in Eq. (5), or equivalently the parameters $\beta_2^{(e)}$ and $\beta_0^{(z)}$ called into play in (10) and (11).

These parameters are directly related to (1) via $\beta_2^{(e)} = n_i b_{ij}^{(e)} n_j$ and $\beta_0^{(z)} = n_i b_{ij}^{(z)} n_j$, in the axisymmetric case where b_{ij}, $b_{ij}^{(e)}$ and $b_{ij}^{(z)}$ are entirely generated by their axial component. The choice of initial parameters $\beta_2^{(e)} = n_i b_{ij}^{(e)} n_j = 0.02$ and $\beta_0^{(z)} = n_i b_{ij}^{(z)} n_j = -0.06$ corresponds to a 'pancake' type of Reynolds stress tensor, with negative $n_i b_{ij} n_j = -0.04$, as the one created by a convergent axisymmetric duct. As an important final remark, 'ballistic' short-time evolution is almost trivial, but the linear one has nothing to do with the 'Brownian' turbulent diffusion law: this behaviour reflects here a pure anisotropic phase-mixing by dispersive inertial waves, and the threshold from quadratic law to linear one is found to be similar to the rotation time-scale (same remark for the plateau).

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