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Impact of a bead on a rotating wall

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Abstract

We study experimentally the impact of a plastic bead on a rotating wall made of steel (velocity Ω; radial position x_0). The results show that the restitution coefficient is directly function of the impact velocity $x_0\Omega$ and is invariant by changing frame reference. The influence of the height of release of the particle on its angular velocity after impact is also studied. We observe an increase of the angular velocity with height followed by a saturation. We propose an interpretation for this evolution considering that the particle may roll without sliding during all the impact. This physical feature is not always taken into account in existing models of impact between rigid bodies. ***To cite this article: F. Rioual et al., C. R. Mecanique 336 (2008).***

Résumé

Impact d'une bille sur une paroi verticale en rotation. Nous étudions expérimentalement l'impact d'une bille plastique sur une paroi verticale en acier soumise à une rotation (vitesse angulaire Ω ; position radiale x_0). Nous montrons que le coefficient de restitution bille/paroi est directement fonction de la vitesse d'impact $x_0\Omega$ et est invariant par changement de référentiel. Nous avons aussi exploré l'influence de la hauteur de laché de la particule sur sa vitesse angulaire aprés impact. Nous observons notamment une augmentation de la vitesse angulaire de la particule avec la hauteur suivie d'une saturation et nous proposons une interprétation en considérant un mouvement de roulement sans glissement de la particule tout au long du choc sur le plan, phénomène qui n'est pas toujours pris en compte dans les théories existantes d'impact entre corps rigides. ***Pour citer cet article : F. Rioual et al., C. R. Mecanique 336 (2008).***

Keywords: Granular media; Rheology; Rigid body theory

Mots-clés : Milieux granulaires ; Rhéologie ; Théorie des corps rigides

1. Introduction

Granular flows are ubiquitous in the industry and nature (food, agriculture, manufacturing, arid deserts etc.). They behave as very peculiar fluids as the energy is only transmitted through the contact between two of their elements by

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friction and impacts. It is thus observed that in various contexts where a dilute phase of a granular flow appears, the rheological properties of the flow may be dependent on the precise nature of the impact between their constituents (see [1] for instance in the context of eolian transport or [2] in particle-laden channel flows). Therefore, much effort has to be devoted in order to develop simple realistic impact models to be implemented in simulations of particle flows. The classical approach to tackle impact problems is the rigid body framework [3]. In this view, the characteristics of the impact are determined simply from balance equations for impulse and momentum before and after impact according to rigid body mechanics. A typical oblique impact is supposed to be instantaneous and characterized by two parameters which encode the constitutive modelling of the deformed materials involved in the impact: a friction coefficient which is used to model the tangential dissipation of energy and the Newtonian normal coefficient of restitution for normal dissipation. For impacts close to head on however, this approach appears to be insufficient: hence the existence in experiments of a negative rebound angle in the oblique impact of a particle on a plate [5,6]. This feature can only be explained taking also into account the tangential compliance over the contact area during the impact event. Different approaches have thus been proposed as in [4,5] and in [6] in order to integrate these compliance effects in a phenomenological way. They require the use of a third parameter; a tangential coefficient of restitution β for rolling or sticking tangential movement distinct from sliding collisions. Some experimental situations are however still out of the scope of the rigid body approach as for oblique impacts with initial angular velocities [8] or grazing impacts on elasto-plastic plates [9]. Even the Newtonian normal coefficient of restitution that is usually considered as a material constant may be ill-defined in certain impact geometries [10].

We study experimentally in this article the impact of a bead released from a height h_0 with a rotating wall at a constant velocity Ω. This situation arises e.g. during the centrifugal spreading [7], in which the energy of a rotating boundary is monitoring a granular flow.

2. Experimental setup and procedure

The beads used in the experiments were spherical plastic BB beads used in air guns. The size of these beads is roughly 6 mm (5.93 mm $\pm$ 0.02 mm). The rotating plate is made in steel and recovered of black painting for imaging purposes. It is massive as compared to the beads and its velocity is not affected by the collision. We used in these experiments two different experimental setups (see Fig. 1):

– In a first setup, a mobile wall of length $l = 20$ cm and width $d = 4$ mm rotates around an axis triggered by an electrical motor. The possible range of angular velocities lies between 100 rpm and 1500 rpm. An electrovane device has been set up in order to release the bead at a given position in space.

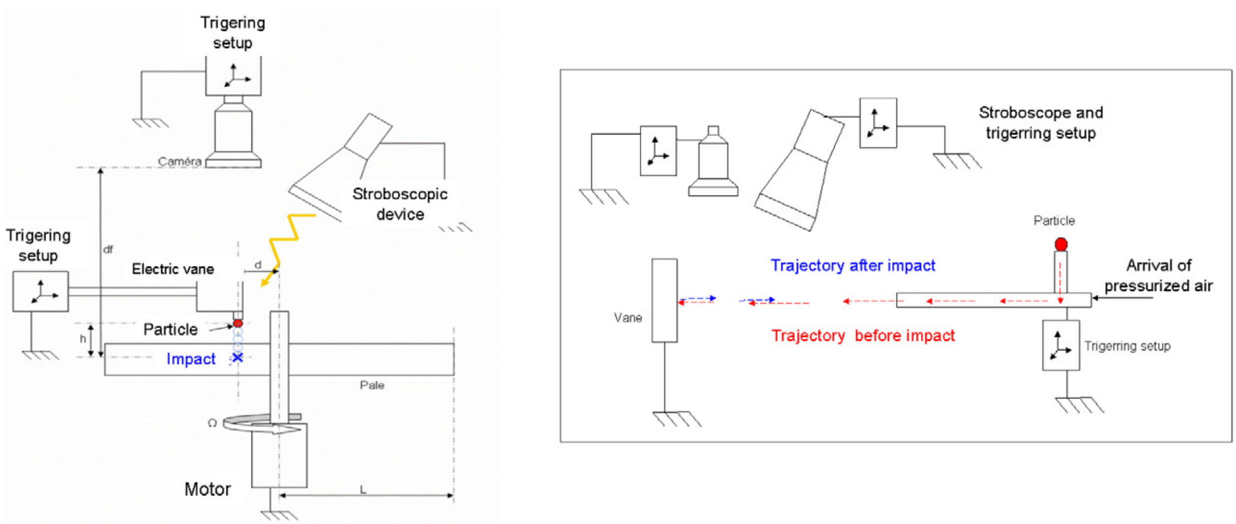

Fig. 1. Two experimental setups used in this study: Rotating setup (left); Fixed setup (right).

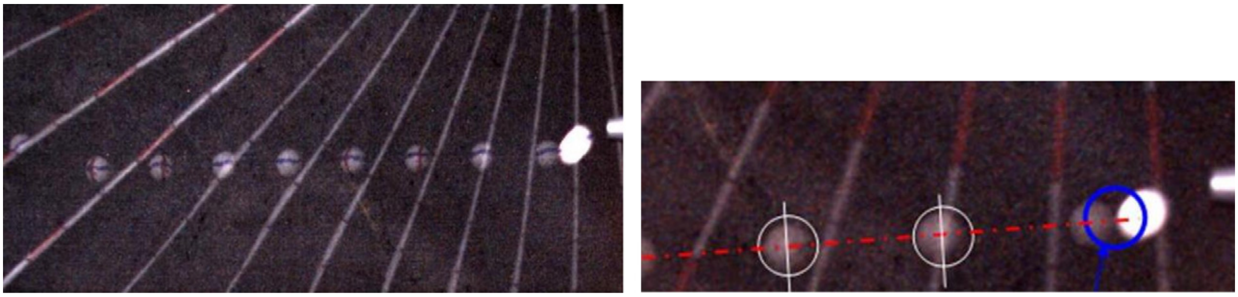

Fig. 2. Left: the bead rotates on itself (redline) around one axis (blue line). Right: region of interest for the impact.

– The second setup consists in an airgun which provides a bead with a controlled velocity before its impact on a fixed wall.

From the pictures of the trajectory of the bead taken by a numerical camera, one can detect by image analysis the centers of the particles and measure the distances between particles in order to deduce the velocities. For the determination of the angular velocity, we designed on each particle a colored cross when preparing the experiment. We track this cross along the trajectory until the bead has done one complete rotation. We deduce thus a framing of the angular velocity of the bead. A statistics on 45 samples has been conducted for each configuration of impact. Each distribution of the results has been interpolated by a Gaussian curve in order to deduce the mean value and the root mean square. The released distance of the particle from the center of rotation of the vane d_0 can be varied from 0 to 300 mm. The angular velocity of the vane Ω can be triggered from 0 to 1500 rpm. The height of release of the particle above the vane h_0 has been varied from 0 to 450 mm. In this study, the width of the vane was maintained fixed equal to $\lambda = 5$ mm. Experimental errors come mainly from the dispersion of the zone of impact. We checked the localization of the impact using a camera at the side of the setup and triggered with an electrovane. The uncertainty on the zone of impact is around 20 mm. As the position of the camera is at 800 mm from the zone of impact, we can estimate this error to be of the order of 2.5%.

3. Results and discussion

Some of the experimental results obtained with a rotating wall (experimental setup 1) and a fixed wall (experimental setup 2) are presented in Fig. 2. In the rotating-wall-experiments, in order to quantify the amount of kinetic energy transmitted by the wall to the bead, we introduce the gain coefficient e_G defined as the ratio between the velocity of the particle after impact and the velocity of the rotating wall at the position of impact: $V_{\text{wall}} = x_0\Omega$. The velocity of the particle after impact was measured for a fixed height of release $h_0 = 50$ mm and various impact positions. From these data, the evolution of the gain coefficient is determined. We also measured the velocity of the particle after impact for a fixed radial position along the wall with various rotation velocities Ω in the range from 100 rpm to 1500 rpm. Note that a very good matching between the two curves is observed (see Fig. 3, left, where the two curves are presented). This implies that the orthoradial velocity of the vane $x_0\Omega$ is the critical parameter in this impact problem. In the fixed-wall-experiments, we used an air-gun to propulse at high velocity a bead against the same wall at a fixed position x_0. We determined from these experiments the restitution coefficient e_r. To present graphically the obtained experimental results in Figs. 3 left and right, we used the following simple relation between the gain coefficient and the restitution coefficient: $e_G = 1 + e_r$ (Fig. 3, right). Indeed, the restitution coefficient in the rotating frame can be denoted e.g. by index $'$ and written as

$$e'_r = \frac{V_{\text{particle}} - x_0\Omega}{0 - x_0\Omega} = 1 - e_G$$

where x_0 is the distance from the axis of rotation. If the restitution coefficient particle/wall is invariant with respect to changing frame then this implies that $e'_r = e_r$. In other words we show that inertial effects (Centrifugal and Coriolis) have no significative influence on the impact in the rotating wall experiments for our plastic particles (see [11]).

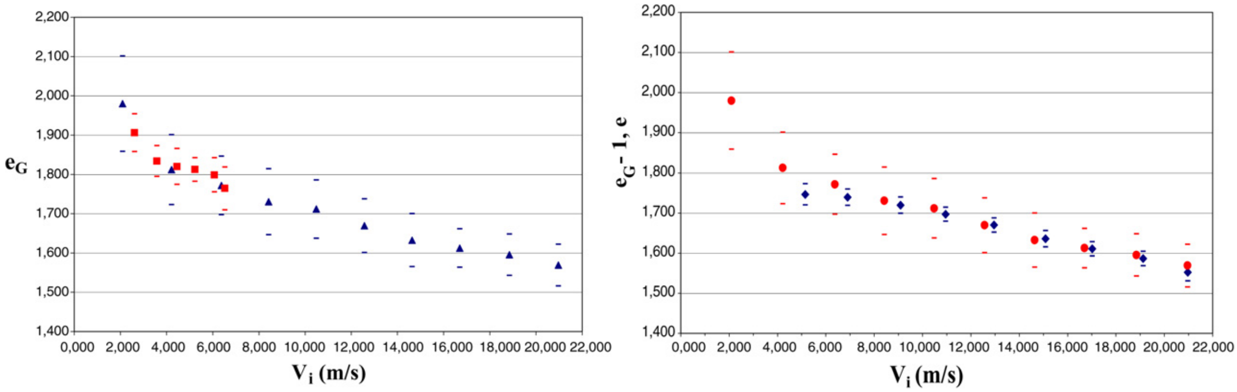

Fig. 3. Left: e_G varying the position x_i (circles) and the rotation velocity Ω (squares). $V_i = x_i\Omega$; Right: Evolution of the restitution coefficient $(1 + e_r)$ and the gain coefficient e_G with V_i.

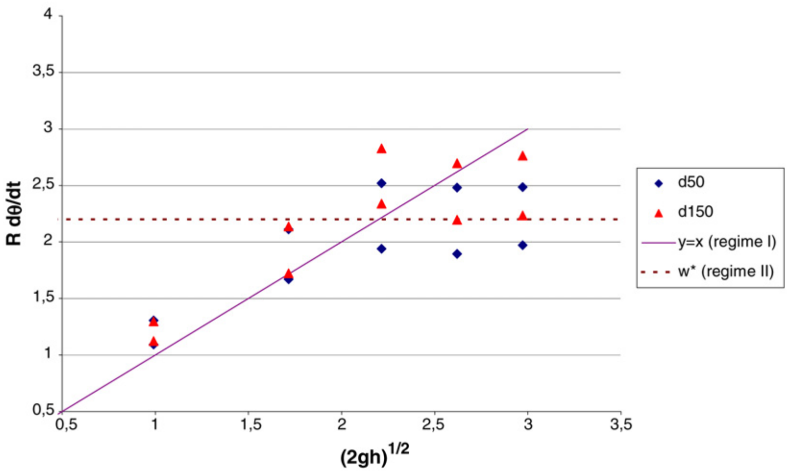

Fig. 4. Evolution of the angular velocity Vs of the particle after impact as a function of falling velocity $\sqrt{2gh}$.

In Fig. 3, left, the triangles represent the evolution of e_G as a function of the impact velocity $V_i = x_i\Omega$ where the position of the impact x_i has been varied. The squares represent the evolution of e_G as a function of impact velocity varying the rotation velocity of the wall Ω.

In Fig. 3, right, the losanges represent the evolution of the gain coefficient e_G with impact velocity measured from the impact with the rotating wall. The disks represent the evolution of the gain coefficient e_G with impact velocity measured from impact experiments on a fixed wall.

In Fig. 4, the losanges represent a framing of the evolution of angular velocity with respect to the fall velocity for an initial impact position at $x_0 = 50$ mm. The triangles represent the same framing for an initial impact position at $x_0 = 150$ mm. We propose a linear interpolation to fit the data for low velocities of fall $\sqrt{2gh}$. The results show a decrease of the gain coefficient e_G with increasing velocity. This observation is consistent with increasing energy dissipation. The following interpolation can be obtained: $e_G = 1 + f(v)$ where $f(v)$ is equal to $f(v) = \alpha V^{\beta}$ with $\alpha \approx 1219$ and $\beta = -\frac{-1}{4}$. The decrease with respect to the impact velocity can thus be described by the theory of Johnson for an inelastic impact [12].

We explored also the influence of the height of release h_0 of the particle which induces a tangential motion of the bead with respect to the normal to the wall and hence an angular velocity on the particle. The experiment has been conducted for two different radial distances: $x_0 = 50$ mm (d50) and $x_0 = 150$ mm (d150). We observe a roughly linear increase of the angular velocity with the initial vertical velocity $\sqrt{2gh}$ (regime II) and then a saturation of the value of the angular velocity (regime I, see Fig. 4). Using a rigid body mechanics description for gross sliding, the particle moves during impact according to the three following equations:

$$Vz_f - Vz_i = -\frac{\mu}{m}\int N\,\mathrm{d}t, \quad \frac{1}{m}\int N\,\mathrm{d}t = \frac{\mathrm{d}y}{\mathrm{d}t}_f - \frac{\mathrm{d}y}{\mathrm{d}t}_i = ex_0 * \Omega, \quad I(w_f - w_i) = R * \mu * \int N\,\mathrm{d}t \tag{1}$$

such that the condition for rolling without sliding at the end of the contact is: $Vz_f = R * w_f$. This gives us $Vz_i = \frac{7}{2}\mu ex_0\Omega$. In Eq. (1), Vz_i and Vz_f are the vertical components of the velocity at the beginning and at the end of the impact, respectively. N is the normal component of the contact force between the bead and the rotating plate (the Coulomb friction law can be written as $T = \mu N$). $\frac{\mathrm{d}y}{\mathrm{d}t}_i$ and $\frac{\mathrm{d}y}{\mathrm{d}t}_f$ are the normal components of the velocity of the bead (with respect to the plane of the rotating plate) at the beginning and the end of the impact, respectively. R denotes the radius of the spherical particle, and I is the moment of inertia of the spherical particle of mass m ($I = \frac{2}{5}mR^2$). w_i and w_f are respectively the angular velocities of the particle at the beginning and at the end of the impact. μ is the Coulomb friction coefficient between the particle and the plate. $\int$ represents the integration over the time of impact of the bead with the rotating wall.

For low sliding velocities, we measured the value of the friction coefficient of the bead by following the trajectory of a raft of 3 beads along the same plate inclined with respect to the vertical which gives us $\mu \approx 0.35$. The value of the restitution coefficient e depends on the impact velocity and can be deduced from the previous experimental curves. e is equal to 0.80 for $x_0 = 50$ mm (from Fig. 3, left). For a distance $x_0 = 50$ mm, using Vz_i, we find that rolling without sliding can appear during impact for vertical velocities lower than $V_{i_{d50}} \approx 3.1$ m/s. This value is coherent with the position of the beginning of regime I (Fig. 3, right).

According to Eq. (1), the rigid body theory in regime (I) predicts that the final angular velocity is independent of the initial vertical velocity and equal to $R.w^\star = \frac{5}{2}\mu ex_0\Omega$. Using the mentioned value of the low velocity friction coefficient gives us a prediction for the angular velocity at $d = 5$ cm: 2.2 m/s; in good agreement with the experimental data available (Fig. 4).

On the other hand, for low values of h_0, the dependence of the angular velocity of the particle on height is a priori unexpected (regime II). Suppose that the bead can roll without any sliding ($R - S$) during all the impact. In this case the velocity of the contact point M between the bead and the wall V_M is equal to zero at every instant. The velocity of a point M belonging to the vane has no component along the vertical axis of the bead and we get: $V_M = \sqrt{2gh} - R\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0$. We can thus predict the angular velocity of the particle after rebound according to the formula $R\frac{\mathrm{d}\theta}{\mathrm{d}t} = \sqrt{2gh}$. For low vertical velocities, this linear dependency between the angular velocity and the fall velocity is in reasonable agreement with experiments (Fig. 3, right).

It suggests a transition between a regime (I) of gross sliding to a regime (II) of rolling without sliding all along the impact. The existence of an intermediate regime between (I) and (II) is also possible in which rolling without sliding starts during the impact. This feature is coherent with Maw's model of impact [5] but does not correspond to Walton's model [4] in which sliding and rolling collisions are mutually exclusive.

4. Conclusion

We have presented in this Note some new experimental results of the impact of a bead on a rotating plate. We observed the conservation of the restitution coefficient by changing frame for our plastic beads such that the inertial effects linked to the rotation of the wall are negligible for our plastic particles. The incident velocity was the crucial control parameter of the impact. Also, a significative influence of the height of release on the angular velocity of the particle after impact was observed. Experiments show that the angular velocity increases with the height of release then saturates above a certain critical height. The constant value of the angular velocity is in agreement with the prediction of a two-parameter-rigid-body model for gross sliding. However, for low heights h_0, the angular velocity increases with h_0. We propose an interpretation for this increase by considering that the particle rolls without sliding during the impact. Note that this specific regime is probable in practice and must be taken into account in problems of collisions involving bodies of very different characteristic dimensions and inertia. More extensive experiments will be carried out in the near future in order to propose a reliable theoretical framework for the impact.

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References

[1] F. Rioual, A. Valance, D. Bideau, Phys. Rev. E 62 (2000) 2450–2459.
[2] M. Sommerfeld, N. Huber, Int. J. Multiph. Flow 25 (1999) 1457–1489.
[3] W. Goldsmith, Impact: The Theory and Behavior of Colliding Solids, E. Arnold (Publishers) Ltd, London, 1960, p. 379.
[4] O.R. Walton, R.L. Braun, J. Rheology 30 (1986) 949–980.
[5] N. Maw, J.R. Barber, J.N. Fawcett, Wear 38 (1976) 101.
[6] S.F. Foester, M.Y. Louge, Hongder Chang, K. Allia, Phys. Fluids 6 (3) (1994) 1108–1115.
[7] A. Le Quiniou, F. Rioual, Y. Lapusta, in: C. Appert, Ph. Gondret, C. Sykes (Eds.), Traffic and Granular Flow '07.
[8] H. Dong, M.H. Moys, Powder Technology 161 (1) (2006) 22–31.
[9] M. Louge, M.E. Adams, Phys. Rev. E 65 (2002) 021303.
[10] C.E. Smith, P.P. Liu, J. Appl. Mech. 59 (1992) 963–969.
[11] A. Le Quiniou, F. Rioual, Europhys. Lett. 82 (3) (2008) 34001.
[12] K.L. Johnson, Contact Mechanics, Cambridge University Press, 1985.