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A nodal boundary elements formulation. Application to the solution of the Laplace equation for irrotational flows

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Abstract

The Note presents an unconventional computational method for irrotational and incompressible fluid flows over lifting bodies. At first, Laplace's equation for the velocity potential is solved with internal Dirichlet conditions expressed at the nodes of the mesh rather than at smooth surface positions. Continuous distributions of surface normal doublets are used, and obtaining the surface velocity field with such distributions becomes straightforward. Secondly, an original Neumann type formulation of the Kutta conditions is proposed. Expressing the minimization of the velocity flux across the wall shows a significant reduction of the discretization impact upon the computed global efforts when compared to local no-load conditions. The method can be applied to 2 or 3-dimensional flows, steady or not. ***To cite this article: P. Ardonceau, C. R. Mecanique 337 (2009).***

Résumé

Une procédure nodale de résolution numérique de l'équation de Laplace. Application aux écoulements à potentiel de vitesse. La Note présente une méthode non-conventionnelle de calcul de l'écoulement de fluide parfait incompressible autour de corps portants. D'abord l'équation de Laplace régissant le potentiel de la vitesse est résolue avec des conditions de Dirichlet internes exprimées aux nœuds du maillage plutôt que sur des parties lisses de la surface. La méthode est applicable moyennant l'utilisation de répartitions surfaciques continues de couches de doublets, avec lesquelles l'obtention du champ de vitesse pariétale devient trivial. Ensuite une formulation originale de type Neumann des conditions de Kutta est proposée. Exprimée sous forme d'une minimisation du flux pariétal de la vitesse, elle entraîne une réduction sensible de l'impact de la discrétisation sur l'estimation des efforts globaux par rapport aux formulations locales. La méthode est applicable aux écoulements bi ou tri-dimensionnels, stationnaires ou non. ***Pour citer cet article : P. Ardonceau, C. R. Mecanique 337 (2009).***

Keywords: Fluid mechanics; Dirichlet; Doublets; Flow; Kutta; Laplace; Nodes; Potential

Mots-clés : Mécanique des fluides ; Dirichlet ; Doublets ; Écoulement ; Kutta ; Laplace ; Nœuds ; Potentiel

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1. Introduction

The vast majority of problems discussed today in fluid mechanics deal with complex viscous flows, possibly detached and compressible. There remain some marginal issues for which the basic inviscid/incompressible fluid flow model is suitable, especially at aero-acoustics and aero-structural interfaces. The work presented in this Note refers to an aerodynamic optimization under structural constraint, the induced drag reduction [1]. This type of drag, which comes from the vortical wake generated by a lifting surface, originates from the inviscid fluid model which is therefore a natural tool to compute it.

The irrotational "and incompressible" flow over lifting or non-lifting bodies is controlled by the Laplace's equation and the boundary element method (BEM) provides an interesting alternative to finite difference or finite element methods by reducing the spatial dimension of the problem [2,3], a crucial aspect for the integration of a code into an iterative process. Applied to fluid dynamics, the boundary element method, generally known as the panel method, became popular at the end of the sixties with the advent of computers [4–6].

The first panel methods were based on the velocity formulation [4]. Point or distributed elements whose induced velocity field is known for a unit intensity, such as source-sinks, vortices, doublets, are distributed over the body surface. The actual intensity is derived from the resolution of a linear system built up by writing the velocity slip condition $\vec{V} \cdot \vec{n} = 0$ over the body surface.

A second family of methods has subsequently emerged, with the velocity potential Φ as the primary variable. In this case, the influence of surface singularities is expressed by their induced potential and the Neumann conditions ($\partial\Phi/\partial n$ at the wall, where n symbolizes the direction normal to the wall) are then replaced by internal Dirichlet conditions ($\Phi_{\mathrm{i}} = Cte$) [7].

With both methods, one (2D) or more (3D), constraints must be added for lifting bodies, the Kutta conditions, in order to select the regular "physical" solution among an infinite set of "mathematical" solutions. In practice there is a wide variety of formulations, and selecting one is not trivial in discretized problems.

The panel codes have been widely used in aerodynamics since the 1970s. Numerous light custom codes coexist with heavier industrial codes such as Panair [8], aimed to deal with complete geometries, such aircrafts. The latter uses fairly sophisticated interpolation or patching techniques, in order to provide accurate outcomes despite relatively coarse meshes.

The development of an intermediate procedure was motivated by the fact that basic codes did not provide the required precision to blend into optimization algorithms, especially evolutionist ones, and industrial codes are not suitable to fit into intensive iterative processes.

The original method described in this paper was designed with two objectives: a fast convergence with mesh refinement, to reduce the computation time, and an easy interfacing with finite element structural codes.

2. Overview of the basic "low order" method

The underlying technique of the present development is a "potential" formulation: like the field to calculate, the potential "influence coefficients" (Green functions in the BEM frame) are of scalar type, hence reducing the data storage compared to multi-dimensional coefficients. In its basic formulation, it can be called "0th order", i.e. constant intensity singularities (per segment or panel). The steady Kutta condition is implicitly fulfilled by extending the surface sheets into a semi-infinite constant intensity wake of doublets, attached to the emission point or line (3D case) [7,9]. This is a strong advantage compared to explicit statements of this condition, as it turns out to prescribe equal pressures on both sides of the body at the very trailing edge rather than at mid-mesh upstream. Another interesting feature of a "potential" formulation is the actual knowledge of the potential, required for unsteady pressure calculations, which involves its time derivative.

This potential formulation however generates several adverse counterparts. At first, in order to obtain the more physical surface velocity and pressure fields, the surface potential has to be differentiated, which is not straightforward in the 3D case. Second, it can stressed that the velocity coming out from the differentiation of the potential, a potential method has to be one order higher than a velocity method. In other words, to get a piecewise C^0 surface velocity field, the scalar method has to be 1st order, i.e. linearly varying surface sheet intensity.

Although the proposed method has been applied even to unsteady 3D cases, the following discussion focuses on the two-dimensional steady version that contains the essential features of the problem, hence avoiding lengthy algebraic expressions and 3D visualizations.

2.1. Continuous formulation

We wish to solve the Laplace equation, $\nabla^2\Phi = 0$, which governs the velocity potential Φ of an irrotational and incompressible flow, such that: $\vec{V} = -\vec{\nabla}\Phi$. The body surface is covered with a double layer (normal doublets) of local intensity $\mu(P)$ at point P. The natural slip condition, $\vec{V} \cdot \vec{n} = 0$, which appears as a Neumann condition for the potential, can be replaced by an internal Dirichlet condition [7,9]. Indeed, with a continuous formulation, imposing a constant internal potential, $\Phi_i(P) = Cte$, $\forall P$, leads to $\Phi_e(P) = \mu(P) + Cte$. The external surface potential is therefore equal to the potential jump across the layer, up to some constant. Thereafter, the constant will be set to 0. The normal velocity, invariant across a layer of normal doublets and identically zero inside the body, must also be zero on the outside surface. The internal Dirichlet condition is therefore equivalent to the Neumann zero normal velocity condition for a continuous problem.

If the body has an edge at some point B along it, the external surface potential is no longer uniquely defined and has a jump $[\![\Phi_{\mathrm{TE}}]\!]$, equal to the circulation Γ of the velocity around the body, for any path binding points B_{in} and B_{ex}, where the indices $()_{\mathrm{in}}$ and $()_{\mathrm{ex}}$, state the lower and the upper surfaces. The downstream potential jump is modelled by a semi infinite line of normal doublets with intensity $[\![\Phi_{\mathrm{TE}}]\!] = \Gamma = \mu_{\mathrm{ex}}(B) - \mu_{\mathrm{in}}(B)$. In steady flows, the doublet wake can extend along the trailing edge bisector or along the unperturbed velocity, the wake is then called “frozen”. It can be aligned with the local velocity, a model termed “relaxed”. In steady three-dimensional flows a “frozen” or “relaxed” doublet sheet replaces the doublet line. Generally the choice has little influence on the solution. In unsteady flows the intensity of the sheet varies. Each point keeps the value of the potential jump, at the time and position of the emission, and must be convected with the local velocity (no load on the wake) [11].

2.2. Discrete formulation

The numerical resolution of the internal Dirichlet problem begins with a discretization of the body. Modeling the surface curvature has not shown a significant improvement of the results, and seriously complicates the procedures. Rectilinear segments are therefore used in this work. The profile is divided into N_P segments, of variable length, adapted to the local curvature and decreasing near the trailing edge. The lengths of the two segments on both sides of the trailing edge are moreover constrained to be strictly identical. At 0th order, each segment I wears a distribution of normal doublets of constant intensity $\mu(I)$ and an internal control point at the center of the associated segment.

We note $C(I, J)$ the potential induced at the control point of the segment I by a segment J, with unitary intensity, and $\Phi_\infty(I)$ the unperturbed velocity potential of the flow at the same point I [5]. The internal Dirichlet condition writes:

$$\sum_{J=1,\ J\neq I}^{N_P+2} C(I,J)\mu(J) + \pi\mu(I) = -2\pi\Phi_\infty(I), \quad I = 1, Np$$

In the simplest case the wake is represented by two superimposed semi-infinite straight lines, with constant intensity. The linear system then consists of N_P equations with $N_P + 2$ unknowns. It has to be completed by two relationships expressing the Kutta condition. It is customary to write:

$$\mu_W(1) = \mu(N_P + 1) = \mu(1)$$
$$\mu_W(2) = \mu(N_P + 2) = \mu(N_P)$$

where $(\cdot)_W$ stands for “wake”.

These two elements can be combined as they are geometrically superimposed. The “socket” model is however preferred for logical considerations.

With a continuous formulation and a zero angle trailing edge, this statement of the Kutta condition is fully justified (Joukowski profiles). Vectors normal to the wall are continuous at the transition from surface to wake, on both sides of the trailing edge. The two previous relationships are equivalent to imposing a zero gradient of the surface potential, which expresses the existence of a stagnation point each side of the trailing edge. In the event of a finite trailing edge angle, the justification is only piecewise because there are four different normals at the same point. Nevertheless, in practice, with a discrete formulation, the procedure works fine.

2.3. Problems associated with the basic method

The method, briefly described above, provides a simple and elegant tool to compute the surface potential field (the overall field can be deduced). It has to be later derived to obtain the wall velocity and pressure fields. A first drawback appears: the potential, indeed finite, is discontinuous at junction between elements. The resulting velocity and pressure fields are therefore neither continuous nor even finite at the wall. Such infinite quantities may be troublesome with configurations such as multiple bodies in strong interaction or thin wing sections.

A second, more technical problem of surface differentiation remains in any case. Positioning the control points at the segment centers (panel centroids 3D) defines the potential at a location where its derivative is rather required. The choice of a contour by straight lines (2D) or planar (3D) elements does not accommodate an intrinsic geometrical representation by means of curvilinear coordinates and makes it difficult to build differentiation algorithms, at least in the 3D case. Experience shows that the three-dimensional wall pressure field depends significantly upon the differentiation algorithm. Moreover the use of unstructured meshes implies a complicated management of a neighbouring table between elements.

3. Development of a higher order model

3.1. Logical construction

The "potential" 0th order model, while providing a simple and fast implementation, has not demonstrated the qualities required for a code to be integrated and interfaced in an optimization loop. The aforementioned problems, sensitivity to the surface derivation method and presence of infinite quantities, can be overcome by shifting to a higher order potential model. To keep the procedure simple and as close as possible to the basic formulation, a 1st order model is selected. The intensity of the normal doublet sheet then becomes a linear (2D) or bi-linear (3D) function of the coordinates on each element and is continuous at the nodes. The potential field is therefore also continuous everywhere. The surface velocity, although discontinuous at the nodes, remains finite and can be objectively defined and easily computed. The problem may now be stated as follows: seek the intensity of the doublet sheet at the N_N mesh nodes that verify the slipping condition over the body. The nodes, which are the natural geometric interface variables, also carry the solution of the fluid problem. The linear system (Section 2.2) can be written:

$$\sum_{J=I}^{N_N+2} C(I,J)\mu(J) = -2\pi\Phi_\infty(I), \quad I = 1, N_N$$

where $C(I,I)$ represents the "self-influence" coefficient, i.e. the influence of the value of the doublet intensity of the surface sheet on the control point I [7] (π for two aligned segments, $\neq \pi$ in the general case).

By definition this approach contains a logical problem: positioning the control points. If they are maintained at the centers (centroïds/3D) of the panels, the resulting linear system is generally undetermined. Given N_P panels, one gets a set of N_P equations, the number of unknowns being N_N, the number of nodes. N_P and N_N are linked to the number of edges N_E by the Euler relation.[1] Depending upon the type of mesh, the three cases $N_N > N_P$, $N_N = N_P$, $N_N < N_P$ may arise.

In practice the case $N_N > N_P$ is more frequently encountered (i.e. under-determined linear systems). Several techniques have been evaluated to overcome this problem. One is to add enough internal control points to provide the required number of constraints [10]. This raises the question of choosing the position of these additional points, which seems to have no objective response. A more sophisticated technique is to build "a" solution based on the singular vectors obtained by singular value decomposition of the matrix system, using an optimization criterion. However such solutions systematically exhibit spatial instabilities and the approach was rejected.

On the contrary, if the control points are set at the mesh nodes, then, for a closed body, the Dirichlet problem is well determined (N_N relations for N_N unknowns). It can be solved if the Green's identity is applied at the angular

[1] $N_N + N_P = N_E + \alpha$ where N_E is the number of edges and α the Euler–Poincaré characteristic ($\alpha = 1$ for a plane, $\alpha = 2$ for a sphere, $\alpha = 2-2t$ for a surface with holes, ...).

points, the vertices of the polyhedron, rather than on the plane walls [9]. Note that this method does not apply to a velocity formulation as the surface velocity is not defined at these points.

To sum up, with this option:

- the unknowns become the value of the intensity the doublet distribution at the mesh nodes;
- the surface velocity is unambiguously defined over each panel and simply follows the local intensity law of the doublet sheet;
- the control points being the nodes, no additional geometrical definition and associated supplementary storage are required;
- in case of a geometrical optimization, the deformation of the body can be done simply by moving the nodes, which supports a simple interface.

The method built on these bases will then be called "nodal". Apart from the tricky calculation of the influence from one node on the control point at the same location, it does not contain any difficulty and the computation of the potential flow over non-lifting bodies is straightforward. Obtaining a fast convergence of the solution with the mesh refinement in the lifting case is however less straightforward and requires a special attention to the formulation of the Kutta condition which is outlined below.

3.2. Kutta conditions

The lift force induced by the body upon the flow implies the existence of some circulation Γ which must be specified on "physical" considerations. The velocity potential cannot be continuous everywhere when $\Gamma \neq 0$ and it is a common practice to insert a jump of Φ across the wake originating at the trailing edge (cf. Section 2.1). Therefore Φ is not uniquely defined at the trailing edge and each node along it must be twined. Thus K additional degrees of freedom appear, $N_N \rightarrow N_N + K$, without corresponding increase in the number of relations since the pair of nodes are geometrically coincident. The linear system has to be supplemented by as many relationships as Kutta double nodes.

Originally designed in the framework of two-dimensional flows conformal mapping [12], the Kutta condition is now less restrictively expressed. It turns out to select a circulation that provides a regular flow at the trailing edge by preventing its circumvention. In discrete versions, this is stated by an absence of local load, zeroing the pressure differential across the wing or the wake, close to the trailing edge. But such a circumvention of the edge cannot be observed at smaller scale than the mesh size. The no-load condition being expressed at the half-mesh, this suggests a strong dependency of the computed lift with the mesh size.

To illustrate the problem, Fig. 1 shows the calculated pressure coefficients over the rear side of a NACA 64212 wing section at 10° angle of attack (Fortran codes are run with real*8 data representation). This slightly cambered wing section is selected for its thin trailing edge, a selective computational case. The continuous curve comes from one 0th order calculation with a large number of segments (1000). It is noticed that this method, even applied with a rough discretization (32 segments), provides a lift coefficient, underestimated by only 3.5% (triangular symbols). The 1st order method (circular symbols) underestimates it more than 12%. The Kutta condition, stating equal pressure coefficients at the center of the two terminal segments, is apparent on the graph. Expressed in terms of equal pressures (bottom/top surfaces), the Kutta condition should actually be written at the very trailing edge and not upstream.

A general goal for a discrete method is its rapid convergence towards the continuous solution with increasing number of elements. The fundamental solution to satisfy this criterion would be to shift the order of the model, but it is prescribed from outer considerations in the present work. To check the accuracy of the process at a given panel number N_p, the solution can be compared to the asymptotic "smooth" solution ($N_p \rightarrow \infty$) or to the "exact" solution at given N_p, i.e. the solution over the N_p faces polygon (Fig. 2). The velocity field over such a polygon is indeed singular at the corners, but the singularity does not induce singular efforts [10]. A polygonal reference solution can be simply computed by means of a standard sub-meshing of the body sides (Fig. 2). This way of doing seems more appropriate as there is an infinite set of smooth profiles in which the N_P polygon fits and there is no valid reason to refer to one rather than another.

Among the many attempts to tackle the formulation the Kutta condition with linear doublet distributions, the most obvious answer is to match the extrapolations to the trailing edge of the wall pressure coefficients from both faces (2nd degree Lagrange extrapolation). The lift coefficient at 10° angle of attack for the previous profile, with an

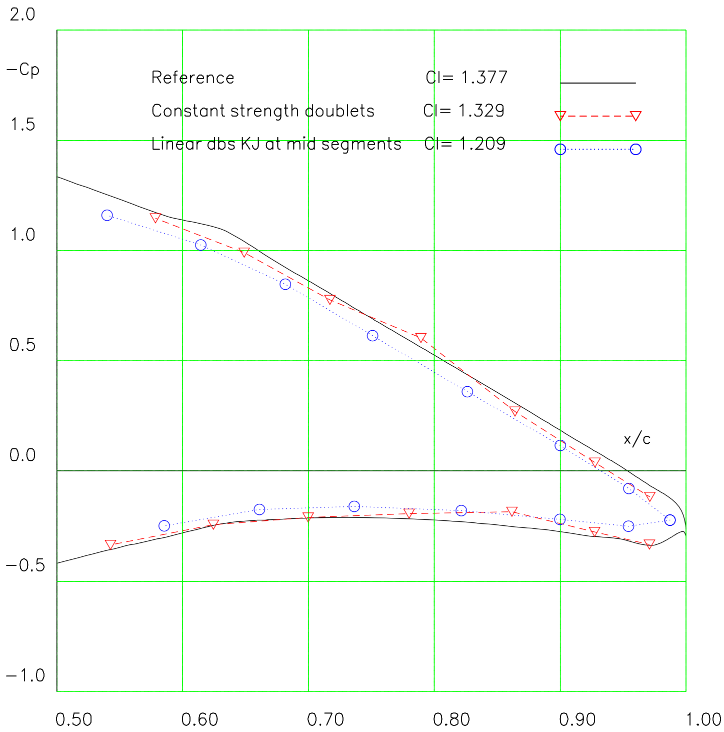

Fig. 1. Pressure coefficients over the rear side of a NACA64212 airfoil at 10° angle of attack. Comparison between the constant (triangles) and linear (circles) doublet distributions with 32 segments. Reference computed with constant doublets and 500 segments (continuous line).

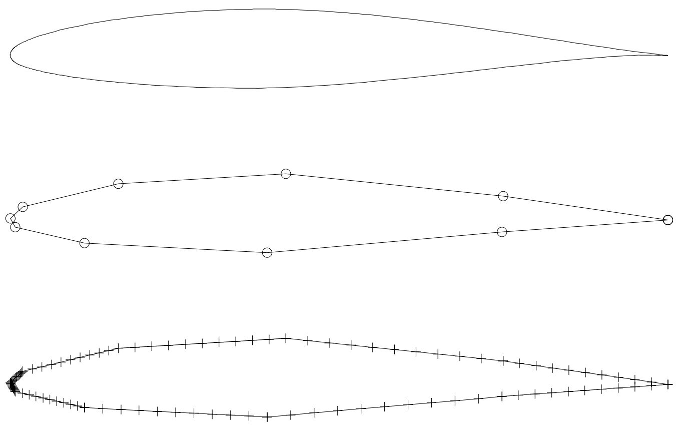

Fig. 2. The smooth NACA64212 (top), the derived 10 segments discretized section (middle) and the submeshed polygon (bottom).

increasing number of nodes, is shown in Fig. 3. This results into a significant improvement but a still unacceptable gap remains with the desired solution (continuous curve). It is recalled that the two-dimensional pressure exhibits a singular evolution at finite trailing edge angles, and obviously no extrapolation can capture this behaviour.

Following various unsuccessful attempts, the problem has been reversed. With fixed parameters (geometry, incidence, given mesh), the evolution of several local and integral quantities has been observed, under prescribed circulations varied around the "correct" value, otherwise known. This strategy was devised with the hope of suggesting a selection criterion for Γ. Fig. 4 represents the evolution of the drag coefficient (theoretically zero in 2D steady flow), the velocity fluxes across the segments adjacent to trailing edge F_{TE}, and the overall squared flow flux over the profile ($F_{V^2} = \oint_C (\vec{V} \cdot \vec{n})^2 \, \mathrm{d}l$ where $\vec{n}$ is the unit local normal vector and $\mathrm{d}l$ the elementary tangent length).

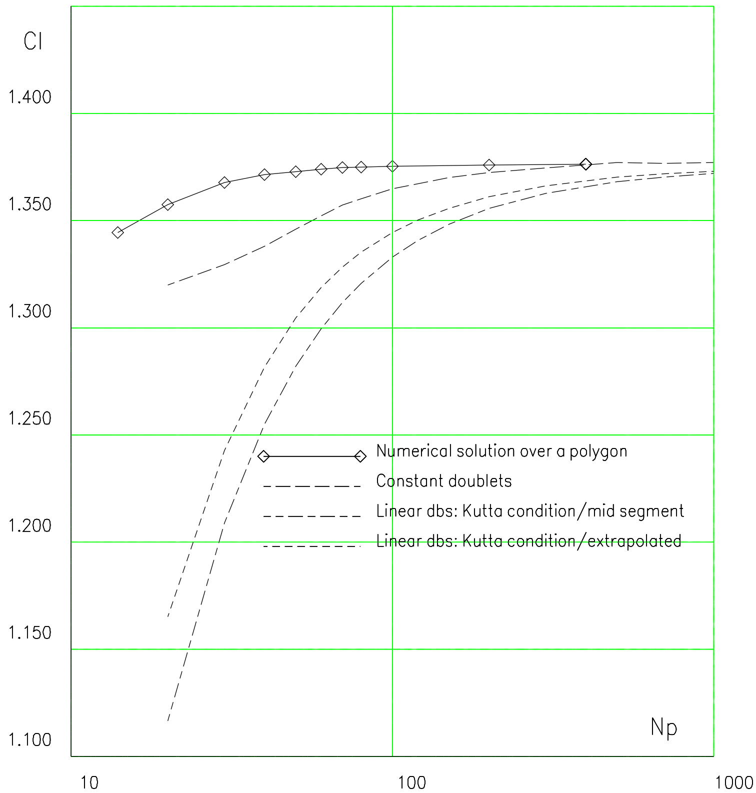

Fig. 3. Evolution of the lift coefficient C_l with the number of nodes (NACA 64212, 10° angle of attack, linear doublet distribution). Comparison between the constant and linear doublet distributions (Kutta condition at mid-segment and extrapolation at trailing edge).

Surprisingly it is observed that this flux and the computed drag show extreme values at the correct circulation Γ whereas the fluxes of $\vec{V}$ across the trailing edge segments are cancelled at the same value. The equivalence between the Neumann and Dirichlet problems mentioned above (Section 2.1) is not therefore systematically verified by the discrete model, except at the adapted circulation. This fundamental feature provides criteria that may be used to formulate alternative Kutta conditions.

As a criterion built on the drag cannot likely be extended to three-dimensional flows which exhibit some induced drag, and even to two-dimensional unsteady flows (the unsteady vortical wake also generates some drag), there remain the procedures based upon velocity fluxes.

Without valid reason to cancel the flux across one of the trailing edge segments rather than the other (bottom/top surfaces), and building an algorithm on a single segment may likely induce asymmetry, so it seems preferable to rely on the algebraic sum: $F_{\mathrm{TE}} = \vec{V}_1 \cdot \vec{n}_1 \, \mathrm{d}l_1 - \vec{V}_{Np} \cdot \vec{n}_{\mathrm{Np}} \, \mathrm{d}l_{\mathrm{Np}}$ (the negative sign comes from the opposite directions of normal unit vectors). A F_{TE} cancelling algorithm can be simply built observing that the velocities, and therefore the fluxes, are linear functions of the circulation. The internal Dirichlet problem may be solved for two arbitrary values (in practice $\Gamma = 0$ and $\Gamma = 1$). The correct value of the circulation Γ and of the intensities μ_I, $I = 1, N_N$, are deduced by linear interpolation.

A similar procedure is applied to the other version of the Kutta condition, the "global flux" minimization. Observing once again that all velocities, especially at the wall, are linear functions of Γ, the quadratic flux F_{V^2} is therefore a second degree function of Γ. The Dirichlet problem has then to be solved for 3 values $(-1, 0, +1)$ and the maximum of the parabolic function $F_{V^2}(\Gamma)$ is algebraically deduced.

Applied to the case already treated (Fig. 3), the two procedures show a rapid convergence towards the exact polygonal solution of the lift coefficient C_l with the number of nodes (Fig. 5), faster than the 0th order method (Fig. 3). The "local" no flux condition, numerically lighter, seems more efficient than the "global" flux minimization. It has been selected for two-dimensional applications. Its application to the three-dimensional case is less obvious because

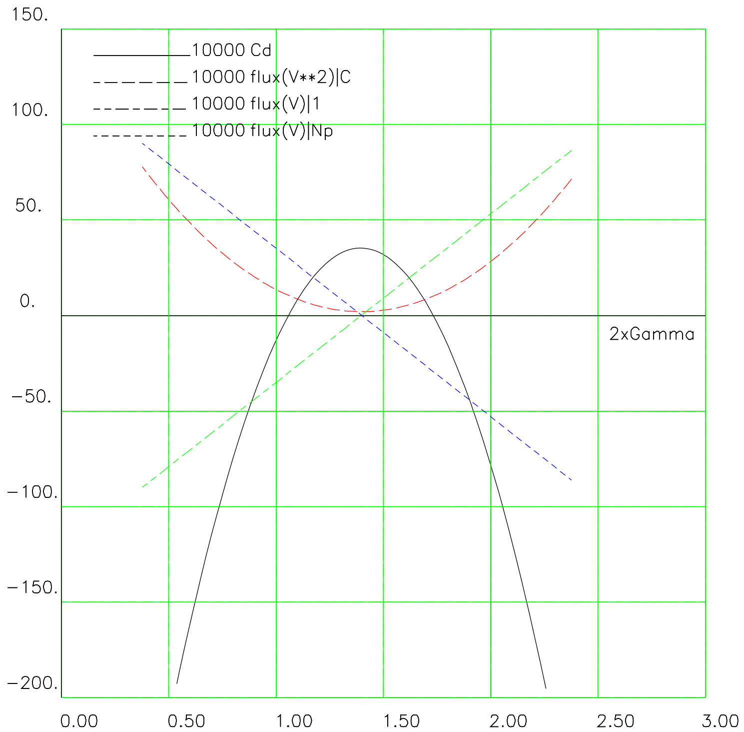

Fig. 4. Variation of the computed drag C_d and various fluxes as the prescribed circulation Γ is varied.

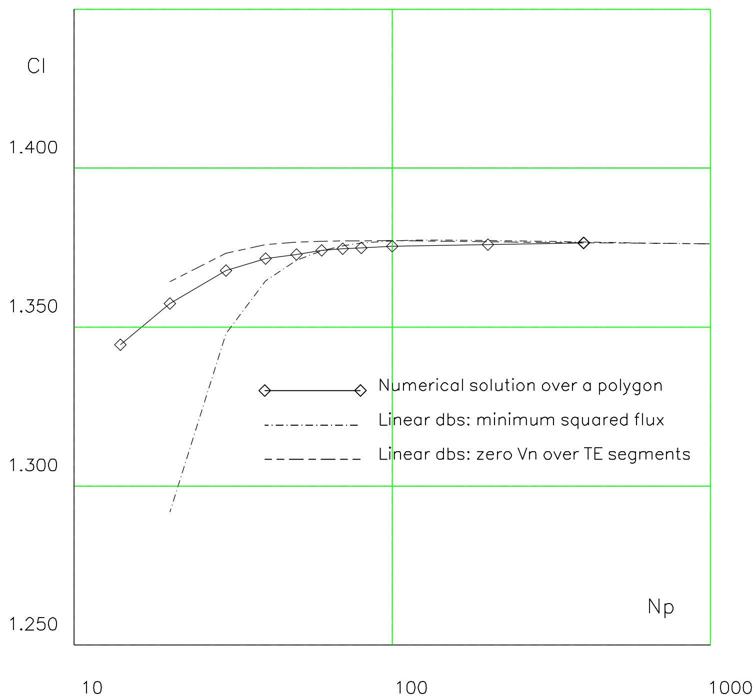

Fig. 5. Convergence of the lift coefficient C_l with the number of nodes (linear doublet distributions). Kutta condition expressed as zero flux across trailing edge segments or minimal quadratic flux across the section.

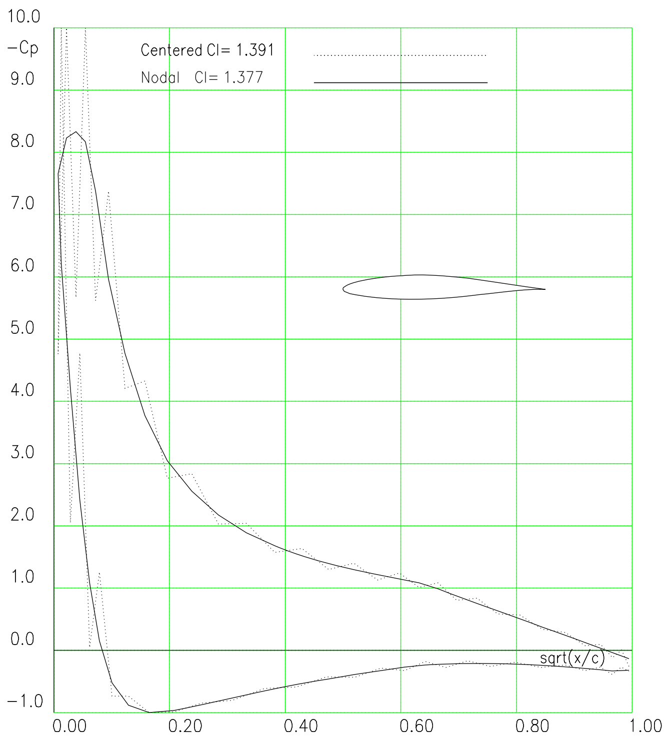

Fig. 6. Comparison between the surface pressure coefficients computed by the centered and nodal methods (NACA64212 wing section, 10° angle of attack, 61 nodes).

each node at the trailing edge belongs to several panels on each side, not necessarily with the same shapes. To avoid arbitrarily selecting one of these panels, a minimization of the overall quadratic flux is preferred. The algorithm is based upon the Lagrange multipliers method. The wing being meshed with N_N nodes, among which K are doubles (those of the trailing edge), $N_N - K$ multipliers are introduced, which leads to a well determined linear system of size $2N_N - K$, obviously larger that the system built for a 0th order method of size N_P.

The present nodal method does not show any evidence of spatial instability, the associated linear system is almost diagonal dominant. An attempt to derive a similar formulation with a more conformist positioning of the control points at the center of the elements was found very unstable. In this case, the potential influence of a segment I on the associated control point I is through the half sum $(\mu(I) + \mu(I+1))/2$ only, where $\mu(I)$ and $\mu(I+1)$ are the intensities of the doublet sheet at the surrounding nodes. The respective effects of $\mu(I)$ and $\mu(I+1)$ possibly come from the adjacent panels induction, provided that the surface has some local curvature. The matrix of the singular system is then nearly singular and this may induce severe instabilities as can be seen on Fig. 6. This observation from 2D experiments can be related to the 3D instabilities mentioned in Section 3.1. The problem can be partially cured by a slight de-centering of the control point, at the expense of some deviation of the produced lift.

Finally the zero internal potential condition has been checked by means of a numerical surface integration of the potential field inside the body. The rms deviation from this condition is typically 5×10^{-5} for realistic numbers of nodes ($N_N > 60$ in the 2D case).

4. Conclusions

The numerical computation of incompressible irrotational fluid flows is performed with a potential formulation, surface doublet sheets of C^1 class, and internal Dirichlet conditions. The use of (bi)linear surface distributions, necessary to obtain finite velocity fields and easy derivation of the surface velocity field, prompts to develop an unconventional methodology. The two main features of the approach are: (1) Dirichlet conditions written at the mesh nodes and (2) an original Kutta condition formulation. Writing the Dirichlet conditions at the nodes rather than at the centroids yields a number of equations equal to the number of unknowns in the non-lifting case. The linear problem is then well-determined. When the body develops some lift, a cutting must be put in place at the trailing edge. This implies double knots and therefore supplementary degrees of freedom. The missing equations are provided by explicitly formulated Kutta conditions. A classical writing of these conditions, expressing the absence of pressure differential across the trailing edge, is seen to induce a systematic lift deficit. This writing at the half-mesh turns out to match the pressures upstream of the very trailing edge. It is observed that the "suitable" circulation is the one which minimizes the velocity fluxes through the wall, reinforcing the no-penetration condition that is not explicitly imposed in the discrete Dirichlet formulation of the method. At the price of a moderate complexity compared to a more conventional 0th order method, we get a significantly improved convergence of the solution with the number of nodes. In addition the calculation of the surface derivatives becomes trivial. Several applications have been validated, bi- and tri-dimensional, stationary or unsteady, the perspective being an iterative matching with structural codes.

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