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A macro-element for a shallow foundation to simulate Soil–Structure Interaction considering uplift

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Abstract

In earthquake engineering, several approaches exist to take into account Soil–Structure Interaction (SSI): the following work is based on the "macro-element" concept. The particularity of the macro-element lies in the fact that the movement of a foundation is entirely described by a system of generalised variables (forces and displacements) defined at the foundation centre. The non-linear behaviour of the soil is reproduced using the classical theory of plasticity. This Note improves an already existing macro-element by adding the uplift behaviour of the foundation according to the plasticity theory. Comparisons with experimental results of a foundation submitted to cyclic loadings show the performance of the approach. ***To cite this article: S. Grange et al., C. R. Mecanique 336 (2008).***

Résumé

Un macro-élément d'Interaction Sol–Structure pour une fondation superficielle prenant en compte un mécanisme de décollement. Dans le domaine du génie parasismique, plusieurs méthodes existent pour considérer l'Interaction Sol–Structure : ce travail est basé sur le concept de « macro-élément ». La particularité du macro-élément réside dans le fait qu'il est formulé en variables globales (forces et déplacements) décrites au centre de la fondation. Le comportement non-linéaire du sol est reproduit selon la théorie de plasticité. Cette Note propose une amélioration d'un macro-élément existant dans la littérature par la prise en compte du décollement de la fondation selon la théorie de la plasticité. Des comparaisons avec des résultats expérimentaux d'une fondation soumise à des chargements cycliques montrent la performance de cette approche. ***Pour citer cet article : S. Grange et al., C. R. Mecanique 336 (2008).***

Keywords: Soils; Soil–structure interaction; Uplift; Macro-element; Foundation; Plasticity

Mots-clés : Sols ; Interaction sol–structure ; Décollement ; Macro-element ; Fondation ; Plasticité

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Dans le domaine du génie parasismique, l'Interaction du Sol avec la Structure (ISS) est un phénomène important à considérer pour espérer rendre compte du comportement réel d'une structure et donc évaluer sa vulnérabilité. Plusieurs méthodes existent pour prendre en compte cette interaction. Basée sur le concept de « macro-élément », la formulation presentée dans nos anciens travaux permettait de modéliser, moyennant un adimensionnement adéquat des variables, une fondation superficielle de forme circulaire reposant sur un massif de sol semi infini en considérant la plasticité du sol [1]. Dans cet article, une amélioration du macro-élément initial est proposée par la prise en compte du décollement de la fondation formulé aussi suivant la théorie de plasticité. La particularité de ce décollement est qu'il est considéré avoir un comportement non-linéaire et non-réversible (Fig. 3(a)) et que les deux sens de chargements (moment positif ou négatif) sont découplés. En d'autres termes, deux surfaces de charge évoluant en chargement et en déchargement peuvent être actives (l'Éq. (6)). L'évolution des variables d'écrouissage est différente en chargement et en déchargement (l'Éq. (5)). Du fait de l'apparition de décollements résiduels dus à la plastification du sol (Fig. 3(b)), le domaine élastique peut totalement disparaitre ce qui conduit à l'activation simultanée des deux mécanismes (Fig. 4).

Le nouveau macro-élément est implémenté dans FEDEASLab, un code élément finis développé dans Matlab [2]. La comparaison avec des résultats expérimentaux d'une fondation soumise à des chargements cycliques [3] montre l'importance de la composante de décollement notamment dans le calcul du soulèvement du centre de la fondation (Fig. 5).

1. Introduction

In the field of earthquake engineering, Soil–Structure Interaction (SSI) is a phenomenon that has to be taken into account in order to reproduce correctly the non-linear behaviour of a structure and thus to be able to predict its relative displacements at the top. Several methods exist: the macro-element approach consists in condensing all non-linearities into a finite domain. It works with generalised variables (forces and displacements) at the centre of the foundation, thus allowing decreasing the necessary degrees of freedom of the numerical model.

Several macro-elements can be found in the literature [4–6]. The macro-element presented in [1] reproduces the behaviour of a shallow circular foundation considering the plasticity of the soil. An extension is introduced in this Note considering the uplift of the foundation according to the plasticity theory. The macro-element is implemented into FEDEASLab, a finite element MATLAB toolbox [2]. After the mathematical description of the macro-element, numerical results compared with experimental tests under cyclic loadings [3] are provided to show the performance of this new numerical tool.

2. Associated generalised variables

As usual is the case for a macro-element, the associated generalised variables (displacement and force vectors) are dimensionless [1]. The are defined hereafter: vertical force V', horizontal forces H'_x, H'_y and moments M'_x, M'_y, but also the corresponding displacements, vertical settlement u'_z, horizontal displacements u'_x, u'_y and rotations θ'_x, θ'_y. Torque moment (M'_z) is not taken into account (Fig. 1).

3. Decomposition of three mechanisms: elasticity plasticity and uplift

The new SSI macro-element takes into account three different mechanisms: elasticity, plasticity of the soil and uplift of the foundation. The total displacement must thus be decomposed as a sum of an elastic, plastic and uplift part:

$$\underline{\mathbf{u}} = \underline{\mathbf{u}}^{\mathbf{el}} + \underline{\mathbf{u}}^{\mathbf{pl}} + \underline{\mathbf{u}}^{\mathbf{up}} \tag{1}$$

Uplift is defined as the negative vertical displacement of the centre of the foundation. It is the result of rocking, i.e. the fact that the foundation rotates around θ_x or θ_y (part of the foundation looses contact with the soil), see Fig. 2. In order to compute uplift, the simple plasticity of the soil is not sufficient and a new non-linear mechanism must be introduced in the macro-element. The reason is that the actual plasticity mechanism of the macro-element can take into account only positive values of the vertical displacement ($u'_z > 0$). Plasticity and uplift are strongly coupled [4].

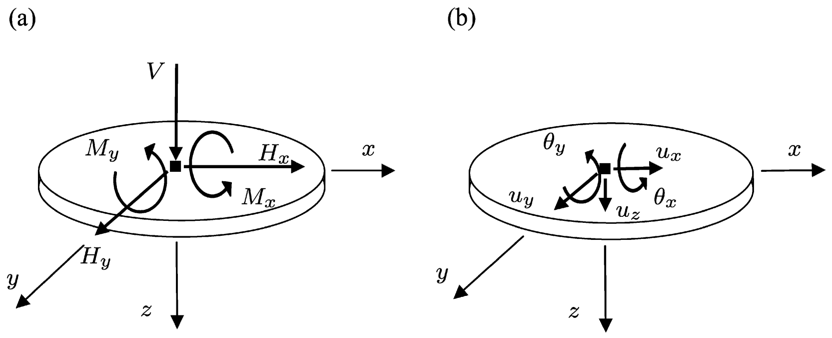

Fig. 1. Generalised variables: (a) forces and (b) displacements for a circular foundation.

Fig. 1. Variables généralisées : (a) forces et (b) déplacements pour une fondation circulaire.

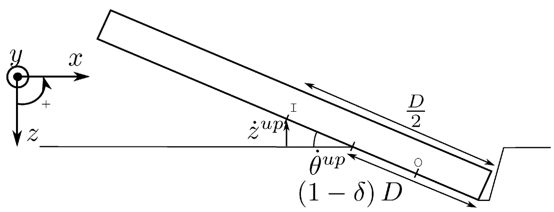

Fig. 2. Kinematics of a foundation for the uplift mechanism.

Fig. 2. Cinématique de la fondation pour un mécanisme de décollement.

4. Mathematical description of the macro-element

4.1. Elastic behaviour and plastic mechanism

The elastic and plastic mechanisms are very briefly described hereafter. For more information the reader is invited to look in [7,1].

The elastic part of the constitutive law is defined as $\underline{\mathbf{F}} = \underline{\underline{\mathbf{K}}}^{\mathbf{el}}\underline{\mathbf{u}}^{\mathbf{el}}$, where the displacement $\underline{\mathbf{u}}^{\mathbf{el}}$ and force vectors $\underline{\mathbf{F}}$ are dimensionless. The elastic stiffness matrix $\underline{\underline{\mathbf{K}}}^{\mathbf{el}}$ is calculated using the real part of the static impedances. It is considered diagonal, i.e. there is not coupling between the different directions of the loading.

For the plastic mechanism, a failure criterion, a loading surface and a flow rule are needed. The failure criterion is defined for an overturning mechanism with uplift. It comes from [8]. The adaptation for a 3D loading is presented in [7]. It relies on the fact that failure criteria are similar for different shapes of foundation when described into the appropriate space of dimensionless variables. Thus, the extension to the case of a circular foundation is straightforward due to the axial symmetry of the problem. For a rectangular foundation however, we assume continuity of the behaviour between the two principal axes of the foundation. This is of course a more questionable assumption that needs improvement in the future. However, due to the nature of the macro-element (being a simplified numerical tool), we consider this hypothesis acceptable. The loading surface follows the same philosophy and one obtains 5D surfaces. A flow rule is finally needed. It is associated in the (H'_x, M'_y, H'_y, M'_x) hyperplane and non-associated in the (H'_x, V'), (M'_y, V'), (H'_y, V'), (M'_x, V') planes [7].

4.2. Uplift behaviour

The mechanism presented hereafter describes in a phenomenological way uplift using a unique state variable δ. This variable represents the percentage of the surface of the uplifted footing [4], Fig. 2 (for any a, $\dot{a}$ defines the derivative with respect to time). Indeed, this is the only way to introduce the influence of the change of the geometry, the macro-element being just a point. We assume that uplift is not influenced by horizontal forces.

For 2D loading, the behaviour of the foundation during uplift is presented in Fig. 3. This figure presents the relation between $M' - \delta$ (M' being M'_x or M'_y), see also [4]. One can clearly see that the behaviour is not reversible and that the

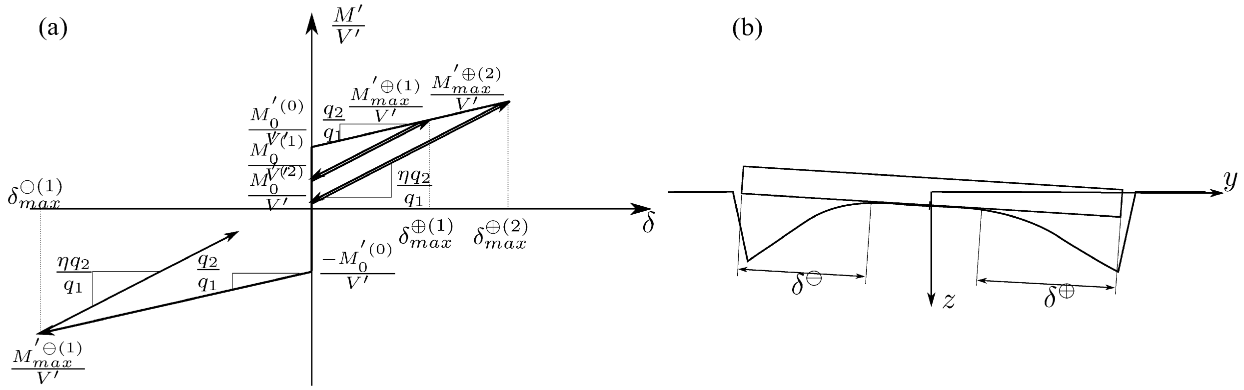

Fig. 3. Moment-δ relationship for a plastic soil.

Fig. 3. Evolution moment-décollement sur sol plastique.

unloading slope ($\eta q_2/q_1$) is increased with respect to the original loading one (q_2/q_1). The two directions of loading ($M' > 0$ or $M' < 0$) are uncoupled. Furthermore, this behaviour is physically translated into a residual uplift at each edge of the foundation, see [1] and Fig. 3(b). The symbols $\oplus$ and $\ominus$ are used to discriminate the directions of loading.

For a circular foundation $q_1 = 6$, $q_2 = 2$, for a rectangular foundation $q_1 = 4$, $q_2 = 1$ and the evolution of η is provided by [4]:

$$\eta = 4 - 3\mathrm{e}^{-4V'} \tag{2}$$

4.2.1. Failure criterion and loading surfaces

For the uplift mechanism, failure occurs when the foundation is completely detached from the soil, i.e. when $\delta = 1$. A simple analytical analysis for a circular foundation lying on elastic soil allows finding the relation $M' = V'/2$ between the overturning moment and the given vertical force. This equation can be actually considered as a failure criterion. On a plastic soil, the relationship between the overturning moment and the vertical force is more complicated [4]. For a loading in two different directions (for $M' > 0$ and $M' < 0$) we obtain [1]:

$$f_\infty \equiv M'^2 - \left(\frac{V'}{q_1}\left(\mathrm{e}^{-AV'} + q_2\right)\right)^2 = 0 \tag{3}$$

where $A = 2.5$ is a dimensionless parameter of the constitutive law.

The uplift behaviour being a non-linear, non-reversible mechanism with the unloading slope increased with respect to the original loading one, the evolutions of the loading surfaces are activated even during unloading. Nevertheless, an initial elastic domain often exists (except if the loading is important leading to high non-linearities in the soil and thus to a total suppression of the elastic domain).

In order to activate the loading surfaces in loading but also in unloading, their mathematical expression is chosen to be always positive whatever the sign of the loading. For the case of the direction $\oplus$, the corresponding hardening variable $\beta^\oplus$ evolves between δ and $\delta_{\max}$ (maximal percent of uplift reached during the loading). It is defined as:

$$\beta^\oplus = \delta_{\max}(1 - \eta) + \eta\delta \tag{4}$$

The evolution of the hardening variable $\beta^\oplus$ is also given by Eq. (5) as a function of the rotation due to uplift θ'^{up} and the rotation θ'_0 related to the initial overturning moment, see also Fig. 2 ($\beta^\oplus_{\max}$ being the maximal value of $\beta^\oplus$ reached during the loading).

$$\begin{cases} \dot{\beta^\oplus} = \dfrac{\dot{\theta}'^{up}}{\theta'_0}\dfrac{(1-\beta^\oplus)^2}{\beta^\oplus(2-\beta^\oplus)} & \text{if } \beta^\oplus = \beta^\oplus_{\max} \\ \dot{\beta^\oplus} = \dfrac{\dot{\theta}'^{up}}{\theta'_0}\eta\dfrac{\left(1-\left(\frac{\beta^\oplus-(1-\eta)\beta^\oplus_{\max}}{\eta}\right)\right)^2}{\frac{\beta^\oplus-(1-\eta)\beta^\oplus_{\max}}{\eta}\left(2-\left(\frac{\beta^\oplus-(1-\eta)\beta^\oplus_{\max}}{\eta}\right)\right)} & \text{if } \beta^\oplus \leqslant \beta^\oplus_{\max} \end{cases} \tag{5}$$

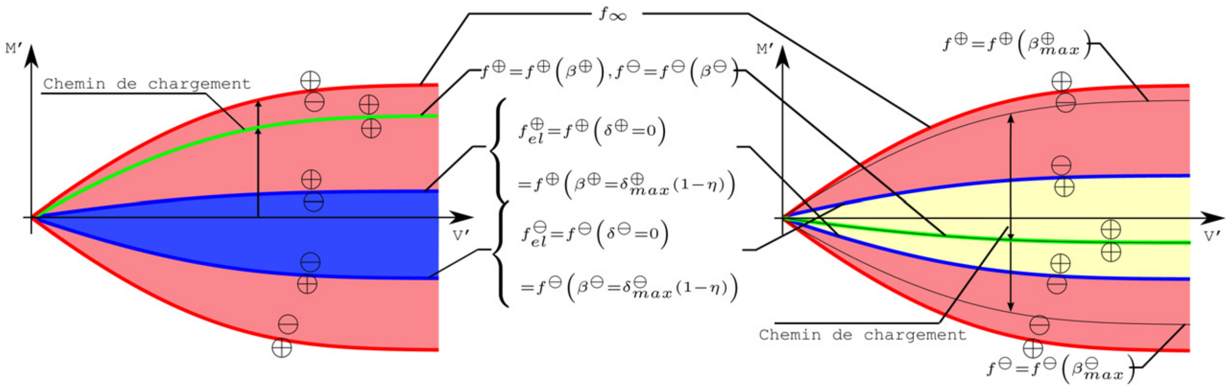

Fig. 4. Loading surfaces, failure criterion and elastic limits with their signs.

Fig. 4. Surfaces de charge, critère de rupture et limites élastiques avec leurs signes.

The mathematical expression of the loading surface for the direction $\oplus$ is finally provided by Eq. (6).

$$f^{\oplus} \equiv \left| M' - \frac{V'}{q_1}\left(\mathrm{e}^{-AV'} + q_2\beta^{\oplus}\right)\right| = 0 \tag{6}$$

It is also necessary to introduce the surface defining the elastic limit zone. The loading surfaces being always positive, a test on the elastic surface allows knowing in which mechanism (uplift or elastic) is the model. The elastic limit is a function of the maximum percentage $\delta_{\max}$. Its mathematical expression is given in Eq. (7):

$$f_{\mathrm{el}}^{\oplus} \equiv M' - \frac{V'}{q_1} q_2 \beta_{\max}^{\oplus}(1-\eta) - \frac{V'}{q_1}\mathrm{e}^{-AV'} = 0 \tag{7}$$

If residual uplift occurs on the $\oplus$ side of the foundation, the elastic domain entirely disappears. Conditions for choosing the activated mechanisms are given by the equations:

$$\begin{cases} f_{\mathrm{el}}^{\oplus}(M', V') \leqslant 0 \quad \text{or} \quad f^{\oplus}(M', V', \beta^{\oplus}) = 0 \ \Rightarrow \ \text{elasticity} \\ f_{\mathrm{el}}^{\oplus}(M', V') > 0 \quad \text{and} \quad f^{\oplus}(M', V', \beta^{\oplus}) > 0 \ \Rightarrow \ \text{uplift} \end{cases} \tag{8}$$

The same equations can be written for the other direction by replacing $\oplus$ with $\ominus$. The graphical representation of all the surfaces is finally given in Fig. 4.

As for the plasticity mechanism, the adaptation of the uplift mechanism in 3D is done assuming continuity of the behaviour between the two principal axes of the foundation (the two directions are coupled considering a projection of the moment in the principal base [1]). Coupling of the plasticity and the uplift mechanisms is done following the classical theory of the multi-mechanisms [1]. The choice and the computation of the normal vectors on the different plasticity mechanisms follow the algorithm presented in [1] and in [9].

4.2.2. Flow rule

The flow rule for the uplift mechanism is found through geometrical considerations, assuming that the centre of rotation of the foundation stays always at the middle of the non-uplifted segment (Fig. 2, [1]). Its mathematical expression takes the following form (β is equal to $\beta^{\oplus}$ or $\beta^{\ominus}$):

$$\begin{cases} \dfrac{\partial g}{\partial M'} = \dfrac{f_g}{|f_g|} \\ \dfrac{\partial g}{\partial V'} = -\dfrac{\delta}{2}\dfrac{f_g}{|f_g|} \end{cases} \quad \text{with } f_g \equiv M' - \frac{V'}{q_1}\left(\mathrm{e}^{-AV'} + q_2\beta\right) = 0 \tag{9}$$

5. Numerical simulations and comparisons with experimental results for a cyclic loading

Within the European program TRISEE, several experimental tests are performed on a shallow 1 m × 1 m rectangular foundation lying on sand [3]. Two tests are presented hereafter concerning different types of sand (High density sand

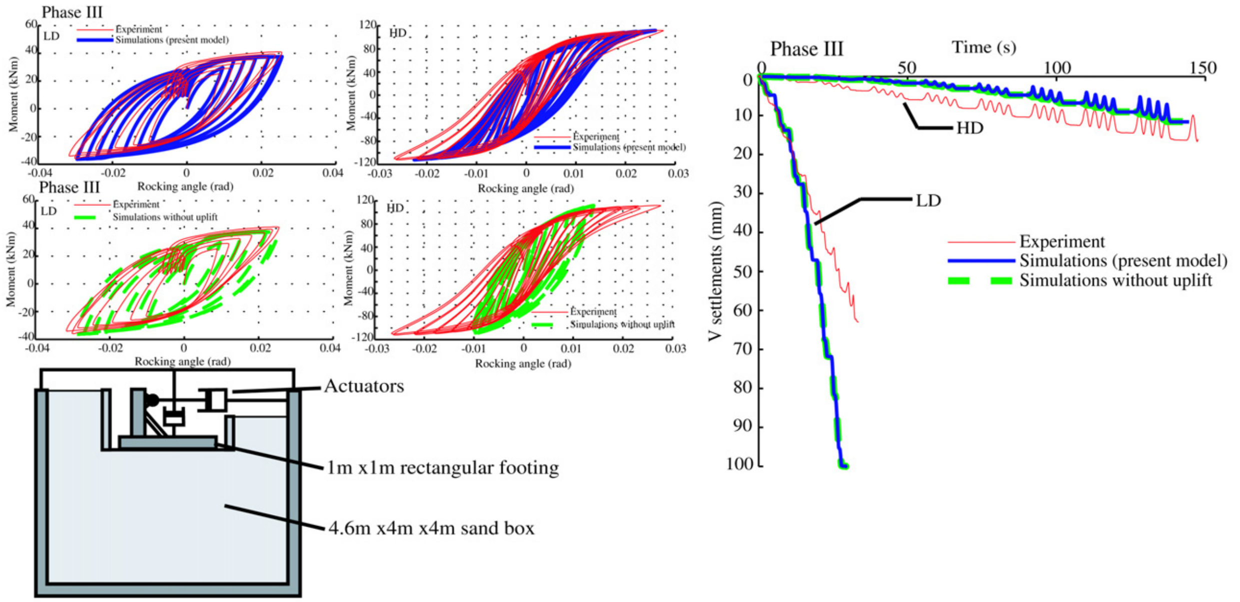

Fig. 5. Tests TRISEE, moment–rotations and time history of the vertical settlements for the High density (HD) and the Low density sand (LD) for the phase III.

Fig. 5. Tests TRISEE, moment–rotations et déplacement vertical en fonction du temps pour le sable haute densité (HD) et le sable de faible densité (LD) pour la phase III.

(HD) and Low density sand (LD)). Phase III (where the loading was the most severe) of the experimental program is simulated using the macro-element. Fig. 5 shows the comparison of the experimental with the numerical results. One can identify the influence of the uplift mechanism looking at the S-shaped moment–rotation curve. Rotations due to the plasticity of the soil are found almost equal to the ones coming from uplift (phase III with HD sand). In other words, uplift and plasticity of the soil have similar contributions on the moment–rocking angle curve. The waves present in the settlement curve are also not reproduced if uplift is not taken into account.

6. Conclusions

The 3D macro-element developed within this work gives satisfactory results for simulating the non-linear behaviour of shallow rigid foundations lying on an infinite space submitted to a cyclic loading. It takes into account the plasticity of the soil and the uplift of the foundation. Using global variables it presents the advantage of inducing low computational costs (couple of minutes for each simulation).

All the results presented in this paper are for loadings in a plane (2D). The 3D behaviour of the element has not been validated due to the difficulty to find experimental results with loadings in 2 horizontal directions. This point should constitute a future validation for the element.

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