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Experimental validation of a homogenized plate model for the yield design of masonry walls

Julien Dallot, Karam Sab *, Olivier Godet

Université Paris-Est, Institut Navier, LAMI (ENPC/LCPC), École des Ponts, 6 & 8 avenue Blaise-Pascal, 77455 Marne-la-Vallée cedex 2, France

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Abstract

The aim of this Note is to provide a comparison between the predictions of a homogenized plate model for the determination of the ultimate loads of out-of-plane loaded masonry walls and the results of a new experimental set up developed for this purpose. ***To cite this article: J. Dallot et al., C. R. Mecanique 336 (2008).***

Résumé

Validation expérimentale d'un modèle de plaque homogénéisée en calcul à la rupture des murs de maçonnerie. L'objet de cette Note est de présenter une comparaison entre les prédictions d'un modèle de plaque homogénéisée pour le calcul à la rupture des murs de maçonnerie chargés hors plan, d'une part, et les résultats d'une expérimentation à échelle réduite conçue à cet effet, d'autre part. ***Pour citer cet article : J. Dallot et al., C. R. Mecanique 336 (2008).***

Keywords: Solids and structures; Homogenization; Strength domain; Masonry; Out-of-plane loads; Love–Kirchhoff plate; Experimental set up

Mots-clés : Solides et structures ; Homogénéisation ; Domaine de résistance ; Maçonnerie ; Chargement hors-plan ; Plaque de Love–Kirchhoff ; Dispositif expérimental

1. Introduction

In a previous paper, Sab [1] suggested a homogenization procedure for the yield design of masonry walls considered as Love–Kirchhoff plates. The periodic brickwork is made of 3D infinitely resistant blocks connected by cohesionless Mohr–Coulomb interfaces (friction angle φ, $0 < \varphi < \frac{\pi}{2}$). The out-of-plane homogenized yield surfaces are obtained semi-analytically in terms of φ and the geometric characteristics of the microstructure (brick dimensions and pattern). They are additional and complementary to the in-plane anisotropic yield surfaces found by de Buhan and de Felice [2]. However, there is a need to compare this theoretical homogenization procedure to experimental results. Hence, the

* Corresponding author.
E-mail addresses: dallot@lami.enpc.fr (J. Dallot), sab@lami.enpc.fr (K. Sab).

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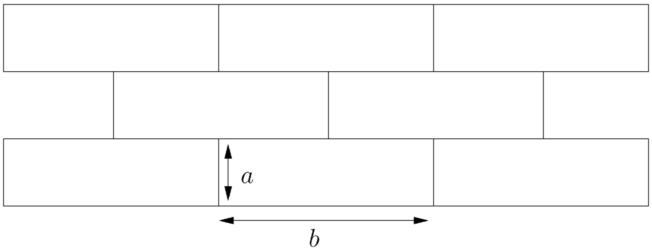


Fig. 1. Masonry texture: Running bond.

purpose of this Note is to adopt the homogenized Love–Kirchhoff model and to compare its results to the ones of a new experimental set up developed for this purpose.

2. The homogenized Love–Kirchhoff plate model

In this section, the homogenized strength domain of a masonry plate under the assumption of infinitely resistant blocks connected by cohesionless Mohr–Coulomb interfaces is recalled. The so-called running bond texture is considered: a is the height, b is the width, t is the thickness of the brick and $m = \frac{2a}{b}$ is the shape parameter; see Fig. 1. The heterogeneous plate under consideration occupies a domain $\Gamma \times]-\frac{t}{2}, \frac{t}{2}[$ where $\Gamma \subset \mathbb{R}^2$ is the middle surface of the plate and t is its thickness. The plate exhibits a periodic structure in both directions 1 and 2, so that it is possible to extract an elementary module which contains all information necessary to completely describe the plate. Under the assumption that the thickness t is of the same order as the other sizes of the unit cell and that t is very small in comparison with the typical size of Γ, it has been proposed by Bourgeois et al. [3] and Sab [1] and then formally proved by Dallot and Sab [4] that the limit analysis of the 3D-body can be performed using a homogeneous Love–Kirchhoff plate theory. Shear effects may be taken into account through a homogenized Reissner–Mindlin plate model suggested in [5,6] and [7].

The following notations are used: Greek indexes $\alpha, \beta = 1, 2$, Latin indexes $i, j = 1, 2, 3$. $\mathbf{N} = (N_{\alpha\beta})$ is the macroscopic in-plane (membrane) stress field for the homogenized plate; $\mathbf{M} = (M_{\alpha\beta})$ is the macroscopic out-of-plane (flexural) stress field; $\mathbf{D} = (D_{\alpha\beta})$ is the corresponding in-plane strain rate field; $\boldsymbol{\chi} = (\chi_{\alpha\beta})$ is the out-of-plane strain rate field; $\mathbf{V} = (V_i)$ is a virtual velocity field. The macroscopic rate fields are related to the macroscopic virtual velocity field components $V_1(x_1, x_2)$, $V_2(x_1, x_2)$ and $V_3(x_1, x_2)$ as follows: $D_{\alpha\beta} = \frac{1}{2}(V_{\alpha,\beta} + V_{\beta,\alpha})$ and $\chi_{\alpha\beta} = -V_{3,\alpha\beta}$.

The kinematic definition of the strength domain of the homogenized Love–Kirchhoff plate, G_p^{hom}, is:

$$(\mathbf{N}, \mathbf{M}) \in G_p^{\text{hom}} \Longleftrightarrow \mathbf{N}\colon \mathbf{D} + \mathbf{M}\colon \boldsymbol{\chi} \leqslant \pi_p^{\text{hom}}(\mathbf{D}, \boldsymbol{\chi}) \quad \text{for all } (\mathbf{D}, \boldsymbol{\chi}) \tag{1}$$

where the homogenized π-function is:

$$\pi_p^{\text{hom}}(\mathbf{D}, \boldsymbol{\chi}) = \operatorname*{Inf}_{\Omega_3} \pi_p(\mathbf{D}, \boldsymbol{\chi}, \Omega_3), \quad \pi_p(\mathbf{D}, \boldsymbol{\chi}, \Omega_3) = \begin{cases} 0 & \text{if (3) and (4)} \\ +\infty & \text{otherwise} \end{cases} \tag{2}$$

with:

$$\begin{aligned} &\tan\varphi \sqrt{(D_{12}^{\varepsilon_1} - \Omega_3)^2 + \left(\frac{a}{2}\chi_{12}\right)^2} \leqslant D_{11}^{\varepsilon_1} \\ &\varepsilon_1 = \pm 1, \qquad D_{\alpha\beta}^{\varepsilon} = D_{\alpha\beta} + \varepsilon \frac{t}{2}\chi_{\alpha\beta} \end{aligned} \tag{3}$$

and[1]

[1] In [1], the analysis was based on an erroneous expression by [8] for the third component of the velocity jump at the horizontal interfaces, and this error propagates to condition (4).

$$\tan\varphi\sqrt{\left(m\left(D_{12}^{\varepsilon_3}+\Omega_3\right)+\varepsilon_2 D_{11}^{\varepsilon_3}\right)^2+\left(m\frac{b}{4}\chi_{12}+\varepsilon_2\frac{b}{4}\chi_{11}\right)^2}-\left(D_{12}^{\varepsilon_3}-\Omega_3\right)\varepsilon_2\leqslant mD_{22}^{\varepsilon_3} \tag{4}$$

$$\varepsilon_2=\pm 1,\qquad \varepsilon_3=\pm 1$$

3. Experimental results

A reduced scale rectangular brickwork-like plate , $\Gamma =]-\frac{L_1}{2}, \frac{L_1}{2}[\times]-\frac{L_2}{2}, \frac{L_2}{2}[$, is subjected to in-plane forces and to an increasing out-of-plane (vertical) x_3-force applied in its center; see Fig. 2. The in-plane horizontal dimensions are $L_1 = 13 \times b$ and $L_2 = 25 \times a$ ($\frac{L_2}{L_1} \approx m$). The bricks are all made of the same wooden material ($a = 19$ mm $b = 49$ mm) and their design has been made very carefully in order to obtain the same roughness for all faces and consequently the same frictional behavior at the interfaces (φ is uniform). The x_1-forces, T_1, and the x_2-forces, T_2, are applied through 4 elastic springs which are attached to two pairs of opposite rods in contact with the boundaries of the panel. The rods are free to rotate around their respective axis and they are simply supported. Three horizontal force ratios $r = T_2/T_1$ are tested (0.66, 1 and 1.5). The two in-plane directions of the plate being the two horizontal directions, the bricks of the plate are actually submitted to their own weight. Hence, a minimum force value $T_1 = T_w$, ($T_2 = rT_w$) is required to set the plate in the horizontal position. This force is an experimental artefact which is sensitive to the initial vertical position and inclination of the rods with respect to the brick panel. It is most likely that an (unknown) initial couple is applied to the panel due to its own weight. For $T_1 > T_w$, an increasing vertical force F is applied on a small rectangular region ($l_1 = L_1/5, l_2 = L_2/5$) which is situated at the center of the plate and the limit load F^* that causes the failure of the panel is measured. Two thicknesses of the bricks, $t = 30$ mm and $t = 40$ mm, are tested. For each t, r and T_1, with $T_1 > T_w$, the average value of F^* is measured for several specimens (at least three) and plotted versus T_1. It is found that F^* is proportional to $T_1 - T_w$ for all tested t and r; see Fig. 3(a). This is consistent with the assumption of cohesionless Mohr–Coulomb interface law between the bricks when they are set in the horizontal position with the in-plane forces $T_1 = T_w$, $T_2 = rT_w$. Therefore, by shifting the $F^* - T_1$ straight line to the origin, one can neglect the effect of the own weight of the bricks. Moreover, a four-hinge failure mode joining the plate center to the corners is observed for all tested t, r and T – Fig. 3(b).

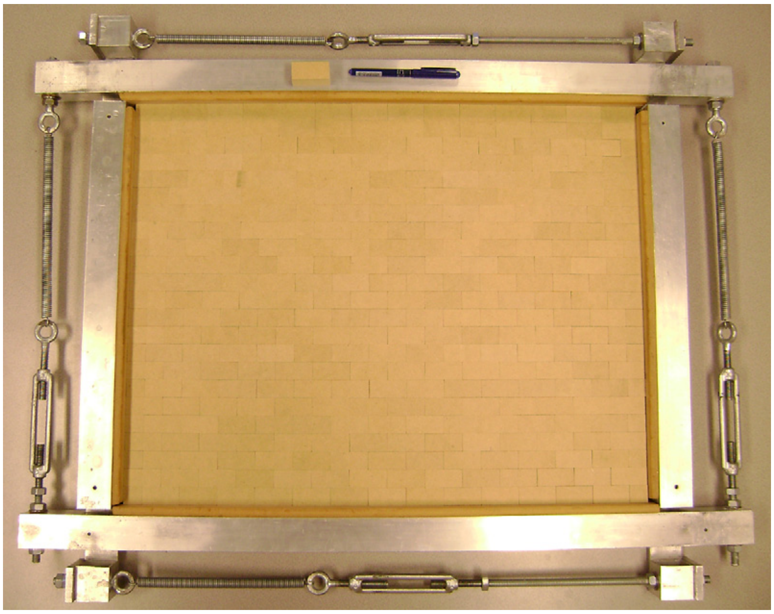


Fig. 2. Experimental set up.

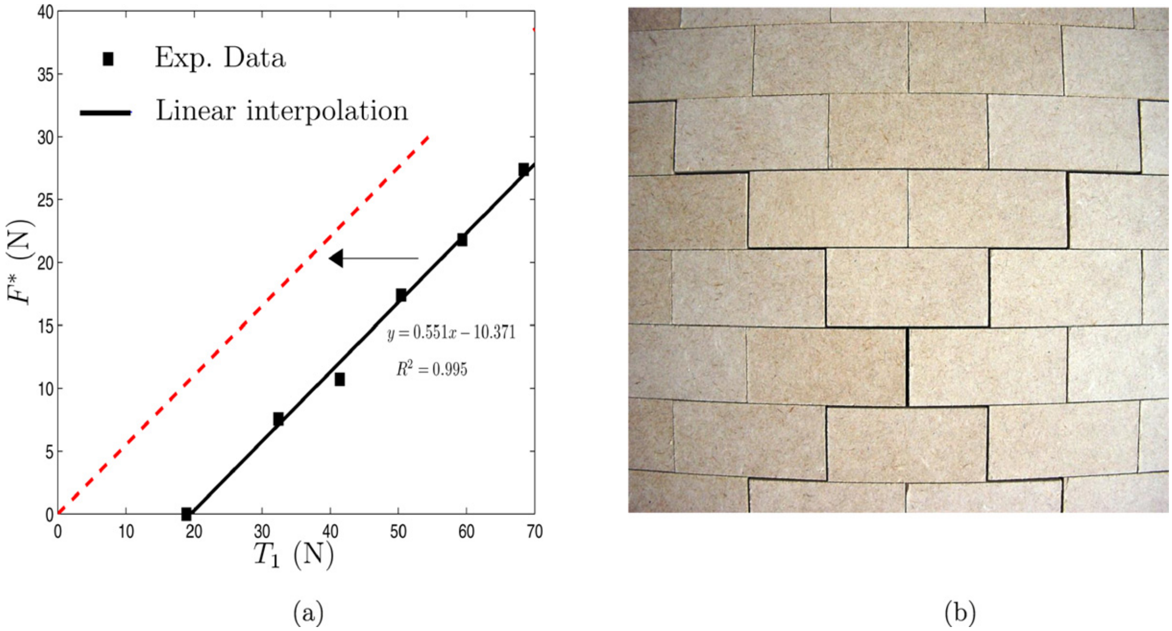


Fig. 3. Experimental results for $t = 40$ mm and $r = 1$: (a) data and linear interpolation; (b) collapse mode: focus on the panel center.

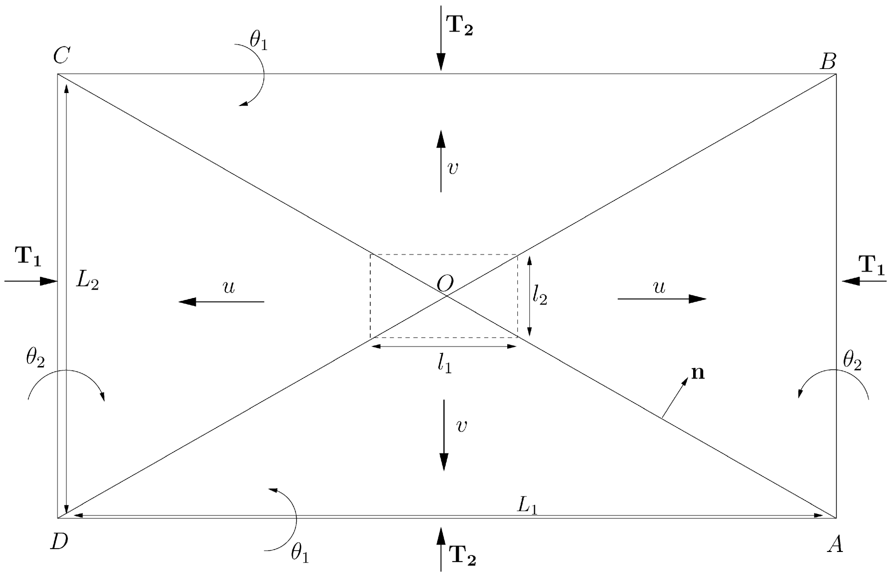
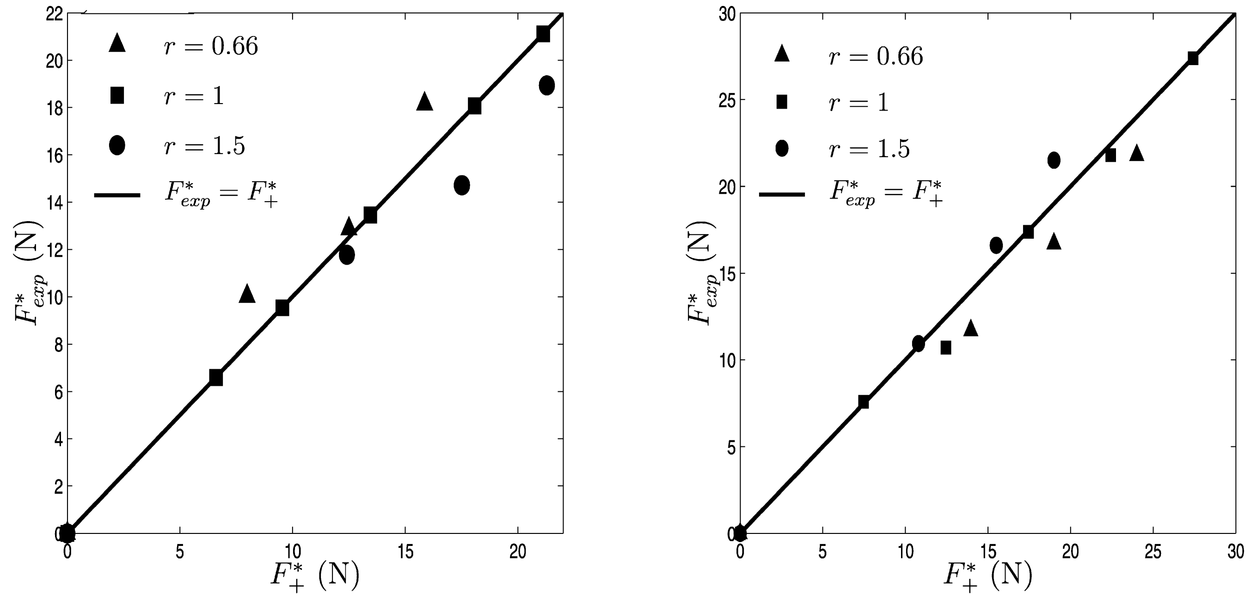


Fig. 4. Notations.

4. The kinematic method

Adopting the homogenized Love–Kirchhoff model, this failure mode is described as follows: The triangular domains OAB, OBC, OCD and ODA of Fig. 4 have the following four rigid body motions:

$$\mathbf{V}(x_1, x_2) = \begin{cases} +u\mathbf{e}_1 - (w - \theta_2 x_1)\mathbf{e}_3 & \text{on } OAB \\ +v\mathbf{e}_2 - (w - \theta_1 x_2)\mathbf{e}_3 & \text{on } OBC \\ -u\mathbf{e}_1 - (w + \theta_2 x_1)\mathbf{e}_3 & \text{on } OCD \\ -v\mathbf{e}_2 - (w + \theta_1 x_2)\mathbf{e}_3 & \text{on } ODA \end{cases} \qquad \theta_1 = \frac{2w}{L_2},\ \theta_2 = \frac{2w}{L_1}$$



(a) $t = 30$ mm (b) $t = 40$ mm

Fig. 5. F^*_+ vs F^*_{exp} for $\tan(\varphi) = 0.918$.

The microscopic scale representation of the macroscopic failure mode described above could be also obtained following [9].

The power of the external forces in this failure mode is:

$$P^{\mathrm{ext}} = \left(1 - \frac{l_1}{3L_1} - \frac{l_2}{3L_2}\right) F w - 2T_1 u - 2T_2 v \tag{5}$$

According to the well-known yield analysis kinematic method [10], and thanks to the symmetry of the considered failure mode, we have:

$$P^{\mathrm{ext}} \leqslant 4 \times \int_{OA} \pi_p^{\mathrm{hom}}(\mathbf{D}_{OA}, \boldsymbol{\chi}_{OA})\,\mathrm{d}s$$

for all (u, v, w) where $(\mathbf{D}_{OA}, \boldsymbol{\chi}_{OA})$ are defined on the plastic hinge OA as:

$$\mathbf{D}_{OA} = (u\mathbf{e}_1 + v\mathbf{e}_2) \otimes^s \mathbf{n}, \qquad \boldsymbol{\chi}_{OA} = (\theta_1\mathbf{e}_1 - \theta_2\mathbf{e}_2) \otimes^s \mathbf{n} \tag{6}$$

Here, $\mathbf{n} = \frac{1}{\sqrt{1+m^2}}(m\mathbf{e}_1 + \mathbf{e}_2)$ is the unit vector normal to OA and $\otimes^s$ denotes the symmetric part of the diadic product operator. Using (2) and (5), we derive the following upper bound for F^*:

$$F^* \leqslant F^*_+ = \min_{u,v,\Omega_3} 2\left(1 - \frac{l_1}{3L_1} - \frac{l_2}{3L_2}\right)^{-1} (T_1 u + T_2 v)$$

where the minimization is over all (u, v, Ω_3) such that the six conditions (3) and (4) are verified for $(\mathbf{D}, \boldsymbol{\chi}) = (\mathbf{D}_{OA}, \boldsymbol{\chi}_{OA})$ given by (6) with $w = 1$.

The MATLAB software has been used to compute F^*_+. It is found that the numerical results fit well with the experimental data when the frictional angle $\varphi = \arctan(0.918)$ is adopted for all tested t, r and T_1 as shown in Fig. 5. It is concluded that the homogenization procedure proposed in [1] is a powerful simple tool for the prediction of the failure of out-of-plane loaded masonry walls when there is no shear effect. Such effect may arise for concentrated loads and restraint boundary conditions.

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