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A discrete thermodynamic approach for modeling anisotropic coupled plasticity-damage behavior in geomaterials

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Abstract

In the present Note, we present a discrete thermodynamic approach for modeling coupled anisotropic plastic flow and damage evolution in geomaterials. The basic idea is to extend the widely-used isotropic coupled elastoplastic damage formulation to the case with induced anisotropy using a discrete approach. The total plastic strain is considered as the consequence of frictional sliding in weak sliding planes randomly distributed in the elastic solid matrix. The effective elastic tensor of damaged material is determined using damage variable associated with each family of weak sliding planes. An example of application is shown for a typical semi-brittle rock. ***To cite this article: Q.-Z. Zhu et al., C. R. Mecanique 336 (2008).***

Résumé

Une approche thermodynamique discrète pour la modélisation du couplage entre la déformation plastique et l'endommagement anisotrope. Dans la présente Note, nous présentons une approche thermodynamique discrète pour la modélisation du couplage entre la déformation plastique et l'endommagement anisotrope induit des géomatériaux. L'idée de base est d'étendre la formulation largement utilisée pour la modélisation du couplage plasticité- endommagement isotrope au cas de l'anisotropie induite. La déformation plastique totale est considérée comme la conséquence du glissement frottant le long des surfaces des plans de faiblesse, qui sont distribués aléatoirement dans la matrice solide élastique. Les propriétés élastiques effectives du matériau endommagé sont déterminées en fonction de l'endommagement lié à l'évolution des plans de faibless. Un exemple d'application à une roche semifragile est illustré. ***Pour citer cet article : Q.-Z. Zhu et al., C. R. Mecanique 336 (2008).***

Keywords: Damage; Discrete approach; Plasticity; Thermodynamic approach; Cracks; Sliding

Mots-clés : Endommagement ; Approche discrète ; Plasticité ; Fissures ; Frottement ; Glissement

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1. Introduction

Plastic deformation and induced damage are two main mechanisms of inelastic behavior in geomaterials like concrete and rocks. Due to specific characteristics of microstructure and its evolution, most geomaterials exhibit inherent (structural) and induced anisotropies. Plastic flow and damage evolution are generally coupled each other. Classically, continuous phenomenological models have been developed for the description of plastic deformation and induced damage, either separately or in coupled way. Tensorial internal variables are used to take into account the anisotropic spatial distribution of damage and plastic hardening variables. However, even with high order tensors, it is not easy to accurately represent actual distribution of damage and plastic hardening states. Further, the mathematical description becomes quickly complex in anisotropic plasticity and damage coupling when unilateral effects should be taken into account. On the other hand, micromechanical models based on linear homogenization techniques have also been developed [1]. These models allow to better account for physical mechanisms involved at the microscopic scale. Strain (or stress) concentration determination and volumetric averaging processes are needed to perform the scale change. Local constitutive behaviors should also be determined for each constituent phase; this is often a difficult task. Limit the present study to some classes of semi-brittle materials with low porosity such as hard rocks, inelastic deformations are essentially concentrated in families of defeats (cracks and interfaces for instance). For convenience and by the extension of cracks, all kinds of defeats leading to possible displacement discontinuity will be called here weak sliding planes (WSPs). The WSPs are randomly embedded and distributed in the representative volume element (RVE). The plastic flow is mainly related to frictional sliding along WSP surfaces while the damage evolution is associated with the growth of WSPs. There are interactions between these two phenomena, leading to coupled macroscopic plastic damage behavior. Based on this analysis, in the present work, we propose to develop an alternative modeling based on a discrete thermodynamic approach. Note that the concept of discrete plasticity has been used in different ways for modeling inelastic behavior of geomaterials, for instance the multi-laminate approach [2] and micro-plane models [3,4]. However, these models are generally not formulated in a proper thermodynamic framework. Furthermore, the damage process related to the evolution of defeats was not taken into account. In the present work, a thermodynamics consistent discrete approach is developed for coupled plastic damage process. A suitable discrete system of orientations is chosen to represent the continuous distribution of weak sliding planes. Total plastic deformations are considered as the consequence of frictional sliding along each family of WSPs. The effective elastic properties of damaged material is deduced as functions of discrete damage variable related to each family of WSPs. Finally, an example of an application to a typical semi-brittle rock is shown. Note that possible plastic strains taking place in the bulk material are not taken into account; this restriction may be not suitable for some classes of materials. In that case, the extension will be needed by considering coupling between matrix plasticity and damage by crack growth. This challenging issue is not discussed here.

2. Thermodynamic framework in the isotropic case

In this section, we briefly recall the thermodynamic framework for isotropic coupled elastoplastic damage formulation, in which scalar-valued internal variables are used. The total strain tensor is first decomposed into an elastic part $\boldsymbol{E}^e$ and a plastic part $\boldsymbol{E}^p$:

$$\boldsymbol{E} = \boldsymbol{E}^e + \boldsymbol{E}^p \tag{1}$$

Under the assumption of small strains and isothermal conditions, the internal variables involved are damage variable d, plastic strain $\boldsymbol{E}^p$ (or elastic strain $\boldsymbol{E}^e$) and plastic hardening variable γ^p. The Helmholtz free energy of damaged materials, which is as a function of the variables $\boldsymbol{E}^e, d$ and γ^p, is assumed to be composed of two additive parts: the elastic free energy $\Psi^e(\boldsymbol{E}^e, d)$ and the locked energy $\Psi^p(\gamma^p, d)$ associated with plastic hardening:

$$\Psi = \frac{1}{2}\boldsymbol{E}^e : \mathbb{C}(d) : \boldsymbol{E}^e + \Psi^p(\gamma^p, d) \tag{2}$$

Note that the two parts are dissipative due to coupled elastic damage process; $\mathbb{C}(d)$ is the fourth order effective elastic stiffness tensor of damaged material. Assuming that the considered material is initially elastic, isotropic and undamaged; its original elastic stiffness tensor $\mathbb{C}^s$ can be characterized by two elastic constants, the bulk modulus k^s

and the shear modulus μ^s. Generally, each modulus is independently degraded by the damage process. The effective elastic stiffness tensor of damaged materials $\mathbb{C}(d)$ is then expressed by:

$$\mathbb{C}(d) = 3k(d)\mathbb{J} + 2\mu(d)\mathbb{K} \tag{3}$$

$\mathbb{J}$ and $\mathbb{K}$ are two fourth order isotropic tensors respectively allow extracting spheric part and deviatoric part of any second order symmetric tensor. From the consideration of the second principle of thermodynamics, the Clausius–Duhem inequality requires $\boldsymbol{\Sigma} : \dot{\boldsymbol{E}} - \dot{\Psi} \geqslant 0$, which follows:

$$\left(\boldsymbol{\Sigma} - \frac{\partial \Psi}{\partial \boldsymbol{E}^e}\right) : \dot{\boldsymbol{E}} + \frac{\partial \Psi}{\partial \boldsymbol{E}^e} : \dot{\boldsymbol{E}}^p - \frac{\partial \Psi}{\partial d}\dot{d} - \frac{\partial \Psi}{\partial \gamma^p}\dot{\gamma}^p \geqslant 0 \tag{4}$$

The inequality (4) must hold for any value of $\dot{\boldsymbol{E}}$, $\dot{\boldsymbol{E}}^p$, $\dot{d}$ and $\dot{\gamma}^p$, which yields the constitutive equations:

$$\boldsymbol{\Sigma} = \frac{\partial \Psi}{\partial \boldsymbol{E}^e} = \mathbb{C}(d) : \left(\boldsymbol{E} - \boldsymbol{E}^p\right) \tag{5}$$

and the thermodynamic forces F^d and α^p, respectively associated with the damage variable d and the hardening variable γ^p:

$$F^d = -\frac{\partial \Psi}{\partial d}, \qquad \alpha^p = \frac{\partial \Psi}{\partial \gamma^p} \tag{6}$$

The plastic strain rate is determined by choosing a yield function $f^p(\boldsymbol{\Sigma}, \gamma^p, d) \leqslant 0$ and for non-associated plastic flow, a plastic potential $g^p(\boldsymbol{\Sigma}, \gamma^p, d) = 0$, as well as a plastic hardening law. For the damage evolution, a damage criterion, function of the damage variable d and its associated thermodynamic force $f^d(F^d, d) \leqslant 0$, is needed.

It is emphasized that in commonly used plastic models, the plastic yield and potential functions are generally expressed with basic stress invariants, via triplet (I_1, J_2, J_3):

$$I_1 = \boldsymbol{\Sigma}_{kk} = 3p, \quad J_2 = \frac{1}{2}\boldsymbol{S} : \boldsymbol{S} = \frac{1}{2}\boldsymbol{\Sigma} : \mathbb{K} : \boldsymbol{\Sigma}, \quad J_3 = \det(\boldsymbol{S}) \tag{7}$$

Based on the decomposition $\boldsymbol{\Sigma} = \mathbb{K} : \boldsymbol{\Sigma} + \mathbb{J} : \boldsymbol{\Sigma}$, we obtain $\boldsymbol{S} = \mathbb{K} : \boldsymbol{\Sigma}$, which is the deviatoric part of stress tensor; and $p = \mathrm{tr}(\mathbb{J} : \boldsymbol{\Sigma})/3$ is the mean stress used to account for the pressure dependency of mechanical behaviors. The same decomposition procedure can be performed on the plastic strain tensor, i.e. $\boldsymbol{E}^p = \mathbb{K} : \boldsymbol{E}^p + \mathbb{J} : \boldsymbol{E}^p$. The usually-used plastic hardening variable, the plastic distortion γ^p, is written in the form:

$$\gamma^p = \int \dot{\gamma}^p, \quad \text{with } \dot{\gamma}^p = \sqrt{\frac{2}{3}\dot{\boldsymbol{E}}^p : \mathbb{K} : \dot{\boldsymbol{E}}^p} \tag{8}$$

3. Extensions to discrete formulations

As mentioned above, in the present work, it is assumed that the plastic deformation and damage in geomaterials are essentially taking place in various defeats called here WSP. The total plastic strain can be considered as an additive consequence of plastic sliding within a number of families of WSPs, which are randomly distributed in the elastic solid matrix. The material damage is associated with the evolution (growth) of these defeats. The term 'plane' is used here to assume that all kinds of defeats may be approximated by plane discontinuity surfaces with thickness being very small and negligible with respect to their size. The notion of WSP is more general than the classic definition of penny shaped microcracks generally used in micromechanical modeling of damage. The orientation of each family of WSPs is identified by its normal vector. Each family contains a number of WSPs with the same normal orientation. For the sake of simplicity, the following hypotheses are also adopted:

- *Hypothesis 1*: The interactions between different WSP families as well as the effects related to their spatial distribution could be neglected. This simplification allows to taking into account the contributions of each WSP family in a separate way.
- *Hypothesis 2*: The material anisotropy is entirely induced by anisotropic distribution of WSPs in space; and the damage in each family of WSPs is described by a scalar variable. Therefore, when the material is weakened by a single family of WSPs, its mechanical behavior is transversely isotropic.

3.1. Discretization of variables

For the sake of simplification in mathematical formulations, we introduce two fourth order tensorial operators $\mathbb{N}$ and $\mathbb{T}$, both function of unit normal vector $\underline{n}$ and with the components:

$$N_{ijkl} = n_i n_j n_k n_l, \qquad T_{ijkl} = \frac{1}{2}(\delta_{ik} n_j n_l + \delta_{il} n_j n_k + \delta_{jk} n_i n_l + \delta_{jl} n_i n_k - 4 n_i n_j n_k n_l) \tag{9}$$

It is verified that there exist the following properties on $\mathbb{N}$ and $\mathbb{T}$:

$$\mathbb{N} : \mathbb{N} = \mathbb{N}, \quad \mathbb{T} : \mathbb{T} = \mathbb{T}, \quad \mathbb{N} : \mathbb{T} = \mathbb{T} : \mathbb{N} = 0 \tag{10}$$

and

$$\mathbb{J} : \mathbb{T} = \mathbb{T} : \mathbb{J} = 0, \qquad \mathbb{K} : \mathbb{T} = \mathbb{T} : \mathbb{K} = \mathbb{T} \tag{11}$$

3.1.1. Discrete description of plastic strain

Denote by $\boldsymbol{\varepsilon}^p(\underline{n})$ the local plastic strain tensor related to the family of defeats with unit normal vector $\underline{n}$. The plane geometrical form of WSPs allows to decompose the strain $\boldsymbol{\varepsilon}^p(\underline{n})$ into two distinct parts: a normal part $\boldsymbol{\varepsilon}^{p,n}$ and a tangent part $\boldsymbol{\varepsilon}^{p,t}$:

$$\boldsymbol{\varepsilon}^p(\underline{n}) = \beta(\underline{n})\underline{n} \otimes \underline{n} + \underline{\gamma}(\underline{n}) \overset{s}{\otimes} \underline{n} \tag{12}$$

with relations:

$$\boldsymbol{\varepsilon}^{p,n}(\underline{n}) = \beta(\underline{n})\underline{n} \otimes \underline{n} = \mathbb{N} : \boldsymbol{\varepsilon}^p(\underline{n}), \qquad \boldsymbol{\varepsilon}^{p,t}(\underline{n}) = \underline{\gamma}(\underline{n}) \overset{s}{\otimes} \underline{n} = \mathbb{T} : \boldsymbol{\varepsilon}^p(\underline{n}) \tag{13}$$

The scalar-valued variable $\beta(\underline{n})$, with $\mathrm{tr}(\boldsymbol{\varepsilon}^p) = \beta(\underline{n})$, is related to plastic volumetric deformation whereas the vector $\underline{\gamma}(\underline{n})$ characterizes plastic shear strains. Note that this decomposition is similar to that used in micromechanical damage analysis for quasi-brittle materials with penny shaped cracks [5]. Note that we are using here a discrete phenomenological approach. As assumed above, the total plastic strains of material are obtained by adding local ones taking place in each family of WSPs. Assuming a continuous spatial distribution of local plastic strains with the unit vector, the total plastic strains can be analytically calculated by making integration of $\boldsymbol{\varepsilon}^p(\underline{n})$ over the surface S^2 of a unit sphere:

$$\boldsymbol{E}^p = \frac{1}{4\pi} \int_{S^2} \boldsymbol{\varepsilon}^p(\underline{n})\, \mathrm{d}S = \frac{1}{4\pi} \int_{S^2} \left[\beta(\underline{n})\underline{n} \otimes \underline{n} + \underline{\gamma}(\underline{n}) \overset{s}{\otimes} \underline{n}\right] \mathrm{d}S \tag{14}$$

However, it is generally impossible to determine the close forms of the distribution function $\boldsymbol{\varepsilon}^p(\underline{n})$ ($\beta(\underline{n})$ and $\underline{\gamma}(\underline{n})$, equivalently), numerical approximation is needed to calculate the integral (14) using a suitable algorithmic scheme. According to the previous works by Elata [6], it is shown that the choice of a system composed of 15 orientations can provide a good approximation of the continuous distribution of strain tensor. Thus, let $\underline{n}^r$, $r = 1, \ldots, 15$, be the chosen integration orientations. It is proved that there exist the following relations:

$$\frac{1}{4\pi} \int_{S^2} \underline{n} \otimes \underline{n}\, \mathrm{d}S = \frac{1}{15} \sum_{r=1}^{15} \underline{n}^r \otimes \underline{n}^r \tag{15}$$

and

$$\frac{1}{4\pi} \int_{S^2} \underline{n} \otimes \underline{n} \otimes \underline{n} \otimes \underline{n}\, \mathrm{d}S = \frac{1}{15} \sum_{r=1}^{15} \underline{n}^r \otimes \underline{n}^r \otimes \underline{n}^r \otimes \underline{n}^r \tag{16}$$

The explicit integrations are then replaced by the summation over the chosen discrete orientations. Note that the coefficient $\frac{1}{15}$ is related to the particular scheme adopted here. The total plastic strain is accordingly obtained by the following discrete form:

$$\boldsymbol{E}^p = \frac{1}{15} \sum_{i=1}^{15} \boldsymbol{E}^{p,i} \tag{17}$$

with

$$\boldsymbol{E}^{p,i} = \beta^i \underline{n}^i \otimes \underline{n}^i + \underline{\gamma}^i \overset{s}{\otimes} \underline{n}^i \tag{18}$$

3.1.2. Decomposition of damage

In this work, we consider only the solid matrix weaken by WSPs in compression regime and unilateral effects are not taken into account. Therefore, the degradation of elastic properties due to damage evolution takes effect only on the shear modulus. Following classic isotropic damage theory, $k(d)$ and $\mu(d)$ involved in (3) are expressed in the form:

$$k(d) = k^s, \qquad \mu(d) = \mu^s(1 - \tilde{\kappa} d) \tag{19}$$

Reporting of (19) into (3) gives:

$$\mathbb{C}(d) = 3k^s \mathbb{J} + 2\mu^s(1 - \tilde{\kappa} d)\mathbb{K} = \mathbb{C}^s - 2\mu^s \tilde{\kappa} d\mathbb{K} \tag{20}$$

Obviously, the term $2\mu^s \tilde{\kappa} d\mathbb{K}$ represents the modification to the initial elastic stiffness tensor $\mathbb{C}^s$ due to damage. In the case of a random distribution of WSPs, the damage state is generally anisotropic due to the propagation of WSPs in some preferred orientations. For the proper description of such anisotropic damage, it is necessary to replace the isotropic damage variable d by a continuous distribution function $\omega(\underline{n})$. In addition, it is well known that the fourth order tensor $\mathbb{T}(\underline{n})$ is directly related to the degradation of the shear modulus [7]. Thus, the term $d\mathbb{K}$ can be evaluated by the following integral form:

$$d\mathbb{K} = \zeta \frac{1}{4\pi} \int_{S^2} \omega(\underline{n}) \mathbb{T}(\underline{n})\, \mathrm{d}S \tag{21}$$

Note that $\frac{1}{4\pi} \int_{S^2} \mathbb{T}(\underline{n})\, \mathrm{d}S = \frac{2}{5}\mathbb{K}$; it follows, from the case of isotropic damage distribution, i.e. $\omega(\underline{n}) = d$, that $\zeta = \frac{5}{2}$.

The effective elastic stiffness tensor $\mathbb{C}$ can then be rewritten as:

$$\mathbb{C}\big(\omega(\underline{n})\big) = \mathbb{C}^s - 2\mu^s \kappa \frac{1}{4\pi} \int_{S^2} \omega(\underline{n}) \mathbb{T}(\underline{n})\, \mathrm{d}S \tag{22}$$

with $\kappa = \frac{5}{2}\tilde{\kappa}$. Again, as it is generally impossible to obtain the close form of this integration, the same approximation scheme as that used for plastic strains is used and the effective elastic tensor is evaluated by the following discrete form:

$$\mathbb{C}(\underline{\omega}) = \mathbb{C}^s - 2\mu^s \frac{\kappa}{15} \sum_{i=1}^{15} \omega^i \mathbb{T}^i = 3k^s \mathbb{J} + 2\mu^s \left(\mathbb{I} - \frac{\kappa}{15} \sum_{i=1}^{15} \omega^i \mathbb{T}^i \right) : \mathbb{K} \tag{23}$$

3.2. Criterions and evolution laws

As previously indicated, the interaction effects between different families of WSPs are neglected in the present work; that leads to apply the discretization procedure to the blocked energy Ψ^p as:

$$\Psi^p = \frac{1}{4\pi} \int_{S^2} \Psi^p\big(\gamma^p(\underline{n}), \omega(\underline{n})\big)\, \mathrm{d}S = \frac{1}{15} \sum_{i=1}^{15} \Psi^{p,i}(\gamma^{p,i}, \omega^i) \tag{24}$$

where $\gamma^{p,i}$ is the hardening variable associated with the family of WSPs with the normal $\underline{n}^i$, and it is defined as

$$\gamma^{p,i} = \int \dot{\gamma}^{p,i}, \quad \dot{\gamma}^{p,i} = \big(2\dot{\boldsymbol{E}}^{p,i} : \mathbb{T}^i : \dot{\boldsymbol{E}}^{p,i}\big)^{1/2} = (\underline{\dot{\gamma}}^i \cdot \underline{\dot{\gamma}}^i)^{1/2} \tag{25}$$

Accordingly, the discrete thermodynamic forces associated with the damage variable ω^i and the hardening variable $\gamma^{p,i}$ are deduced by:

$$F^{\omega,i} = -\frac{\partial \Psi}{\partial \omega^i}, \qquad \alpha^{p,i} = \frac{\partial \Psi^{p,i}}{\partial \gamma^{p,i}} \tag{26}$$

The plastic strain rate $\dot{\boldsymbol{E}}^{p,i}$ can be calculated by means of plastic flow rule for each family of WSPs. For most geomaterials, a non-associated plastic flow rule is usually needed. In order to formulate plastic functions for each family of WSPs, the stress tensor $\boldsymbol{\Sigma}$ is projected onto the corresponding directions $\underline{n}^i$ as follows:

$$\boldsymbol{\Sigma}^{n,i} = \mathbb{N}^i : \boldsymbol{\Sigma}, \qquad \boldsymbol{\Sigma}^{t,i} = \mathbb{T}^i : \boldsymbol{\Sigma} \tag{27}$$

As in isotropic formulations, the following stress invariant for both $\boldsymbol{\Sigma}^{n,i}$ and $\boldsymbol{\Sigma}^{t,i}$ are proposed:

$$\sigma_n^i = (\boldsymbol{\Sigma}^{n,i} : \boldsymbol{\Sigma}^{n,i})^{1/2} = (\boldsymbol{\Sigma} : \mathbb{N}^i : \boldsymbol{\Sigma})^{1/2}, \qquad \sigma_t^i = \left(\frac{1}{2}\boldsymbol{\Sigma}^{t,i} : \boldsymbol{\Sigma}^{t,i}\right)^{1/2} = \left(\frac{1}{2}\boldsymbol{\Sigma} : \mathbb{T}^i : \boldsymbol{\Sigma}\right)^{1/2} \tag{28}$$

Let us define the unit vector $\underline{t}^i = \frac{1}{\sigma_t^i}\boldsymbol{\Sigma} \cdot \underline{n}^i \cdot (\boldsymbol{\delta} - \underline{\boldsymbol{n}}^i \otimes \underline{\boldsymbol{n}}^i)$ as the plastic shear direction for the ith family. It is proved that the projection procedure adopted here verifies the following relation between the set of local stress invariants $(\sigma_n(\underline{n}), \sigma_t(\underline{n}))$ and the macroscopic stress tensor $\boldsymbol{\Sigma}$:

$$\frac{3}{4\pi}\int_{S^2} \left[\sigma_n(\underline{n})\underline{n} \otimes \underline{n} + \sigma_t(\underline{n})\underline{t} \overset{s}{\otimes} \underline{n}\right] \mathrm{d}S = \boldsymbol{\Sigma} \tag{29}$$

The discrete formulation for plastic yield function $f^{p,i}$, plastic potential $g^{p,i}$ and damage criterion $f^{\omega,i}$ can be given as follows:

$$f^{p,i}(\sigma_n^i, \sigma_t^i, \gamma^{p,i}, \omega^i) \leqslant 0 \tag{30}$$

$$g^{p,i}(\sigma_n^i, \sigma_t^i, \gamma^{p,i}, \omega^i) = 0 \tag{31}$$

$$f^{\omega,i}\left(F^{\omega,i}, \omega^i\right) \leqslant 0 \tag{32}$$

The plastic flow and damage evolution rules for each family of WSPs are given as follows:

$$\dot{\boldsymbol{E}}^{p,i} = \dot{\lambda}^{p,i}\frac{\partial g^{p,i}}{\partial \boldsymbol{\Sigma}} = \dot{\lambda}^{p,i}\left(\frac{\partial g^{p,i}}{\partial \sigma_n^i}\frac{\partial \sigma_n^i}{\partial \boldsymbol{\Sigma}} + \frac{\partial g^{p,i}}{\partial \sigma_t^i}\frac{\partial \sigma_t^i}{\partial \boldsymbol{\Sigma}}\right), \quad \text{and} \quad \dot{\omega}^i = \dot{\lambda}^{\omega,i}\frac{\partial f^{\omega,i}}{\partial \omega^i} \tag{33}$$

with

$$\frac{\partial \sigma_n^i}{\partial \boldsymbol{\Sigma}} = \underline{n}^i \otimes \underline{n}^i, \qquad \frac{\partial \sigma_t^i}{\partial \boldsymbol{\Sigma}} = \underline{t}^i \overset{s}{\otimes} \underline{n}^i \tag{34}$$

On the other hand, we can directly obtain from (18) the following expression

$$\dot{\boldsymbol{E}}^{p,i} = \dot{\beta}^i(\underline{n}^i \otimes \underline{n}^i) + \dot{\underline{\gamma}}^i \overset{s}{\otimes} \underline{n}^i \tag{35}$$

Comparison between (33) and (35) leads to the evolution rate of the variables β^i and $\underline{\gamma}^i$:

$$\dot{\beta}^i = \dot{\lambda}^{p,i}\frac{\partial g^{p,i}}{\partial \sigma_n^i}, \qquad \dot{\underline{\gamma}}^i = \dot{\lambda}^{p,i}\frac{\partial g^{p,i}}{\partial \sigma_t^i}\underline{t}^i \tag{36}$$

In the case of stationary damage state (no damage evolutions) ($\dot{\omega}^i = 0, i = 1, \ldots, 15$), the consistency condition $\dot{f}^{p,i} = 0$ reads:

$$\dot{f}^{p,i} = \frac{\partial f^{p,i}}{\partial \boldsymbol{\Sigma}} : \dot{\boldsymbol{\Sigma}} + \frac{\partial f^{p,i}}{\partial \gamma^{p,i}}\dot{\gamma}^{p,i} = 0 \tag{37}$$

with

$$\dot{\boldsymbol{\Sigma}} = \mathbb{C} : \dot{\boldsymbol{E}} - \frac{1}{15}\mathbb{C} : \sum_{k=1}^{15}\dot{\boldsymbol{E}}^{p,k} = \mathbb{C} : \dot{\boldsymbol{E}} - \frac{1}{15}\mathbb{C} : \sum_{k=1}^{15}\dot{\lambda}^{p,k}\frac{\partial g^{p,k}}{\partial \boldsymbol{\Sigma}} \tag{38}$$

and

$$\dot{\gamma}^{p,i} = \left(2\dot{\boldsymbol{E}}^{p,i} : \mathbb{T}^i : \dot{\boldsymbol{E}}^{p,i}\right)^{1/2} = \dot{\lambda}^{p,i}\frac{\partial g^{p,i}}{\partial \sigma_t^i} \tag{39}$$

Let us denote $\{\dot{\lambda}^{p,\cdot}\}_{15\times 1}$ the column matrix of plastic multipliers $\dot{\lambda}^{p,I}$, $I = 1, \dots, 15$, and $[\mathbf{M}^p]_{15\times 15}$ the coefficient matrix with components

$$M_{IJ}^{p} = \frac{1}{15}\frac{\partial f^{p,I}}{\partial \boldsymbol{\Sigma}} : \mathbb{C} : \frac{\partial g^{p,J}}{\partial \boldsymbol{\Sigma}} - \frac{\partial f^{p,I}}{\partial \gamma^{p,I}}\frac{\partial g^{p,I}}{\partial \sigma_t^I} \tag{40}$$

The combination of the consistency conditions $\dot{f}^{p,i} = 0, i = 1, \dots, 15$, for all the families leads to the following matrix form:

$$\left[\mathbf{M}^p\right]\{\dot{\lambda}^{p,\cdot}\} = \left\{\mathbb{C} : \frac{\partial f^{p,\cdot}}{\partial \boldsymbol{\Sigma}}\right\} : \dot{\boldsymbol{E}} \tag{41}$$

The column matrix of plastic multipliers is then expressed as:

$$\{\dot{\lambda}^{p,\cdot}\} = \left[\mathbf{M}^p\right]^{-1}\left\{\mathbb{C} : \frac{\partial f^{p,\cdot}}{\partial \boldsymbol{\Sigma}}\right\} : \dot{\boldsymbol{E}} \tag{42}$$

It follows that the fourth order tangent tensor relating the rate of the stress tensor to that of the total strain by $\dot{\boldsymbol{\Sigma}} = \mathbb{C}^{\tan} : \dot{\boldsymbol{E}}$ can be given by:

$$\mathbb{C}^{\tan} = \mathbb{C} - \frac{1}{15}\left\{\mathbb{C} : \frac{\partial g^{p,\cdot}}{\partial \boldsymbol{\Sigma}}\right\}^T \left[\mathbf{M}^p\right]^{-1}\left\{\mathbb{C} : \frac{\partial f^{p,\cdot}}{\partial \boldsymbol{\Sigma}}\right\} \tag{43}$$

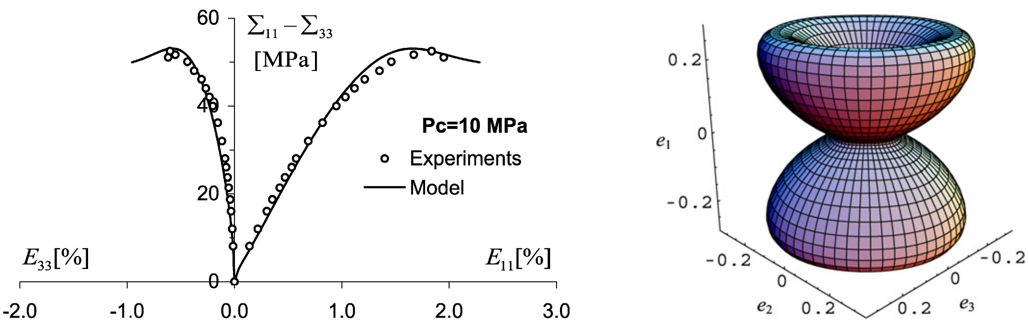
with the components:

$$\mathbb{C}_{ijkl}^{\tan} = \mathbb{C}_{ijkl} - \frac{1}{15}\left\{\mathbb{C}_{ijrs}\frac{\partial g^{p,I}}{\partial \boldsymbol{\Sigma}_{rs}}\right\}^T \left[\mathbf{M}^p\right]_{IJ}^{-1}\left\{\mathbb{C}_{klpq}\frac{\partial f^{p,J}}{\partial \boldsymbol{\Sigma}_{pq}}\right\} \tag{44}$$

As in the classical isotropic formulation, the tangent operator $\mathbb{C}^{\tan}$ is asymmetric for a non-associated plastic flow.

4. Example of applications to typical semi-brittle rock

To illustrate the predictive capability of the proposed discrete elastoplastic damage model, an example of application to hard clay is presented here. To complete the model formulation, a Coulomb-type yield function is adopted for plastic sliding in the WSP families; plastic hardening and damage softening are considered. Based on experimental results, a specific plastic potential is proposed for non-associated plastic flow. In addition, the plastic shear strains in the weak sliding planes are considered as the unique mechanism for damage evolution (the detailed description of the functions is not given here due to the limited length of paper). In Fig. 1(a), we show the comparison between the model prediction and experimental data for a conventional triaxial compression test with a confining pressure of 10 MPa. Fig. 1(b) shows the three-dimensional spatial representation of the hardening variable $\gamma^p(\underline{n})$ in the rosette form. Obviously, one can see that the plastic deformation is anisotropic in space; no plastic flow occurs in the plane with the normal vector parallel to the axial direction; the plastic deformations are mainly concentrated in the WSPs inclined at around 40 degrees with respect to the axial loading direction. The model's simulations reproduce the main



(a) (b)

Fig. 1. (a) Comparisons of the predicted stress–strain curve with experimental data; (b) Distribution on rosette of $\gamma^p(\underline{n})$ [%].

features of rock behaviors such as non-linear responses, volumetric dilatancy, strain softening and degradation of elastic properties.

5. Conclusions

In this study, we have proposed a new discrete thermodynamic approach for modeling anisotropic plastic deformation and damage in semi-brittle geomaterials. The commonly-used isotropic coupled elastoplastic damage formulation is extended to anisotropic case using a discrete phenomenological formulation. This new approach provides an easy way to take into account the coupling between plasticity and anisotropic damage. Although some restrictions used, it was found that the proposed model can reproduce main features observed in mechanical behaviors of some classes of geomaterials. The extension of the present work will include the consideration of unilateral effects, plastic deformation in bulk material and modeling of poromechanical behavior of saturated media by including pore pressure effects on plastic flow and damage evolution.

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