Available online at www.sciencedirect.com
ScienceDirect
C. R. Mecanique 336 (2008) 612–615
COMPTES RENDUS MECANIQUE
http://france.elsevier.com/direct/CRAS2B/

Transition of unsteady flows of evaporation to steady state

Amah d'Almeida

Département de mathématiques, Université de Lomé, BP 1515 Lomé, Togo

Received 14 December 2005; accepted after revision 14 April 2008

Available online 9 May 2008

Presented by Évartiste Sanchez-Palencia

Abstract

We investigate the half-space problem of evaporation and condensation in the scope of discrete kinetic theory. Exact solutions are found to the boundary value problem and the initial boundary value problems of the flow in the half space for a discrete velocity model. The results are used to analyze the transition of the unsteady solutions towards steady states. ***To cite this article: A. d'Almeida, C. R. Mecanique 336 (2008).***

Résumé

Transition vers l'état stationnaire d'écoulements instationnaires d'évaporation. On étudie dans le cadre de la théorie cinétique discrète les phénomènes d'évaporation et de condensation dans le demi-espace. Des solutions exactes sont trouvées pour le problème aux valeurs initiales et aux limites de l'écoulement dans le demi-espace pour un modèle discret et on analyse leur convergence vers les solutions du problème aux limites stationnaire. ***Pour citer cet article : A. d'Almeida, C. R. Mecanique 336 (2008).***

Keywords: Fluid mechanics; Exact solution; Half-space; Evaporation; Condensation

Mots-clés : Mécanique des fluides ; Solutions exactes ; Demi-espace ; Evaporation ; Condensation

1. Introduction

Consider a gas and its condensed phase respectively located in the region $y > 0$ and $y < 0$. Depending on the thermodynamical conditions of the two phases, evaporation or condensation occurs at the interface. This problem is known as the half space problem of evaporation and condensation. Various methods of solution have been used. Interesting results have been obtained using the linearization of the classical Boltzmann equation or its continuous models around a uniform flow at infinity or extensive numerical analysis of the BKW equation for the long time behavior of time dependent studies of the half-space problem [1–4]. The nonlinear problem is studied only approximatively or numerically. However it is necessary to solve the boundary value problem of the nonlinear Boltzmann equation to obtain the relations required for the existence of the steady solution [5]. The modelling via discrete models is recent [10,11].

E-mail address: salmeida@tg.refer.org.

1631-0721/$ – see front matter
doi:10.1016/j.crme.2008.04.006

A discrete velocity gas is a medium composed of particles whose velocities belong to a *given* set of vectors. Exact and numerical solutions have been found and analytical form of the relation between the macroscopic variables on the interface and at infinity have been derived for some discrete models [6–8]. In this paper we study, in the scope of discrete kinetic theory, the long time behavior of the solution of the time dependent half-space problem. We show in Section 2 that there is no solution for the steady half-space problem of evaporation for the symmetrical model C_1 [9,7]. In Section 3 we find exact solutions to the unsteady half-space problem of evaporation and condensation and we analyze their convergence for large times to solutions of the corresponding steady half-space problem.

2. Steady solutions

The model C_1 is a ten velocity three dimensional discrete model. When we assume the distribution of the velocities of the model to be symmetrical with respect to the axis normal to the interface, the flow is one-dimensional and the number of unknown densities is reduced to four. They are the microscopic densities: N_i, $i = 1, 2, 9, 10$. We shall look, by analogy with the studies of the subject in classical kinetic theory of gases [5] for a solution of the half-space problem under the additional assumption that the microscopic densities depend only upon the space variable y. Therefore, the solution is a special case of the one obtained in [7] when only binary collisions are retained. The macroscopic variables of the flow are the total density $N = 4(N_1 + N_3) + N_9 + N_{10}$, the normal component $V = 4(N_1 - N_3) + N_9 - N_{10}$ of the macroscopic velocity $\mathbf{U}$ and the total energy E. We denote by n_w, n_∞ and l_∞ respectively the saturation density of the vapor corresponding to the temperature and the pressure of the condensed phase, the total density and the mean free path at infinity. Then we introduce the nondimensional variables $n = N/n_\infty$, $v = V/c$, $e = E/mc^2$, $n_i = N_i/n_\infty$, $i = 1, \dots, 10$, $\nu_w = n_w/n_\infty$, $\tilde{y} = y/l_\infty$ and $Kn = (Sn_\infty l_\infty)^{-1}$. The solution for n_1 and n_9 is:

$$n_1(\tilde{y}) = \frac{(1+v_\infty)(e_w - e_\infty)}{8}\exp\left(\frac{\sqrt{6}v_\infty}{2Kn}\tilde{y}\right) + \frac{(1+v_\infty)(2e_\infty - 1)}{16}$$

$$n_3 = \frac{v_\infty(1-2e_\infty)}{8} + n_1, \quad n_9 = \frac{1+v_\infty}{2} - 4n_1, \quad n_{10} = \frac{1+v_\infty(2e_\infty - 2)}{2} - 4n_1 \tag{1}$$

The relation required for the existence of the steady solution is $\nu_w(1+v_w) = 1 + v_\infty$. The accommodation coefficient ν_w is thus given as a function of the normal velocities at the interface and at infinity. The solution blows up at infinity for $v_\infty > 0$. *So the solution of the steady half-space for the model C_1 only describes condensation on the interface for* $v_\infty \neq 0$. For $v_\infty = 0$ the vapor is in Maxwellian equilibrium with the condensed phase and the flux of incoming particles with respect to the gas phase is balanced by the flux of outgoing particles.

3. Unsteady solutions

We denote by t_∞ the average time between collisions in the steady state at infinity and in addition to those defined in the previous section, we introduce the nondimensional variables $\tilde{t} = t/t_\infty$, $\gamma = ct_\infty/l_\infty$. For the symmetrical ten velocity model studied in the previous section the kinetic equations reduce to:

$$\left(\frac{\partial}{\partial \tilde{t}} + \gamma\frac{\partial}{\partial y}\right)n_1 = -\left(\frac{\partial}{\partial \tilde{t}} - \gamma\frac{\partial}{\partial \tilde{y}}\right)n_3 = \frac{\sqrt{6}}{2\gamma Kn}(n_3 n_9 - n_1 n_{10})$$

$$\left(\frac{\partial}{\partial \tilde{t}} + \gamma\frac{\partial}{\partial \tilde{y}}\right)n_9 = -\left(\frac{\partial}{\partial \tilde{t}} - \gamma\frac{\partial}{\partial \tilde{y}}\right)n_{10} = 2\frac{\sqrt{6}}{\gamma Kn}(n_3 n_9 - n_1 n_{10}) \tag{2}$$

We solve them for the general initial and boundary conditions $n_i(0, \tilde{y}) = h_i(\tilde{y})$, $i = 1, 3, 9, 10$, and $n_i(\tilde{t}, 0) = k_j(\tilde{t})$, $j = 1, 9$, $\lim_{\tilde{y}\to\infty} n_j(\tilde{t}, \tilde{y}) = l_j(\tilde{t})$, $j = 1, 3, 9, 10$. The general solutions are:

$$n_9(\tau,\eta) = a_1\left[G(\tau,\eta) + \mu\right], \quad n_{10}(\tau,\eta) = a_2\left[\frac{B}{K}G(\tau,\eta) + \mu\right], \quad 8n_1 = a_1 - 2n_9, \quad 8n_3 = a_2 - 2n_{10}$$

$$G = \exp\left[K\int_0^\eta a_1(s)\,\mathrm{d}s + B\int_0^\tau a_2(s)\,\mathrm{d}s\right], \quad a_1(\eta) = (8h1 + 2h_9)(\eta), \quad a_2(\tau) = (8h_3 + 2h_{10})(\tau)$$

$$\tau=\tilde{y}+\gamma\tilde{t},\quad \eta=\tilde{y}-\gamma\tilde{t},\quad K=\frac{\sqrt{6}B}{\sqrt{6}+4B\gamma\,Kn},$$
$$2n(\tilde{t},\tilde{y})=a_1(\eta)+a_2(\tau),\quad 2nv(\tilde{t},\tilde{y})=a_1(\eta)-a_2(\tau) \tag{3}$$

B and μ are integration constants. The solutions (3) is the sum of a Maxwellian part proportional to a_i, $i=1,2$, and a nonequilibrium part proportional to a_iG, $i=1,2$. The forms (3) of the solution and of the number of initial and boundary conditions impose compatibility relations between the data. A discussion of these conditions is made in [9]. Here we solve the problem for the particular case

$$n_i(0,\tilde{y})=\nu_0 n_{i0}\big[1+w(\tilde{y})\big],\quad \lim_{\tilde{y}\to\infty} n_i(\tilde{t},\tilde{y})=n_{i\infty}\big[1+w(\tilde{t})\big],\quad i=1,3,9,10$$
$$n_i(\tilde{t},0)=\nu_w n_{iw}\big[1+w(\tilde{t})\big],\quad i=1,9,\quad \lim_{|x|\to 0} w(x)=\lim_{|x|\to\infty} w(x)=0$$
$$n_{1\alpha}=\frac{(1+v_\alpha)(2e_\alpha-1)}{16},\quad n_{3\alpha}=\frac{(1-v_\alpha)(2e_\alpha-1)}{16},\quad n_{9\alpha}=\frac{(1+v_\alpha)(3-2e_\alpha)}{4}$$
$$n_{10\alpha}=\frac{(1-v_\alpha)(3-2e_\alpha)}{4},\quad \alpha=0,w,\infty \tag{4}$$

The n_{i0}, n_{iw} and $n_{i\infty}$ are respectively the densities of an arbitrary Maxwellian state of the gas phase taken as the initial state of the vapor, the densities of the discrete gas in equilibrium with the condensed phase and the densities of the Maxwellian state of the vapor far away from the interphase. When $\tilde{t}$ tends to infinity $w(\tilde{t})$ vanishes and we obtain the boundary conditions of the steady half-space problem [7,6]. The initial and boundary conditions imply the continuity of a_1. Hence from $\lim_{\tilde{t}\to 0}(\lim_{\tilde{y}\to 0} a_1(\eta))=\lim_{\tilde{y}\to 0}(\lim_{\tilde{t}\to 0} a_1(\eta))$ and $\lim_{\tilde{t}\to\infty}(\lim_{\tilde{y}\to\infty} a_1(\eta))=\lim_{\tilde{y}\to\infty}(\lim_{\tilde{t}\to\infty} a_1(\eta))$ we deduce the relation:

$$\nu_0(1+v_0)=\nu_w(1+v_w)=(1+v_\infty) \tag{5}$$

which gives the accommodation coefficients ν_w and ν_0. The solutions for n_9 and n_{10} are the solutions (3) with:

$$G(\tau,\eta)=\frac{(e_\infty-e_0)}{2}\exp\big[-2K(1+v_\infty)\gamma\tilde{t}\big]\exp\left[K(1+v_\infty)\int_\tau^\eta w(s)\,\mathrm{d}s\right]$$
$$\mu=\frac{3-2e_\infty}{4},\qquad B=\frac{\sqrt{6}[\nu_0(1-v_0)+v_\infty-1](3-2e_0)}{8\gamma\,Kn(1-v_\infty)(e_\infty-e_0)} \tag{6}$$

The geometry of the model imposes $|v_\infty|<1$ and the solution (6) is bounded at infinity provided $K>0$ and $\int_\tau^\eta w(s)\,\mathrm{d}s$ is bounded at infinity. In the case of condensation we prove in [9] the convergence of the solution (6) to the solution (1) of the steady half-space problem of condensation. When $e_0=e_\infty$ the solution (6) is a Maxwellian state of the model. Otherwise the solution is not Maxwellian and depends on the initial and boundary conditions.

An unsteady solution exists for evaporation or condensation for $w(x)=0\ \forall x\in\mathbb{R}$ and $e_0=e_\infty$ and converges to a steady state as $\tilde{t}$ tends to infinity provided the condition (5) is satisfied. The vapor is in the Maxwellian equilibrium state associated to 1, v_∞ and e_∞ whatever v_∞. For $v_\infty=0$ we have the case where the incoming flux of matter balances the outgoing one.

4. Conclusion

We find an exact solution to the unsteady half-space problem of condensation and evaporation for model C_1. The fact that the flow results from a superposition of two waves is clearly shown by the solution. We establish the relation (5) of the macroscopic variables at interface and at infinity that allows a steady flow of evaporation and condensation from the time dependent analysis of the half-space problem. This is in agreement with numerical results obtained with continuous models of the Boltzmann equation [5,1–3]. For the model we use for the computations, the steady half-space problem has no solution for evaporation for nonzero v_∞. However, unsteady solutions exist for evaporation in the half-space for nonzero v_∞. Moreover they converge when $\tilde{t}$ tends towards infinity to a steady state of the flow provided the relation between the macroscopic variables of the flow at interface and at infinity is satisfied. The reason for this result seems to be the dependence of the solution of the unsteady half-space problem upon the initial conditions which determine the type of the transition to the steady state at infinity. Unlike the case of condensation, the existence of a steady state for evaporation needs additional relations linking the macroscopic variables of the initial and the final states of the vapor which cannot be satisfied in the time independent analysis of the half-space problem for model C_1.

Acknowledgements

Thanks to Prof. Renée Gatignol who suggested the work and invited the author to the Laboratoire de Modélisation en Mécanique of Université Paris 6 where a part of the computation was done.

References

[1] Y. Sone, K. Aoki, Y. Yamashita, in: V. Boffi, C. Cercignani (Eds.), Rarefied Gas Dynamics, vol. 2, Teubner, Stuttgart, 1986, pp. 323–333.
[2] Y. Sone, H. Sugimoto, in: G.E.A. Meier, P.A. Thompson (Eds.), Adiabatic Waves in Liquid–Vapor Systems, Springer, Berlin, 1990, pp. 293–304.
[3] K. Aoki, K. Nishino, Y. Sone, H. Sugimoto, Phys. Fluids A 3 (1991) 2260–2275.
[4] K. Aoki, Y. Sone, T. Yamada, Phys. Fluids A 2 (1990) 1867–1878.
[5] Y. Sone, S. Takata, F. Golse, Phys. Fluids 13 (2001) 324–334.
[6] A. d'Almeida, J. Rech. Sci. Univ. Lomé 5 (2001) 167–174.
[7] A. d'Almeida, R. Gatignol, M3AS 13 (2003) 99–119.
[8] R. Gatignol, in: N. Bellomo, R. Gatignol (Eds.), Lecture Notes on the Discretization of the Boltzmann Equation, World Scientific, 2003, pp. 289–294.
[9] A. d'Almeida, Mech. Res. Com. 34 (2007) 405–409.
[10] H. Cabannes, Eur. J. Mech. Fluids B 13 (1994) 685–699.
[11] H. Cornille, A. d'Almeida, Eur. J. Mech. Fluids B 21 (2002) 355–370.