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Fatigue growth of embedded elliptical cracks using Paris-type law in a hybrid weight function approach

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Abstract

A hybrid weight function method (HWFM), improving the calculation of the stress intensity factor (SIF) in mode I, has recently been proposed and validated in the static case [B.K. Hachi, S. Rechak, M. Haboussi, M. Taghite, Modélisation des fissures elliptiques internes par hybridation de fonctions de poids, C. R. Mecanique 334 (2006) 83–90]. In the present Note, the hybridization approach is presented for the fatigue crack growth prediction of embedded elliptical crack in infinite bodies. Hence, Paris's law of crack propagation is incorporated into the developed hybridization-based computer code, along with two degrees of freedom technique for managing the crack evolution and the cracked structure fatigue life. Simulations of the evolution of elliptical cracks (in infinite bodies) of different configurations (ellipse axes ratio, maximum crack advance) corresponding to fatigue and brittle fracture have been conducted. Comparisons with other numerical methods such as the classical weight function method (WFM) or the extended finite element methods (X-FEM) show the pertinence of the HWFM in the treatment of an aspect of fatigue cracking problems. ***To cite this article: B.K. Hachi et al., C. R. Mecanique 336 (2008).***

Résumé

Propagation de fissures elliptiques internes avec la loi de Paris dans l'approche d'hybridation des fonctions de poids. Une méthode, basée sur l'hybridation des fonctions de poids, a récemment été proposée pour améliorer le calcul du facteur d'intensité de contrainte (FIC) en mode I sous chargement statique, [B.K. Hachi, S. Rechak, M. Haboussi, M. Taghite, Modélisation des fissures elliptiques internes par hybridation de fonctions de poids, C. R. Mecanique 334 (2006) 83–90]. On se propose dans cette présente Note, d'appliquer l'approche d'hybridation pour décrire la propagation de fissures elliptiques internes dans un milieu infini. La loi de propagation de Paris est alors incorporée dans le code de calcul développé sur la base de l'hybridation, dans le cadre d'une approche à deux degrés de liberté pour gérer l'évolution de la forme de la fissure. Pour valider cette méthode, des simulations de l'évolution des fissures elliptiques internes de différentes configurations (rapport des axes, avance maximum de la fissure) correspondant à la rupture par fatigue ou fragile ont été réalisées. Elles ont permis de montrer, après comparaisons avec d'autres méthodes numériques telles que la méthode des fonctions de poids ou la méthode des éléments finis étendue, la pertinence

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de l'approche d'hybridation dans le traitement d'un aspect de la fissuration en fatigue. ***Pour citer cet article : B.K. Hachi et al., C. R. Mecanique 336 (2008).***

Keywords: Fatigue; Fatigue elliptical crack growth; Hybridization; Weight function; Paris law; Two degrees of freedom method

Mots-clés : Fatigue ; Propagation de fissure elliptique par fatigue ; Hybridation ; Fonction de poids ; Loi de Paris ; Méthode à de deux degrés de liberté

Version française abrégée

Une méthode, basée sur l'hybridation des fonctions de poids, a été proposée récemment pour améliorer le calcul du facteur d'intensité de contrainte (FIC) de fissures elliptiques en mode I sous chargement statique, [1,2]. Le FIC est alors défini, selon l'Éq. (1), par l'intégrale sur l'aire de la fissure, du chargement de la fissure pondérée par une fonction de poids. Dans cette étude et selon notre idée d'hybridation, la fonction de poids est soit celle d'Oore et Burns [10], donnée par Éq. (2), valable dans la partie de la fissure désignée par Zone (II) sur la Fig. 1 ou celle de Krasowsky et al. [7], donnée par Éq. (3), valable dans la zone (I) de cette même fissure. Ces fonctions de poids sont ainsi utilisées dans la zone de la fissure où elles sont les plus efficaces. La proportion entre les zones (I) et (II) est-elle déterminée de manière à atténuer les effets de singularités liées au calcul des intégrales présentes dans les équations (2) et (3).

Comparée à d'autres méthodes numériques de la littérature, telles que la méthode des éléments finis standards (FEM) [3,4], la méthode des éléments de frontière (BEM) [5], la méthode des fonctions de poids standard (WFM) [6,7], et la méthode des éléments finis étendus (X-FEM) [8], cette méthode s'est révélée précise et numériquement peu coûteuse. On se propose ici d'étendre l'utilisation de l'approche d'hybridation à la prédiction de la propagation de fissures elliptiques en mode I sous chargement cyclique. Pour ce faire, cette méthode sera associée à une loi de propagation comme celle de Paris [9] dans le cadre d'une approche à double degrés de liberté. La méthode d'hybridation nous fournira donc les valeurs des facteurs d'intensité de contrainte en deux endroits du front de fissures, l'avance maximum de la fissure est affectée à l'endroit où le FIC est maximum et l'avance de l'autre point du front de fissure est déterminée en utilisant la loi de propagation. Pour la valider, cette méthode est utilisée pour simuler la propagation de fissures elliptiques, aux différentes caractéristiques géométriques (rapports des axes de l'ellipse) dans les cas d'une rupture par fatigue ou fragile. Les résultats trouvés montrent par exemple, que :

i) Dans le cas d'une fissure interne initialement circulaire ($\alpha_0 = a_0/b_0 = 1.0$) dans un milieu infini en rupture par fatigue ($m = 2.1$ dans la loi de Paris), pour lequel on dispose d'une solution analytique, la méthode d'hybridation (désignée par HWFun) est plus précise que la X-FEM couplée avec une méthode de type FMM (Fast Marching Method) utilisée par Sukumar et al. [8] pour résoudre le même problème, Fig. 2.
ii) Dans le cas d'une fissure initialement non circulaire ($\alpha_0 = 0.4$) dans un milieu infini correspondant à une rupture fragile ($m = 50$), celle-ci tend à devenir circulaire. Ceci a également été constaté pour le cas d'une rupture par fatigue (Exemple non présenté ici). Pour le même problème, Lazarus [12] prévoit une déviation par rapport au profil elliptique avant de revenir à celui-ci avec, curieusement, une inversion du rapport des axes, Fig. 4.

En faite, pour les trois méthodes utilisées pour résoudre ce problème, Fig. 4, α augmente rapidement lors de la propagation de la fissure, il se stabilise, avantageusement, autour de 1 pour l'approche d'hybridation, oscille avant de se stabiliser autour de 1 pour l'approche basée sur la fonction de poids de Krasowsky [7] et se stabilise légèrement au-dessus de 1 pour l'approche basée sur la fonction de poids de Lazarus [12].

Ce travail montre que l'approche d'hybridation de fonctions de poids se positionne favorablement parmi les méthodes de prédiction de propagation des fissures elliptiques par fatigue. S'appuyant sur une procédure à doubles degrés de liberté, la propagation de fissures elliptiques a été simulée de manière satisfaisante. De plus, des résultats probants ont été obtenus concernant par exemple le nombre de cycles de chargement caractérisant la durée de vie de la structure fissurée.

Dans les exemples traités, seuls des chargements de traction uniforme, présentant une double symétrie (par rapport aux axes Ox et à Oy du repère, voir Fig. 1), ont été retenus pour rester cohérent avec la forme elliptique de la fissure.

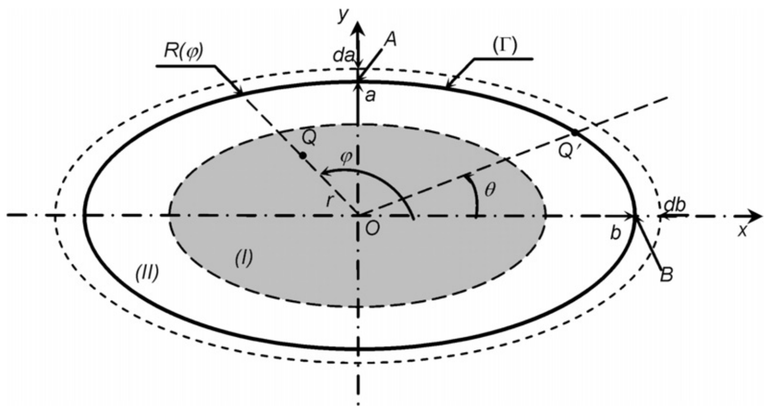


Fig. 1. Geometrical parameters, subdivision and the elliptical crack growth.

Fig. 1. Paramètres géométriques, subdivision et propagation elliptique de la fissure.

D'autres chargements à simple symétrie (par rapport à Oy), non présentés ici, ont été favorablement testés dans le cas de fissures semi-elliptiques (débouchantes).

1. Introduction

A method based on the hybridization of weight functions (HWFM) has recently been proposed for the computation of the stress intensity factor (SIF) of elliptical cracks under static load in mode I, [1,2]. Compared to other numerical methods such as the regular or the extended finite element method (FEM or X-FEM), the boundary element method (BEM) and the classical weight function method (WFM) in the literature, [3–8], the HWFM is revealed to be numerically efficient. In this Note, the extension of the approach to the embedded elliptical crack growth prediction in mode I is attempted. For such a purpose, a propagating law like the Paris law, [9], is used in conjunction with a two degrees of freedom algorithm of crack evolution. The methodology consists in computing the SIF at two specific points of the ellipse (A and B on Fig. 1), then attributing a maximal crack growth to the point of maximal SIF. The crack growth of the other point is calculated by using the propagation law. In order to validate the method, simulations dealing with the growth of elliptical crack of different geometrical characteristics (ellipse axes ratio) and material properties (in fatigue and in brittle fracture), have been conducted. Comparisons with other numerical methods (WFM, X-FEM) have been presented to show the efficiency and the accuracy of the method in the treatment of an aspect of fatigue cracking problems.

The outline of this Note is as follows. In Section 2, the hybridization technique is briefly presented including its numerical implementation in fatigue. In Section 3, we report the results obtained via the developed computer code HWFun. We end this paper by drawing some concluding remarks.

2. Presentation of the hybridization method

In the numerical methods based on weight functions, the stress intensity factor (SIF) in mode I of an elliptical crack in an infinite body (Fig. 1) is defined as follows [10]:

$$K_{IQ'} = \int_S W_{QQ'} q(Q)\,\mathrm{d}S \tag{1}$$

where $K_{IQ'}$ is the stress intensity factor in mode I at the Q' point of the crack front. $W_{QQ'}$ is the weight function related to the problem, it corresponds to $K_{IQ'}$ when a unit concentrated and symmetrical force $q(Q)$ is applied to the arbitrary Q point of the crack. S is the crack area.

The hybridization method which we recently developed, [1,2], for the modelling of elliptical cracks subjected to static loadings, consists in using the weight functions defined by:

$$W_{QQ'} = \frac{\sqrt{2}}{\pi l_{QQ'}^2 \sqrt{\int_\Gamma (1/(\rho_Q)^2)\,\mathrm{d}\Gamma}} \tag{2}$$

and

$$W_{QQ'} = \frac{2\Pi^{1/4}(\theta)}{\sqrt{\pi a(1 - r^2(\varphi)/R^2(\varphi))} l_{QQ'}^2 \int_\Gamma (1/(\rho_Q)^2)\,\mathrm{d}\Gamma} \tag{3}$$

due to Oore and Burns [10] and Krasowsky [7], respectively. These weight functions (2) and (3) are used in the area of the ellipse designed in Fig. 1 by zone (II) and zone (I) respectively. It is understood that these two weight functions are used in the area of the crack where they are more efficient.

In expressions (2) and (3), as shown in Fig. 1, the parameters r and φ are the polar coordinates of an arbitrary point Q, $R(\varphi)$ is the distance between the origin O and the crack front when passing through the Q point, $l_{QQ'}$ is the distance between the Q' point and the arbitrary Q point. (Γ) is the curve of the ellipse (the crack front) and ρ_Q is the distance between the Q point and the elementary segment $\mathrm{d}\Gamma$, θ is the angular position of the Q' point, $\alpha = a/b$ is the ratio of the ellipse axes, and Π is a function defined as $\Pi(\theta) = (\sin^2\theta + \alpha^4\cos^2\theta)/(\sin^2\theta + \alpha^2\cos^2\theta)$.

The principle of hybridization is to divide, as shown in Fig. 1, the elliptical crack into two zones, an internal zone I (ellipse in grey) and an external zone II (in white), then to use each of the two weight functions in the area where it is more efficient. The two zones are defined by the following relations:

$$\begin{cases} (\frac{x}{b'})^2 + (\frac{y}{a'})^2 \leqslant 1 & \text{(zone I)} \\ (\frac{x}{b'})^2 + (\frac{y}{a'})^2 > 1 & \text{(zone II)} \end{cases} \tag{4}$$

where a' and b' are such as $a'/a = b'/b = \beta$ and $\beta \in [0, 1]$, β being the proportion between the two zones and a, b are the axes of ellipse (Fig. 1). In order to minimize the singularity effects within the surface and the contour integrals present in Eqs. (1), (2) and (3), the weight function given by Eq. (3) is used in zone I, and the weight function given by Eq. (2) in zone II. The reader could find more details about the hybridization approach in the reference [2].

The hybridization method has been well validated in divers applications subjected to static loadings. In the present work, we intend to apply it to fatigue problems, i.e. case of cyclic loadings. The method is hence associated to a practical and currently used crack propagation law namely the Paris's law, [3,8,11,12]:

$$\frac{\mathrm{d}a}{\mathrm{d}N_c} = C(\Delta K_I)^m \tag{5}$$

where a is the crack characteristic length, N_c, the number of the loading cycles and $\Delta K_I = K_I(\sigma_{\max}) - K_I(\sigma_{\min})$ is the mode I SIF range for one loading cycle. In the expression of ΔK_I, $\sigma_{\max}$ and $\sigma_{\min}$ are the maximal and minimal stresses for one fatigue cycle. The coefficients C and m are the parameters related to the material in its environment (temperature, humidity, ...). It is worth noting that large values of m correspond to the case of brittle materials and the coefficient C is affected by the edge effects [3] and consequently its value depends on whether it is the case of a plane stress or a plane strain. However, for the case of an infinite body and far from the edge effects, the coefficient C takes a constant value [3,8].

Based on the hypothesis that the elliptical crack propagates in its plan area, the fatigue crack growth computation is achieved by first calculating the SIF on two characteristic points A and B of the crack (Fig. 1). A maximum propagation advance $\Delta a_{\max}$ is thus affected to the point of maximum SIF, at the same time, the propagation length of the other point using Eq. (6) which is deduced from Eq. (5) is evaluated:

$$\frac{\Delta a}{\Delta a_{\max}} = \left(\frac{\Delta K_I}{\Delta K_I^{\max}}\right)^m \tag{6}$$

From the new elliptical crack profile, one computes the SIF for the new two points A and B, thus the maximal crack growth $\Delta a_{\max}$ is again affected to the point for which K_I is maximum. The computations are thus carried out up to failure.

This approach named as the two degrees of freedom (DOF) method became popular thanks to the works of Newman and Raju [3]. This algorithm has been adopted for two main reasons. The first one is that in order to follow the evolution of the crack, one just needs to compute the SIF on two points of the elliptical crack contour. This of course leads to an appreciable gain in computing time since the same computation is repeated up to failure. The second one is directly related to the usage of this procedure with the hybrid weight function approach, in which the elliptical crack profile is conserved during propagation. This comes from the fact that only the two points A and B of the profile change their respective positions. Therefore, the ellipse axes ratio may change but the crack keeps its elliptical shape.

This hypothesis regarding the constancy of the elliptical shape using the two-DOF algorithm is proved to be very realistic. In fact, it is reported in the works of [11,12] based on a multiple DOF approach, that an arbitrary crack profile tends to an elliptical shape during propagation (for loading conditions close to those adopted in the present study).

In order to validate the extension of the hybrid approach to fatigue problems, different applications dealing with fatigue crack growth of the consulted literature have been treated.

3. Applications

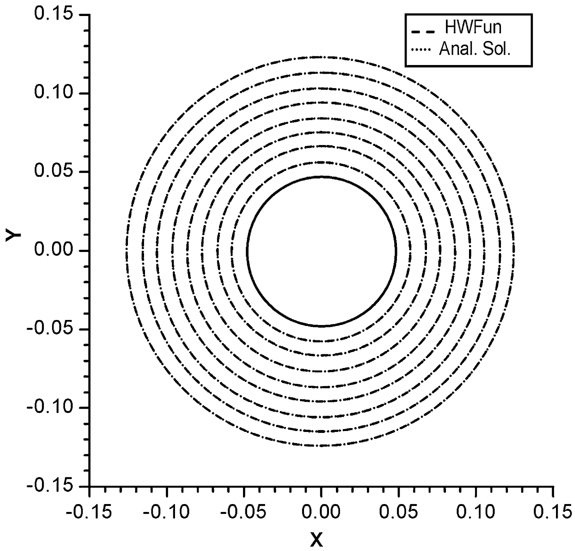
Two specific applications, corresponding to fatigue ($m = 2.1$) and brittle fracture ($m = 50$), have been carried out in order to validate the present hybrid approach. They deal with the propagation of embedded penny-shaped cracks in infinite bodies subjected to a cyclic tensile loading of stress range $\Delta\sigma_t$.

3.1. Infinite body with embedded penny-shaped crack in fatigue

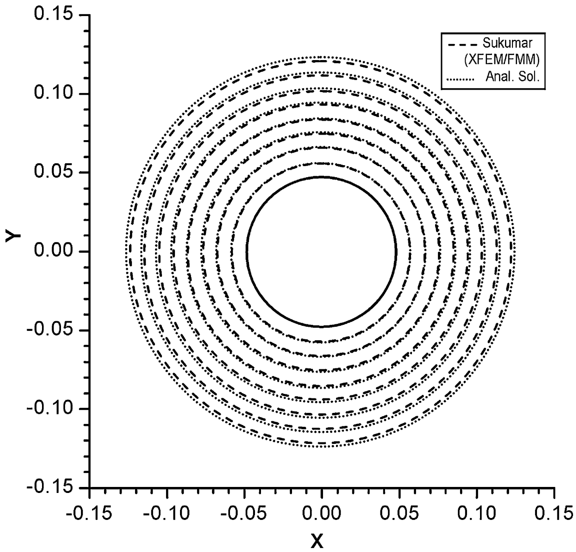
The crack initial state is such as $a_0 = 0.05$ cm, $\alpha_0 = a_0/b_0 = 1.0$, the maximal crack growth is equal to $\Delta a_{\max} = 0.2a_0$ in fatigue ($m = 2.1$).

This application has been treated by Sukumar [8] using the extended finite element (X-FEM) associated with a Fast Marching Method (FMM).

For the sake of clarity, we ought to present the actual results and those of [8] on separate figures (Figs. 2(a) and 2(b)). Both methods were compared to the reference solution, namely the analytical one. From those two figures, one can observe that the initial circular crack keeps, as expected from the model, its shape during propagation even



(a)



(b)

Fig. 2. Evolution of a circular shape crack in the plane (Oxy) and comparison with the analytical solution: (a) The present approach, (b) Sukumar et al. [8].

Fig. 2. Evolution d'une fissure circulaire dans le plan (Oxy) et comparaison de la solution analytique avec la solution de : (a) La présente approche, (b) Sukumar et al. [8].

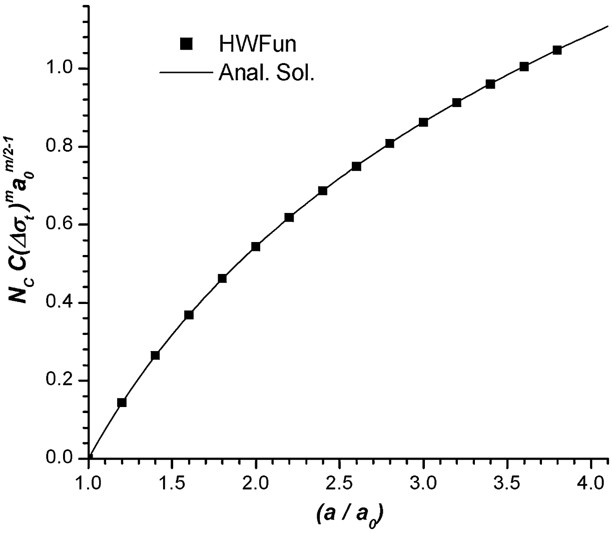


Fig. 3. Variation of the number of loading cycles along with the propagation of the penny-shaped crack when $m = 2$ (fracture in fatigue).

Fig. 3. Variation du nombre de cycles de chargement en fonction de l'avance pour une fissure circulaire lorsque $m = 2$ (rupture par fatigue).

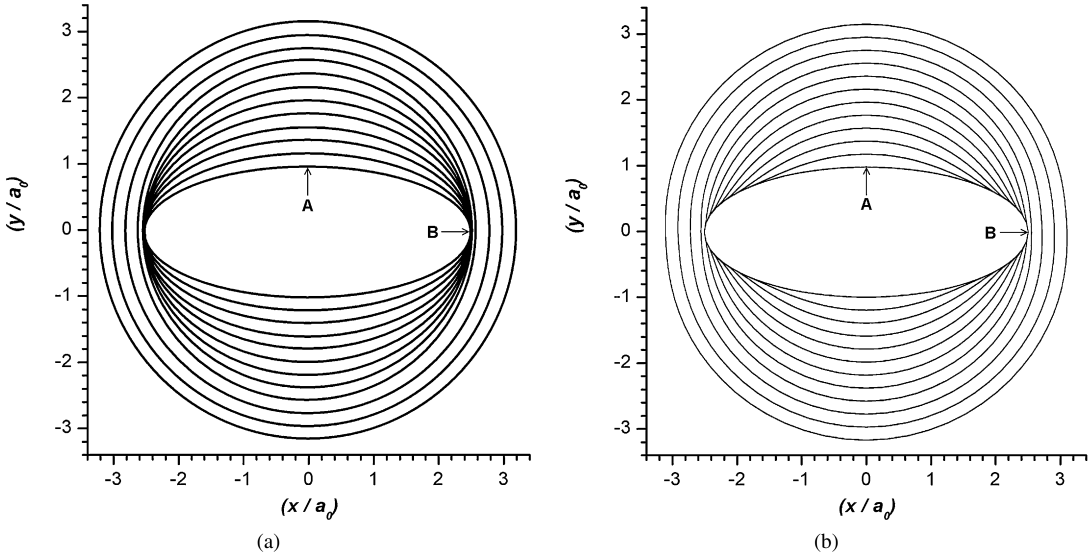


Fig. 4. Evolution of an elliptical crack ($\alpha_0 = 0.4$) in the plane (Oxy) for $m = 50$ (brittle fracture): (a) The present approach, (b) Lazarus [12].

Fig. 4. Evolution d'une fissure elliptique ($\alpha_0 = 0.4$) dans le plan (Oxy) pour $m = 50$ (rupture fragile) obtenue par : (a) La présente approche, (b) Lazarus [12].

for relatively large values of $\Delta a_{\max}$. Numerical comparisons show that the actual HWFun predictions are better than those of [8]. This could probably come from the efficiency of the hybridization approach in the evaluation of the stress intensity factor (result more justified in reference [2]).

The evolution of the loading cycle characterized by the dimensionless quantity $N_c C(\Delta\sigma_t)^m a_0^{m/2-1}$, obtained by the present approach and analytically, is reported in Fig. 3. A good agreement is observed between both results. We recall that the abovementioned analytical solution (7) was obtained by [12] after integrating the relation (5) (with $m = 2$) and the analytical expression $\sigma_t\sqrt{a/\pi}$ of the penny-shaped SIF.

$$N_c C(\Delta\sigma_t)^m a_0^{m/2-1} = N_c C(\Delta\sigma_t)^2 = \frac{\pi}{4}\ln\frac{a}{a_0} \tag{7}$$

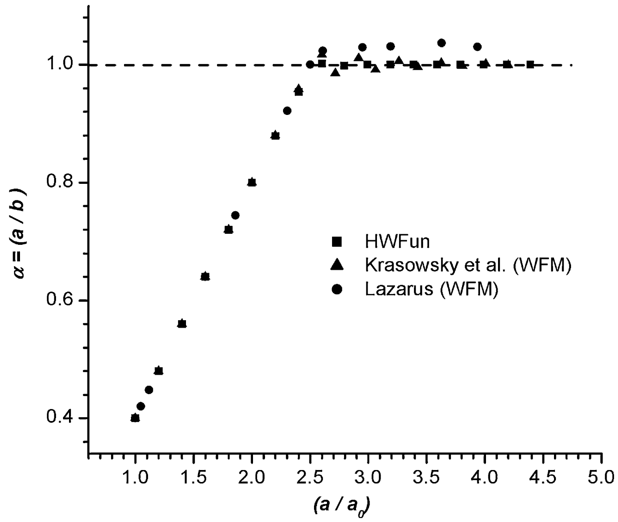


Fig. 5. Evolution of the elliptical crack shape ($\alpha_0 = 0.4$) by different approaches.

Fig. 5. Evolution de la forme d'une fissure elliptique ($\alpha_0 = 0,4$) en fonction de l'avance obtenue par les différentes approches.

3.2. *Infinite body with embedded penny-shaped crack in brittle fracture*

The configuration of the crack is similar to the previous one except that the initial aspect ratio α_0 of the elliptical crack is equal to 0.4 in brittle fracture ($m = 50$).

This has been reported by Lazarus [12] using the weight function method (WFM) inspired from of the works of Rice [13] and Bower and Ortiz [14]. Figs. 4(a) and (b) show the progression of the propagating crack obtained by both approaches, the hybridization and the WFM, from which one can observe that an initial elliptical crack shape tends to a circular one during propagation. From Fig. 4(b), one can additionally remark, that, at its first propagation stage, the crack shape deviates from the elliptical shape then comes back to it [12], whereas the hybridization approach shows that the elliptical shape is by hypothesis saved.

Fig. 5 shows the evolution of the ratio α of the elliptical crack during its propagation. It is noticed that for the three approaches, the ratio α rapidly increases and stabilizes around unity for the hybridization approach, oscillates before stabilizing around unity for the Krasowsky's weight function approach [7] and slightly stabilizes up to unity for Lazarus weight function approach [12]. This justifies the interest of the hybridization approach in the study of crack growth propagation.

4. Concluding remarks

The present study shows that the hybridization weight function approach is favourably established among other numerical methods in the prediction of elliptical crack growth in fatigue. Along with a two-degrees-of-freedom procedure, this approach satisfactorily describes the evolution of the elliptical crack. Moreover, the number of loading cycles characterizing the cracked structure fatigue life is also well predicted.

In the applications treated, only loadings presenting a double symmetry (with respect to Ox and Oy) were considered, Fig. 1, in order to stay consistent with the elliptical shape of the crack. Other loading conditions presenting a simple symmetry with respect to Oy (not presented here) have been successfully tested in the case of semi elliptical cracks.

At the end, it worthwhile to precise that the apparent improvement of the numerical performance of the present approach, compared to the more general evocated approaches which are the XFEM [8] or the WFM [12], is tightly dependent on the conservation of the elliptical profile of the crack, an assumption which could be reasonably adopted after crack propagation case (see [11,12]).

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