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Wax deposit accumulation in a "cylindrical Couette" geometry

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Abstract

Models used to predict wax deposits overestimate the deposit thickness, and they require fitting parameters to match computational results to experimental data.

A new approach is proposed. Waxy crude oil is considered as a viscoplastic Bingham fluid in which both viscosity and yield stress depend on temperature and quantity of wax crystals. Numerical simulations of the flow in a "cylindrical Couette" geometry were carried out. The numerical results highlight the influence of wax crystal content on the flow pattern, especially when comparing yielded and unyielded regions. A static layer region appears near the colder wall, representing the deposit. ***To cite this article: A. Benallal et al., C. R. Mecanique 336 (2008).***

Résumé

L'accumulation des dépôts de paraffine dans une géométrie « Couette cylindrique ». Les modèles utilisés pour prédire la formation d'un dépôt de paraffine surestiment l'épaisseur de celui-ci et exigent des ajustements de paramètres pour réconcilier calculs et données expérimentales.

Nous proposons une approche complémentaire pour la formation de dépôt. Le brut paraffinique est un fluide de Bingham dont les paramètres dépendent de la température et de la quantité de cristaux. Des simulations numériques d'écoulement dans une géométrie « Couette cylindrique » ont été réalisées.

Les résultats montrent le rôle du mécanisme d'enrichissement et l'influence de la quantité de cristaux sur la structure de l'écoulement, en terme de zones cisaillées/non cisaillées. Une couche statique représentant le dépôt apparaît près de la paroi refroidie. ***Pour citer cet article : A. Benallal et al., C. R. Mecanique 336 (2008).***

Keywords: Rheology; Wax crystal; Unyielded regions; Viscoplastic Bingham fluid

Mots-clés : Rhéologie ; Mécanismes d'enrichissement ; Cristaux de paraffine ; Fluide de Bingham ; Zones non-cisaillées

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1. Introduction

The physical properties of Waxy Crude Oils (WCO) cause trouble during production and pipeline transportation, particularly when the ambient temperature is low enough to make the higher molecular-weight n-paraffins crystallize. The temperature at which the first crystals of n-paraffins appear is called the Wax Appearance Temperature (WAT). During the flow of WCO, the main issue is the accumulation of wax deposits along the pipeline wall. Since normal paraffin solubility decreases dramatically with temperature, when the environment temperature is lower than the WAT the normal paraffins crystals appear and migrate to the pipeline wall to form a wax deposit. Consequently, production capacity is reduced, or even blocked.

Noteworthy advancements in the understanding of wax deposits were achieved in the early 1980s [1]. It is commonly accepted in the open literature that molecular diffusion is the main mechanism responsible for wax deposition. It is induced by a spontaneous movement of dissolved particles from an area with a high concentration of dissolved molecules to an area with a low concentration of dissolved molecules. When the pipe wall temperature is less than the WAT, wax crystals appear at the wall which induces a radial gradient of the dissolved normal paraffin concentration in the pipeline, leading to molecular diffusion from the central core of the flow to the pipe wall. This approach considers the wax deposit to consist entirely of solidified normal paraffin. However, experimental observations note that wax deposits consist of a normal paraffin crystals and crude oil blend. The flow effect on the deposit formation is not taken into account [2].

We postulate that the deposit is a subtle combination of flow patterns caused by rheological factors, thermodynamic properties of waxy crude oils, and diffusion mechanisms from high wax concentration regions to low wax concentration regions. Waxy crude oil is considered as a viscoplastic fluid, an extension of the classical Bingham model, in which both viscosity and yield stress depend on temperature and quantity of wax crystals. The model points out the progressive development of an unyielded region near the wall which corresponds to the wax deposit.

2. Governing equations

In a pipeline, convective and radial diffusion are responsible for the displacement of the dissolved normal paraffin. In order to simplify the problem, we consider a "cylindrical Couette" geometry in which only radial diffusion is considered. Although the calculations for the "cylindrical Couette" are quite different from the pipeline situation where the crude oil is constantly renewed at the pipeline inlet with a constant dissolved normal paraffin concentration, the principle of the enrichment mechanism does not change. Moreover, in this geometry, the wax deposit is formed in the area where the shear stress is maximum as in the pipeline situation.

The governing equations for a temperature dependent viscoplastic flow with an enrichment mechanism in a "cylindrical Couette" geometry can be written as follows:

– The mass conservation and momentum equations:

$$\begin{cases} \vec{\nabla} \cdot \vec{U} = 0 \\ \rho \frac{\partial \vec{U}}{\partial t} + \vec{\nabla} P = \vec{\nabla} \cdot \underline{\underline{\tau}} \end{cases} \tag{1}$$

where ρ, $\vec{U}$, P and τ are density, velocity vector, pressure and deviatoric stress tensor, respectively.

– The modified Bingham constitutive equation is considered. The consistency, μ, and the yield stress, τ_0, depend on temperature Θ and solidified normal paraffin content C_s:

$$\begin{cases} \underline{\underline{\tau}} = 2\mu(\Theta, C_s) \cdot \underline{\underline{D}} + \dfrac{D}{\|D\|} \cdot \tau_0(\Theta, C_s) & \text{if } \|\tau\| > \tau_0(\Theta, C_s) \\ D = 0 & \text{if } \|\tau\| \leqslant \tau_0(\Theta, C_s) \end{cases} \quad \text{if } \Theta \leqslant \Theta_{cc} \tag{2}$$

$$C_s = f(\Theta, C_l)$$

$$\tau_0(\Theta, C_s) = 0 \quad \text{if } \Theta > \Theta_{cc} \tag{3}$$

where $\underline{\underline{D}}$ is the strain tensor, Θ_{cc} is the Wax Appearance Temperature, f represents the thermodynamic solid/liquid equilibrium function and $\| \cdot \|$ is the Euclidean norm.

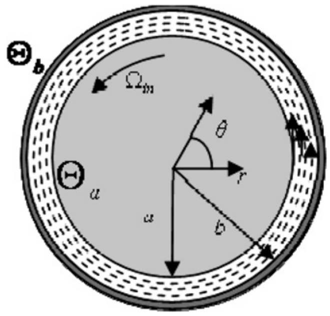

Fig. 1. “Cylindrical Couette” geometry and issue boundary conditions.

Fig. 1. Géométrie de « Couette cylindrique » et conditions aux limites du problème.

– The temperature balance equation can be written as follows:

$$\rho C_f \frac{\partial \Theta}{\partial t} = \lambda_f \nabla^2 \Theta \tag{4}$$

C_f and λ_f are respectively constant heat capacity and thermal conductivity. Viscous dissipation is neglected.

– The Fick diffusion equation for the dissolved normal paraffin is:

$$\frac{\partial C_l}{\partial t} = \vec{\nabla} \cdot (D_m \vec{\nabla} C_l) - Pu \quad \text{and} \quad Pu = \frac{\partial C_s}{\partial t} \tag{5}$$

C_l and C_s are the dissolved and solidified normal paraffins concentration, respectively, D_m is the constant normal paraffin diffusion coefficient, and Pu is the sink term. This term is equal to the kinetic appearance of solidified normal paraffin $\frac{\partial C_s}{\partial t}$, and is generally proportional to C_s.

The “cylindrical Couette” geometry is considered (see Fig. 1). A 1D approach is used because the flow is axisymetric, axially homogeneous which means that the border effects are neglected.

Initially, the fluid and the outer cylinder wall have the same temperature $\Theta_b > \Theta_{cc}$. The inner cylinder wall at temperature $\Theta_a \leqslant \Theta_{cc}$ has a constant rotation velocity Ω_{in}. The outer cylinder is static. No slip conditions are imposed at both cylinder walls. The initial concentration of dissolved normal paraffin is C_0. The wall cylinders are impermeable which means that: $\frac{\partial C_l}{\partial r}(a,t) = \frac{\partial C_l}{\partial r}(b,t) = 0$.

3. Numerical resolution

The unknowns of Eqs. (1)–(5) are the velocity, pressure, temperature and concentration of dissolved normal paraffin. The constitutive equation presents a discontinuity which makes difficult to evaluate the distribution of stress in the unyielded regions. A resolution method based on an augmented Lagrangian allows one to account for the Bingham law discontinuity via Lagrangian multipliers. Thus, the unyielded regions are characterized by a rate tensor perfectly equal to zero and they can be identified exactly [3].

The Stokes problem is solved by an Uzawa algorithm associated with the augmented Lagrangian method. The Marchuk–Yanenko method is used to subdivide the unsteady term of the Fick equation (5) [4]. This splits the global problem into several subproblems to decouple difficulties.

4. The data for the Bingham flow “cylindrical Couette” geometry enrichment mechanism

The Bingham flow with the enrichment mechanism is computed with the following simplified data:

– The yield stress depends arbitrarily on the solidified normal paraffin content as follows: $\tau_0(C_s) = BC_s^2$ where B is a constant. as first step, the temperature dependence is ignored.

– The viscosity follows the Arhenius law: $\mu(\Theta) = \mu_0 \exp(\frac{E_a}{R}(\frac{1}{\Theta} - \frac{1}{\Theta_0}))$ where E_a, R and μ_0 are respectively activation energy, gas constant and reference viscosity at Θ_0. The influence of solidified paraffin content on viscosity is neglected.

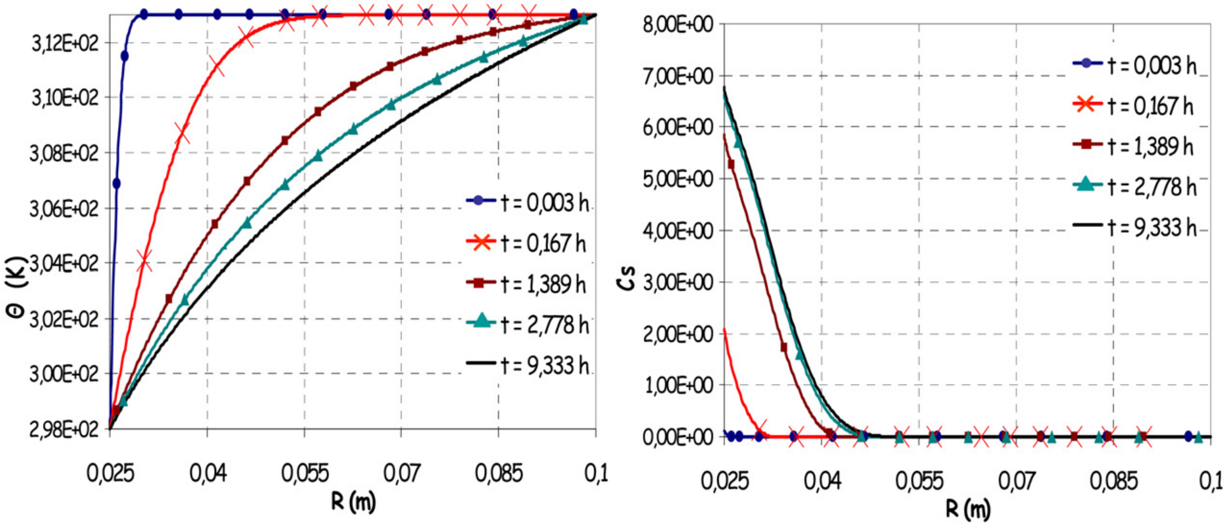

Fig. 2. Left side: Temperature evolution in the gap vs. time. Right side: Solidified normal paraffin distribution in the gap vs. time(s).

Fig. 2. A gauche : Évolution de la température dans l'entrefer pour différents temps. A droite : Distribution de la paraffine solide dans l'entrefer pour différents temps.

– An arbitrary thermodynamic model is used to determine the solidified normal paraffin vs. temperature: $C_s = AC_l(\Theta^{-2} - \Theta_{cc}^{-2})$ if $\Theta \leqslant \Theta_{cc}$ where A is a constant.
– The sink term is expressed as: $Pu(C_l^{n+1}) = k' C_s^{n+1}(\Theta, C_l^{n+1})$ where k' is a constant which may be determined using thermodynamic experiments. An arbitrary value has been chosen here.

5. Results and discussions

The temperature profile within the gap at successive time steps is presented Fig. 2 left. The steady regime is reached at about 9 hours and 30 minutes.

There is a deficit of dissolved normal paraffin near the inner cylinder. Thus the enrichment mechanism of dissolved normal paraffin starts from the outer cylinder ("hot" surface) to the inner cylinder ("cold" surface). The distribution of solidified normal paraffin in the gap is presented in Fig. 2 right. It is observed that the normal paraffin, initially dissolved in the crude oil, is solidified entirely near the inner cylinder wall after about 3 hours.

Fig. 3 left represents the velocity profile in the gap as a function of time. A layer develops progressively which turns at the same angular velocity as the inner cylinder. This region behaves as a solid and can be considered as a wax deposit.

The deposit thickness and the location of the Θ_{cc} isotherm within the Couette gap as a function of time are plotted in Fig. 3 right. It is noticed that the wax deposit accumulates progressively and reaches a constant thickness after about 3 hours. This corresponds to the time it takes for the liquid paraffin to change into solidified paraffin. It is noted that this thickness is less than the one obtained by looking at the Θ_{cc} isotherm evolution. In addition, if one considers that the deposit is only constituted of crystallized paraffin, the thickness value is $e_0 = 0.013$ m which is less than the computed asymptotic value. This confirms, as mentioned in the introduction, that the deposit is a blend of solidified paraffin and crude oil.

6. Conclusions

An advanced approach to predict the wax deposit thickness in crude oil transportation has been proposed.

A one dimensional model is proposed in a "cylindrical Couette" geometry where the convective terms may be neglected. A Bingham viscoplastic behaviour is considered, that depends on temperature and solidified paraffin content. The augmented Lagrangian method is used to model the discontinuity of the Bingham law and therefore the unsheared regions are described exactly. Moreover the Marshuk–Yanenko method is used to solve the Fick law with a sink term.

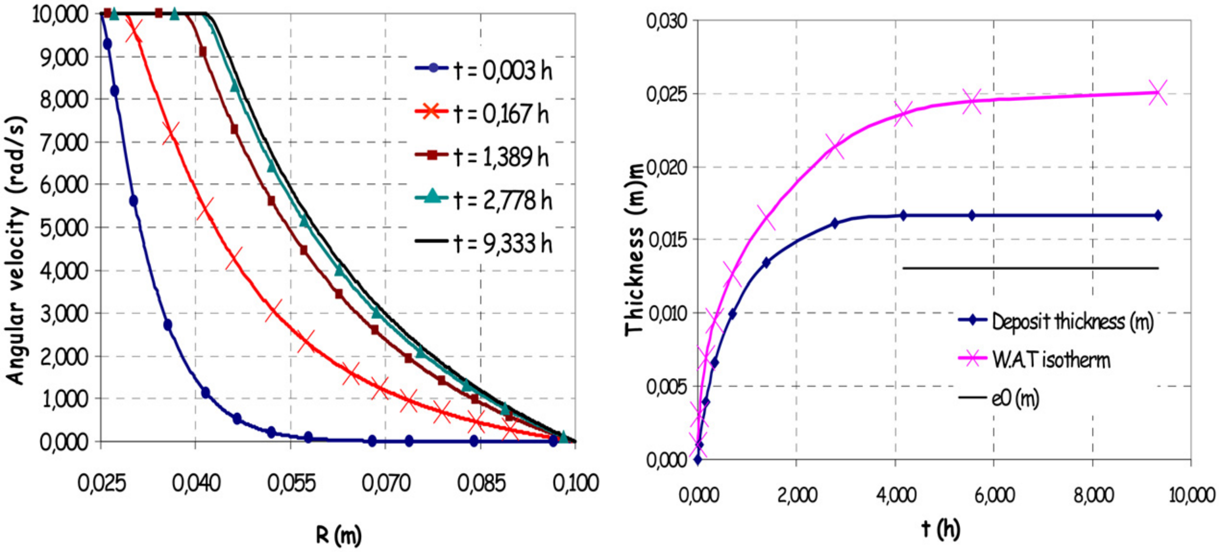

Fig. 3. Left: velocity profile in the gap as a function of time. Right: deposit thickness and location of the Θ_{cc} isotherm within the gap as a function of time.

Fig. 3. A gauche : Évolution de la vitesse angulaire dans l'entrefer pour différents temps. A droite : Évolution de l'épaisseur du dépôt de paraffine et de la position de l'isotherme Θ_{cc} dans l'entrefer de la géométrie de Couette en fonction du temps.

The enrichment mechanism works well: the dissolved normal paraffin concentration is diminished from the crude oil, and the paraffin crystals concentration is augmented near the "cold" wall, changing locally the crude oil behaviour. Moreover, the wax deposit accumulates with time along the "cold" wall until reaching a constant thickness when the dissolved normal paraffin is completely crystallized.

For further developments, the pipeline geometry will be considered which requires a 2D model and coupling between radial diffusion and convective diffusion for the dissolved paraffin.

References

[1] E.D. Burger, T.K. Perkins, J.H. Striegler, Studies of wax deposition in the Trans Alaska pipeline, Journal of Petroleum Technology 33 (1981) 1075–1086.

[2] Ph. Maurel, P. Gâteau, F. Brucy, A. Barbey, N. Bonnet, E. Peuriere, Bruts paraffiniques, Maîtrise des dépôts, Tests en boucle, 2000, rapport interne IFP.

[3] G. Vinay, A. Wachs, J.F. Agassant, Simulation numérique d'un écoulement de fluide viscoplastique thermodépendant : application au transport des bruts paraffiniques, in : 17éme Congrès Français de Mécanique à Troyes, France, 2005.

[4] G.I. Marchuk, Methods of Numerical Mathematics, Springer, 1975.