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A two-scale damage model with material length

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Abstract

The Note presents the formulation of a class of two-scale damage models involving a micro-structural length. A homogenization method based on asymptotic developments is employed to deduce the macroscopic damage equations. The damage model completely results from energy-based micro-crack propagation laws, without supplementary phenomenological assumptions.

We show that the resulting two-scale model has the property of capturing micro-structural lengths. When damage evolves, the micro-structural length is given by the ratio of the surface density of energy dissipated during the micro-crack growth and the macroscopic damage energy release rate per unit volume of the material.

The use of fracture criteria based on resistance curves or power laws for sub-critical growth of micro-cracks leads to quasi-brittle and, respectively, time-dependent damage models. ***To cite this article: C. Dascalu, C. R. Mecanique 337 (2009).***

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1. Introduction

Local theories of damage generally do not involve characteristic lengths. For energy based models (e.g. [1]), the damage evolution is controlled by the rate of energy released per unit volume of the material.

The energy approach of fracture considers the rate of energy released per unit crack surface to characterize the crack growth (e.g. [2]). Fracture is a size-dependent phenomenon and models of size effects for fracture have been proposed (e.g. [3]) to account for the experimental observations.

Many attempts to obtain local damage models from micro-crack growth analysis by change of scale techniques have been performed in the last decades (e.g. [4–7]). Although micro-structural lengths are sometimes involved in the upscaling procedures, the final damage formulations generally do not involve characteristic lengths. Very often, the micromechanical models are based on some phenomenological assumptions, at the macro-scale, that supplement the homogenization analysis.

The aim of this Note is at proposing a general formulation of damage that completely results from the change-of-scale analysis and the appropriate choice of the damage parameter, without supplementary assumptions on the macroscopic damage response. The resulting two-scale theory will involve both microscopic surface energy density

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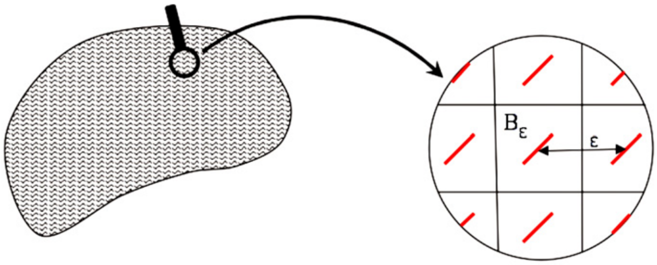

Fig. 1. Fissured medium with locally periodic microstructure.

of fracture and macroscopic volume density of energy released by damage. We show that the derived damage criterion involves the ratio of the two energy densities which is a material characteristic length. Such characteristic length has been introduced previously by different authors (e.g. [8–10]) in different contexts. Here, it appears naturally in the damage criterion as a consequence of the homogenization analysis.

When damage evolves, we show that this material length corresponds to micro-structural size parameters like the mutual distance between neighbor micro-cracks, the size of the quasi-brittle process zone for micro-cracks or other material lengths governing failure at the micro-scale. The proposed homogenization approach naturally captures the microscopic lengths and the presence of a characteristic length in the damage criterion is showed to be an intrinsic property of the new model.

The present formulation extends our recent results [11,12] that concerned the purely brittle case. We focus on the identification of intrinsic features of the macroscopic model, independent of particular sizes of the microstructure. The homogenized damage equations are deduced in the limit of indefinitely small microscopic periods, as it is proper to the asymptotic homogenization approach [13].

Starting from micro-fracture criteria based on fracture resistance curve and power laws for sub-critical growth, we deduce quasi-brittle and, respectively, time-dependent damage models. To our knowledge, such damage models have not been obtained before through asymptotic homogenization.

2. Effective elastic behavior of a micro-fractured body

The effective elastic behavior for bodies with cracks was studied, for instance, in [14,13,11] in the framework of the asymptotic homogenization theory for periodic media. We recall here some basic results that are necessary for development of the damage model.

Consider a two-dimensional isotropic elastic medium containing a large number of small cracks. A (locally) periodic distribution of micro-cracks is assumed, so as one can locally find a periodicity cell, of length ε, containing one crack (see Fig. 1). The length ε, also representing the mutual distance between centers of neighbor micro-cracks, characterizes the size of the microstructure. The cracks are assumed to be straight and of length d^ε.

As it is proper to the asymptotic homogenization method, we consider a suite of periodic structures, indexed over ε, and we evaluate the limit, as $\varepsilon \to 0$, of the corresponding solutions of the mechanical problems. In what follows, we assume that the length d^ε and orientation of micro-cracks are such that, at the limit when ε approaches 0, they vary smoothly inside the elastic body.

Let $\mathcal{B}$ denote the initial heterogeneous medium represented by a bounded two-dimensional domain with a smooth external boundary and $\mathcal{C}$ the union of all the microcracks inside $\mathcal{B}$. In the solid part $\mathcal{B}_s = \mathcal{B} \setminus \mathcal{C}$, we have the equilibrium equations

$$\frac{\partial \sigma^\varepsilon_{ij}}{\partial x_j} = 0, \qquad \sigma^\varepsilon_{ij} = a_{ijkl} e_{xkl}\big(\mathbf{u}^\varepsilon\big) \tag{1}$$

where $\mathbf{u}^\varepsilon$ and $\boldsymbol{\sigma}^\varepsilon$ are the displacement and the stress fields, a_{ijkl} are the elasticity coefficients and where we considered the strain tensor $e_{xij}(\mathbf{u}^\varepsilon) = \frac{1}{2}(\frac{\partial u^\varepsilon_i}{\partial x_j} + \frac{\partial u^\varepsilon_j}{\partial x_i})$ with respect to x coordinates. We assume that the cracks are open and traction-free: $\boldsymbol{\sigma}^\varepsilon \mathbf{N} = 0$, where $\mathbf{N}$ is the unit normal vector to the crack faces, as in Fig. 2. We remark that the case of fully closed cracks, with frictionless contact, could also be considered [14,13,11], leading to a different overall

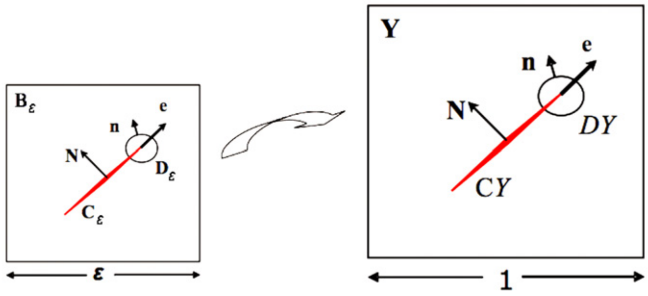

Fig. 2. Rescaling of the unit cell to the microstructural period of the material.

behavior. These two states for micro-cracks lead to different homogenized coefficients in tension and compression [14,11].

The locally periodic microstructure which is reproduced from the unit cell $Y = [0,1] \times [0,1]$ by rescaling with the small parameter ε so that the period of the material is εY, as in Fig. 2. The two distinct scales are represented by the *macroscopic variables* $\mathbf{x}$ and the *microscopic variables* $\mathbf{y} = \mathbf{x}/\varepsilon$. In the unit cell Y we denote the crack by CY and solid part by $Y_s = Y \setminus CY$. The length of CY is $d = d^\varepsilon/\varepsilon$. According to the method of asymptotic homogenization (e.g. [13]), we look for expansion of $\mathbf{u}^\varepsilon$ in the form

$$\mathbf{u}^\varepsilon(\mathbf{x},t) = \mathbf{u}^{(0)}(\mathbf{x},\mathbf{y},t) + \varepsilon\mathbf{u}^{(1)}(\mathbf{x},\mathbf{y},t) + \varepsilon^2\mathbf{u}^{(2)}(\mathbf{x},\mathbf{y},t) + \cdots \tag{2}$$

where $\mathbf{u}^{(i)}(\mathbf{x},\mathbf{y},t)$, $\mathbf{x} \in \mathcal{B}_s$, $\mathbf{y} \in Y$ are smooth functions and Y-periodic in $\mathbf{y}$.

Substituting the expansion (2) into Eq. (1) and the traction-free boundary conditions we obtain boundary value problems for the different orders of ε, formulated on the unit cell Y. It can be shown (e.g. [14,13]) that the function $\mathbf{u}^{(0)} = \mathbf{u}^{(0)}(\mathbf{x},t)$ is independent of $\mathbf{y}$ variable, representing the macroscopic displacement field.

For given $\mathbf{u}^{(0)}(\mathbf{x},t)$, for open traction-free cracks, we deduce [14,13] the following boundary-value problem for the function $\mathbf{u}^{(1)}$:

$$\frac{\partial}{\partial y_j}\left(a_{ijkl}e_{ykl}\left(\mathbf{u}^{(1)}\right)\right) = 0, \quad \text{in } Y_s \tag{3}$$

$$a_{ijkl}e_{ykl}\left(\mathbf{u}^{(1)}\right)N_j = -a_{ijkl}e_{xkl}\left(\mathbf{u}^{(0)}\right)N_j, \quad \text{on } CY \tag{4}$$

and with periodicity boundary conditions on the external boundary of the cell.

The microscopic correction $\mathbf{u}^{(1)}$ has a linear dependence of the macroscopic deformations: $\mathbf{u}^{(1)}(\mathbf{x},\mathbf{y},t) = \boldsymbol{\xi}^{pq}(\mathbf{y})e_{xpq}(\mathbf{u}^{(0)})(\mathbf{x},t)$. Here, the characteristic functions $\boldsymbol{\xi}^{pq}(\mathbf{y})$ are elementary solutions of (3)–(4), for the particular macroscopic deformations $e_{xpq}(\mathbf{u}^{(0)}) = \delta_{pq}$.

By introducing the mean value operator $\langle\cdot\rangle = \frac{1}{|Y|}\int_{Y_s} \cdot \, \mathrm{d}y$, where $|Y|$ is the measure of Y, we can prove [14,13] that

$$\Sigma_{ij}^{(0)} \equiv \left\langle\sigma_{ij}^{(0)}\right\rangle = C_{ijkl}(d)e_{xkl}\left(\mathbf{u}^{(0)}\right) \tag{5}$$

where $\sigma_{ij}^{(0)} = a_{ijkl}(e_{xkl}(\mathbf{u}^{(0)}) + e_{ykl}(\mathbf{u}^{(1)}))$ and

$$C_{ijkl}(d) = \frac{1}{|Y|}\int_{Y_s}\left(a_{ijkl} + a_{ijmn}e_{ymn}\left(\boldsymbol{\xi}^{kl}\right)\right)\mathrm{d}y \tag{6}$$

are the homogenized coefficients. The effective constitutive relation (5) should be used in the macroscopic equilibrium equation.

The previous formulae are valid for every crack micro-orientation in the periodicity cell. If we denote by θ the angle made by the crack line with the horizontal direction, then the effective coefficients are functions of d and θ: $C_{ijkl} = C_{ijkl}(d,\theta)$. The couple (d,θ) completely characterizes the state of damage at a given macroscopic point. These coefficients can be computed by solving the unit cell problem for every d and θ and the functions (2) can be obtained by interpolation of the coefficients computed for a certain number of orientations and lengths of the crack.

3. Homogenization of energy balance: Damage equations

For the description of the evolution of damage the previous equilibrium problem should be completed with damage equations. In this section we obtain a general damage equation as the result of homogenization of the microscopic balance of energy for propagating micro-cracks.

For the initial problem, the *fracture energy release rate* during crack extension can be expressed as

$$\mathcal{G}_\varepsilon = \lim_{D_\epsilon \to O} \int_{\partial D_\varepsilon} \mathbf{e} \cdot \mathbf{b}\big(\mathbf{u}^\varepsilon\big)\mathbf{n}\,\mathrm{d}s \tag{7}$$

where D_ε is a disk of infinitesimal radius, surrounding the crack tip O, with $\mathbf{n}$ the outward normal to the disk D_ϵ, $\mathbf{e}$ is the unit vector in the propagation direction (see Fig. 2) and $b_{ij}(\mathbf{u}^\varepsilon) = \frac{1}{2}a_{mnkl}e_{xkl}(\mathbf{u}^\varepsilon)e_{xmn}(\mathbf{u}^\varepsilon)\delta_{ij} - \sigma^\varepsilon_{jk}u^\varepsilon_{k,i}$ is the configurational stress tensor.

The energy-release rate $\mathcal{G}_\varepsilon$ depend on the crack length l and the orientation θ. The crack propagation is described by the following laws:

$$\mathcal{G}_\varepsilon \leqslant \mathcal{G}_f; \qquad \dot{l} \geqslant 0; \qquad \dot{l}(\mathcal{G}_\varepsilon - \mathcal{G}_f) = 0 \tag{8}$$

where $\mathcal{G}_f$ is the critical fracture energy of the material. These relations should be completed with the reduced dissipation inequality:

$$\mathcal{D}_f \equiv \mathcal{G}_\varepsilon \dot{l} \geqslant 0 \tag{9}$$

We will assume that the critical fracture energy $\mathcal{G}_f$ may depend on the micro-structural size, the crack length and velocity.

As concerns the dependence of θ, the existing criteria are generally independent of those concerning the extension (e.g. [2]). A simple choice, made in [11], is to consider that microcracks are initiated in the direction of maximum energy release and that this direction is maintained after. Pre-existing micro-cracks will extend in their own direction. One can construct more complex behaviors in which, for different loadings, different families of microcracks may be activated. These are beyond the objectives of this paper. In what follows, we assume that the microcracks maintain their orientation during the propagation. This orientation may vary with respect to the macroscopic variable $\mathbf{x}$.

Assuming the symmetric extension of micro-cracks, from (3)–(4) and the periodicity conditions we deduce the global balance of energy on the unit cell:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{Y_s} \frac{1}{2}a_{ijkl}e_{ykl}\big(\mathbf{u}^{(1)}\big)e_{yij}\big(\mathbf{u}^{(1)}\big)\,\mathrm{d}y + \frac{\mathcal{G}_\varepsilon}{\varepsilon}\frac{\mathrm{d}d}{\mathrm{d}t} = 0 \tag{10}$$

Using this relation and following the results in Dascalu et al. [11] it can be *proved* that, for $\dot{d} \neq 0$, we have

$$\frac{\mathcal{G}_\varepsilon}{\varepsilon} = -\frac{1}{2}\frac{\mathrm{d}C_{ijkl}(d)}{\mathrm{d}d}e_{xkl}\big(\mathbf{u}^{(0)}\big)e_{xij}\big(\mathbf{u}^{(0)}\big) \tag{11}$$

where the right member $Y_d \equiv -\frac{1}{2}\frac{\mathrm{d}C_{ijkl}(d)}{\mathrm{d}d}e_{xkl}(\mathbf{u}^{(0)})e_{xij}(\mathbf{u}^{(0)})$ is the *damage energy release rate*. We note that this relation is entirely deduced from microstructural assumptions, without any assumptions on the scaling of energy. This scaling with ε is naturally appearing in the derivation of the damage equation (11). For evolving damage, the previous relation shows that the microstructural length ε makes the link between the surface energy dissipated during micro-crack propagation and damage energy dissipated per unit volume. This energy scaling property will assure the presence of the internal length ε in the damage law.

Using (11) from the micro-crack evolution laws (8) we deduce the damage laws

$$Y_d \leqslant \frac{\mathcal{G}_f}{\varepsilon}; \qquad \dot{d} \geqslant 0; \qquad \dot{d}\left(Y_d - \frac{\mathcal{G}_f}{\varepsilon}\right) = 0 \tag{12}$$

$$\mathcal{D}_d \equiv Y_d\dot{d} \geqslant 0 \tag{13}$$

The presence of a material length in the damage model given by the relations (12), or in the limit form by (16), appears to be new for first order micromechanical damage models. In this respect, the present model extends the one for the brittle case in [11].

In this contribution we assume that the critical fracture energy is given as a constitutive function of the microscopic size ε, the crack length l and velocity $\dot{l}$. In terms of the damage variable $d = l/\varepsilon$, the fracture energy can be expressed as

$$\mathcal{G}_f = \mathcal{G}_f(d, \dot{d}, \varepsilon) \tag{14}$$

Eqs. (12)–(14) correspond to a finite micro-structural parameter $\varepsilon > 0$. Following the asymptotic homogenization approach (e.g. [13]) we consider a family of elasto-damage problems, for different length-scales ε, and look for the limit of the sequence of solutions to these problems as $\varepsilon \to 0$ and the damage equations fulfilled by this "homogeneous" limit solution. It is not our aim, in this Note, to formulate the mathematical framework and to give details of the proof of the convergence, but just to sketch out the way in which one may construct a local damage model with internal length by homogenization of the energy balance law for evolving micro-cracks.

At the limit $\varepsilon \to 0$ in Eqs. (12), in order to have a finite fracture energy dissipation it is necessary that

$$\lim_{\varepsilon \to 0} \frac{\mathcal{G}_f(d, \dot{d}, \varepsilon)}{\varepsilon} = \frac{\mathcal{G}_0(d, \dot{d})}{l_0} \tag{15}$$

where the limit fracture energy $\mathcal{G}_0(d, \dot{d}) > 0$ and the internal length $l_0 > 0$ assures dimensional homogeneity.

If the condition (15) is fulfilled, we can take the limit in (12) to obtain

$$Y_d \leqslant \frac{\mathcal{G}_0}{l_0}; \qquad \dot{d} \geqslant 0; \qquad \dot{d}\left(Y_d - \frac{\mathcal{G}_0}{l_0}\right) = 0 \tag{16}$$

This limit result reveals the presence of a length-scale parameter as an intrinsic property of the homogenized damage laws. The last relation in (16) shows that when damage evolves ($\dot{d} \neq 0$) the internal length becomes equal to the ratio of micro-fracture surface energy $\mathcal{G}_0$ ($\mathrm{J/m^2}$) and macro-damage volume energy Y_d ($\mathrm{J/m^3}$) dissipations:

$$l_0 = \frac{\mathcal{G}_0}{Y_d} \tag{17}$$

We remark that this characteristic length, expressed as the ratio of fracture and damage energies, has been used previously by different authors (e.g. [8–10]) in different contexts. Here it appears naturally in the damage criterion (16) as a consequence of the homogenization analysis.

A consistency condition associated with the last relation in (16) may be obtained by using (11) and (15), yielding the damage evolution law:

$$\frac{\partial \mathcal{G}_0}{\partial \dot{d}}\ddot{d} + \left(\frac{\partial \mathcal{G}_0}{\partial d} + \frac{l_0}{2}\frac{\mathrm{d}^2 C_{ijkl}}{\mathrm{d}d^2} e_{xkl}\big(\mathbf{u}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big)\right)\dot{d} + l_0 \frac{\mathrm{d}C_{ijkl}}{\mathrm{d}d} e_{xkl}\big(\dot{\mathbf{u}}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big) = 0 \tag{18}$$

Note that for constant $\mathcal{G}_0$ the length parameter l_0 does not appear explicitly in this law, but only in the criterion (16).

4. Brittle, quasi-brittle and subcritical evolutions

In this section we consider particular fracture criteria for micro-cracks and we obtain the corresponding damage laws. For each damage model, we give conditions under which the relations (15–16), (18) are fulfilled.

Our first example is the brittle damage model. In a recent paper [11], we showed that a brittle damage model is obtained through homogenization if a Griffith fracture criterion is assumed for micro-cracks. The critical fracture energy was considered to be a constant, independent on the internal length parameter. Here we consider the case of a critical fracture energy which is a constitutive function of the internal length $\mathcal{G}_f = \mathcal{G}_f(\varepsilon)$. In this case, the limit condition (15) is fulfilled for a linear dependence on ε: $\mathcal{G}_f = \frac{\mathcal{G}_c \varepsilon}{c_f}$, where $\mathcal{G}_c$ is constant fracture energy and the length c_f assures dimensional homogeneity. The damage evolution law then becomes

$$\left(\frac{1}{2}\frac{\mathrm{d}^2 C_{ijkl}}{\mathrm{d}d^2} e_{xkl}\big(\mathbf{u}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big)\right)\dot{d} + \frac{\mathrm{d}C_{ijkl}}{\mathrm{d}d} e_{xkl}\big(\dot{\mathbf{u}}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big) = 0 \tag{19}$$

The length c_f is not appearing in this evolution law; its influence on the damage response is given by the criterion (16).

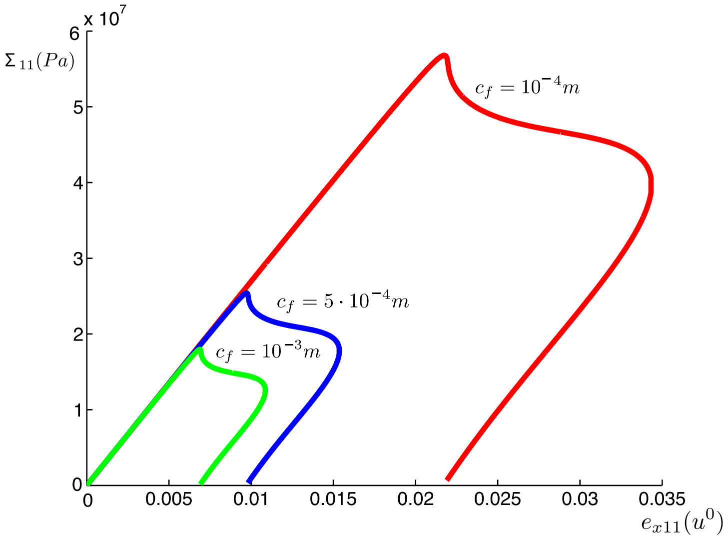

Fig. 3. 1D quasi-brittle elasto-damage response in tension. The curves, for different material lengths c_f, show micro-structural size effects. Snap-back behavior is obtained for values of d close to 1.

For finite-length microscopic periods, the role of c_f is taken by the distance between neighbors micro-cracks ε, in (12). Numerical examples, in the case of constant $\mathcal{G}_f$ were presented in [11,12], showing size effects for the damage response.

The second example concerns the quasi-brittle failure at the small scale. To model the quasi-brittle effects, we adopt the point of view of [10,3] and consider the equivalent crack in an elastic matrix but which propagation is controlled by a resistance curve. Consider the critical fracture energy given by [3]:

$$\mathcal{G}_f(d,\varepsilon) = \mathcal{G}_c \frac{\varepsilon(d-d_{0)}}{c_f} \tag{20}$$

where c_f could be related to the size of the process zone, d_0 is the initial value of damage (normalized crack-length) and $\mathcal{G}_c$ is the corresponding initial fracture resistance. We consider c_f and $\mathcal{G}_c$ as constant parameters of the propagation criterion, independent on ε.

At the limit in (15) becomes

$$\lim_{\varepsilon\to 0} \frac{\mathcal{G}_f(d,\varepsilon)}{\varepsilon} = \frac{\mathcal{G}_c(d-d_0)}{c_f} \tag{21}$$

The parameter c_f now becomes the material length in the homogenized model. The corresponding damage evolution law (18) takes the form:

$$\left(\mathcal{G}_c + \frac{c_f}{2}\frac{\mathrm{d}^2 C_{ijkl}}{\mathrm{d}d^2} e_{xkl}\big(\mathbf{u}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big)\right)\dot{d} + c_f \frac{\mathrm{d}C_{ijkl}}{\mathrm{d}d} e_{xkl}\big(\dot{\mathbf{u}}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big) = 0 \tag{22}$$

We note that the process zone characteristic length c_f is naturally captured by the damage equations.

The one-dimensional response of the deduced elasto-damage model is presented in Fig. 3. The material parameters are: Young modulus $E = 2e9$ Pa, Poisson ratio $\nu = 0.3$, the critical fracture energy $\mathcal{G}_c = 100\ \mathrm{J/m^2}$. The three curves correspond to different values of the material length c_f, showing micro-structural size effects for the homogenized response. We also note the snap-back behavior that occurs for values of d close to 1. It can be shown that this regime corresponds to that of unstable propagation of micro-cracks.

For the last example we assume a sub-critical propagation criterion for micro-cracks. Consider the mode I growth described by a law of the type of that proposed by Charles [15,16]:

$$\frac{\mathrm{d}l}{\mathrm{d}t} = \frac{c_f}{\tau_0}\left(\frac{K_I}{K_0}\right)^2 \tag{23}$$

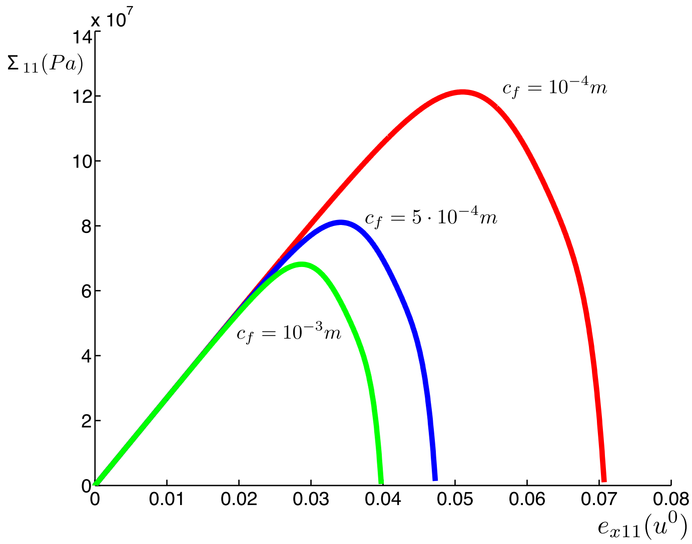

Fig. 4. 1D subcritical damage response, in tension, for different values of the micro-structural parameter c_f.

where K_0 is a limit value of the mode I stress intensity factor K_I and where we followed Salganik et al. [16] by introducing the size of the process zone c_f and the characteristic time τ_0. The sub-critical exponent was taken equal to 2. Using the relation between the stress intensity factor and the energy-release rate we obtain for the critical fracture energy:

$$\mathcal{G}_f(\dot{d}, \varepsilon) = \frac{\varepsilon \mathcal{G}_c \tau_0}{c_f} \dot{d}; \qquad \mathcal{G}_c = \frac{K_0^2}{E'} \tag{24}$$

where $E' = \frac{E}{1-\nu^2}$. As for the previous example, we assume that K_0, τ_0 and c_f are constant parameters of the subcritical propagation law, independent of the size ε. In this case, at the limit $\varepsilon \to 0$, we get

$$\lim_{\varepsilon \to 0} \frac{\mathcal{G}_f(\dot{d}, \varepsilon)}{\varepsilon} = \frac{\mathcal{G}_c \tau_0}{c_f} \dot{d} \tag{25}$$

showing the presence of c_f which becomes the material length of the homogenized model. The damage evolution law (18) now reads:

$$\mathcal{G}_c \tau_0 \ddot{d} + \frac{c_f}{2} \frac{\mathrm{d}^2 C_{ijkl}}{\mathrm{d}d^2} e_{xkl}\big(\mathbf{u}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big) \dot{d} + c_f \frac{\mathrm{d}C_{ijkl}}{\mathrm{d}d} e_{xkl}\big(\dot{\mathbf{u}}^{(0)}\big) e_{xij}\big(\mathbf{u}^{(0)}\big) = 0 \tag{26}$$

The 1D response, in tension, predicted by the elasto-damage model is represented in Fig. 4, for different values of the material length c_f. The material parameters are: Young modulus $E = 2e9$ Pa, Poisson ratio $\nu = 0.3$, the critical fracture energy $\mathcal{G}_c = 100\ \mathrm{J/m^2}$, process zone size $c_f = 10^{-3}$ m and characteristic time $\tau_0 = 10^7$ s.

For a given material, the interaction between the microscopic internal length and a characteristic size of the macro-structure will lead to macro-structural size effects.

5. Conclusions

We constructed a two-scale damage model for locally periodic micro-crack distributions using asymptotic developments homogenization. At the microscopic level an energy-based propagation criterion was considered and the macroscopic damage equations were deduced exclusively through the change of scale procedure. It has been showed that the new damage evolution equations naturally capture microscopic lengths. Their presence in the macroscopic damage law leads to size effects. The general two-scale approach was illustrated for quasi-brittle and, respectively, sub-critical damage models, which appear to be deduced for the first time within the asymptotic homogenization method.

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