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Modelling of orthogonal cutting by incremental elastoplastic analysis and meshless method

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Abstract

This Note introduces an application of the meshless method to the case of machining simulation in small deformations, which is still subjected to numerical limitations. The treatment of the contact problem at the tool/chip interface is presented, and highlights the interest of the coupling of the contact law with friction. Validation results are detailed through typical example. ***To cite this article: E. Boudaia et al., C. R. Mecanique 337 (2009).***

Résumé

Modélisation de la coupe orthogonale par l'analyse élastoplastique incrémentale et la méthode sans maillage. Nous introduisons dans cette Note une application de la méthode sans maillage au cas de la simulation numérique de la coupe en petites déformations, qui se trouve encore confrontée à des limitations d'ordre numérique. La gestion du problème du contact à l'interface outil/copeau est présentée, et l'intérêt du couplage entre le contact et frottement pour ce problème est mis en valeur. Les résultats de la validation, effectuée sur un exemple typique, sont détaillés. ***Pour citer cet article : E. Boudaia et al., C. R. Mecanique 337 (2009).***

Keywords: Computational solid mechanics; Machining simulation; Meshless method; Friction contact

Mots-clés : Mécanique des solides numérique ; Simulation numérique de la coupe ; Méthode sans maillage ; Contact frottant

1. Introduction

The meshless techniques are still under development and much attention has been given to overcoming some of their drawbacks. For instance, when solving boundary value problems, the imposition of the essential boundary con-

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ditions may be a problem since some of the meshless shape functions do not always satisfy the Kronecker delta condition. Over the last decade, some researchers have proposed the use of techniques such as the Lagrange multiplier in [1], without additional Lagrange multiplier in [2], the penalty method in [3] and FEM coupling (see [4]) in an attempt to overcome this drawback. Nevertheless, the use of such techniques may bring about undesired minor issues, such as the increased number of unknowns in the system because of the Lagrange multiplier technique and the uncertainty involved in finding a suitable value for the penalty parameter when using the penalty method.

Basing on the shear plane method, the mechanics of metal cutting has its foundation in the works of Time [5] and Briks [6]. Recently, a number of meshfree methods have also been proposed to solve elastoplastic problems (see [7,8]). In this work, the boundary condition for contact law with Coulomb's friction in the interface is taken into account and is described by the bipotential concept leading us to minimize only one principle of minimum. In Section 2, an overview on the Moving Least Squares (MLS) approximation is given and the transformation method is proposed to impose the essential boundary conditions [9]. Elastoplastic evolution with frictional contact is presented in Section 3. The variational formulation and the implementation of the MLS discretization are, respectively, discussed in Sections 4 and 5. The performance of the proposed methods is examined in Section 6, and a conclusion is given in Section 7.

2. Moving least squares approximation

An excellent description of MLS is given by Lancaster and Salkauskas in [10]. The MLS approximation $u^h(x)$ is defined in the domain Ω by

$$u^h(x) = \sum_{j=1}^{nb} p_j(x)a_j(x) = p^T(x)a(x) \tag{1}$$

where $p(x)$ is the basis function, nb is the number of terms in the basis function, and the coefficients $a_j(x)$ are also functions of x, which are obtained at any point x by minimizing a weighted discrete L_2 norm of:

$$J = \sum_{i=1}^{m} w(x-x_i)\big(p^T(x_i)a(x_i) - u_i\big)^2 \tag{2}$$

where u_i is the nodal value parameter of $u(x)$ at node x_i, and m is the number of nodes in the neighborhood of x for which the weight function $w_i(x) = w(x-x_i) \neq 0$. Many kinds of weight functions have been used in meshless methods. The quartic spline weight function is used in this paper,

$$w(r) = \begin{cases} 1 - 6r^2 + 8r^3 - 3r^4 & \text{if } |r| \leqslant 1 \\ 0 & \text{if } 1 < |r| \end{cases} \tag{3}$$

where $r = \|x_i - x\|/d_{\max}$ is the normalized radius and $d_{\max}$ is the size of influence domain of point x_i.

Using the stationary condition for J with respect to $a(x)$, we can solve $a(x)$. And then, substituting it into Eq. (1), we have

$$u^h(x) = \sum_{i=1}^{m} \phi_i(x)u_i \tag{4}$$

where the MLS shape function $\phi_i(x)$ is defined by

$$\phi_i(x) = \sum_{j=1}^{nb} p_j(x)\big(A^{-1}(x)B(x)\big)_{ji} \tag{5}$$

in the above equation, the matrices $A(x)$ (moment matrix) and $B(x)$ are given by

$$A_{jk} = \sum_{i=1}^{m} B_{ij}\,p_k(x_i); \qquad B_{ij} = w_i(x)\,p_j(x_i) \tag{6}$$

The MLS shape functions given in Eq. (5) do not, in general, satisfy the Kronecker's delta property, i.e., $\phi_i(x_j) \neq \delta_{ij}$. In order to overcome this difficulty, we use the transformation method whose the transformation matrix

Λ is formed by establishing the relationship between the nodal value $u_j^h(x_k) \equiv \widehat{u}_{jk}$ and the "generalized" displacement u_{ij} by

$$u_j^h(x) = \sum_{i=1}^{m} \phi_i(x) u_{ji} \tag{7}$$

$$u_{ji} = \sum_{i=1}^{m} \Lambda_{ik}^{-1} \widehat{u}_{jk} \tag{8}$$

where $\Lambda_{ik} = \phi_i(x_k)$; by substituting Eq. (8) into Eq. (7), one can obtain

$$u_j^h(x) = \sum_{i=1}^{m} \sum_{k=1}^{m} \phi_i(x) \Lambda_{ki}^{-1} \widehat{u}_{jk} \equiv \sum_{k=1}^{m} \bar{\phi}_i(x) \widehat{u}_{jk} \tag{9}$$

where $\bar{\phi}_k(x) = \sum_{i=1}^{m} \Lambda_{ki}^{-1} \phi_i(x)$; note that

$$\bar{\phi}_k(x_j) = \sum_{i=1}^{m} \Lambda_{ki}^{-1} \phi_i(x_j) = \sum_{i=1}^{m} \Lambda_{ik}^{-1} \Lambda_{kj} = \delta_{ij} \tag{10}$$

and u^h and δu^h satisfy the following boundary conditions:

$$u_j^h(x_i) = \sum_{j=1}^{m} \bar{\phi}_j(x_i) \widehat{u}_{ij} \quad \text{and} \quad \delta u_j^h(x_i) = \sum_{j=1}^{m} \bar{\phi}_j(x_i) \delta \widehat{u}_{ij}; \quad \forall i \in \eta_{\bar{u}_i} \tag{11}$$

where $\eta_{\bar{u}_i}$ denotes a set of particle numbers in which the associated particles are located on boundary Γ_u. From Eq. (10), we directly obtain

$$\widehat{u}_{ji} = \bar{u}_j(x_i) \quad \text{and} \quad \delta \widehat{u}_{ji} = 0; \quad \forall i \in \eta_{\bar{u}_i} \tag{12}$$

3. Elastoplastic evolution with frictional contact

3.1. Elastoplastic analysis

The total strain increment can be decomposed into elastic and plastic parts:

$$\Delta\varepsilon = \Delta\varepsilon^e + \Delta\varepsilon^p \tag{13}$$

where $\Delta\varepsilon^e$ is the elastic strain increment defined by the Hooke's law and $\Delta\varepsilon^p$ is the plastic strain increment.

Let us consider the following incremental notations:

$$\Delta\tau = \tau_1 - \tau_0; \qquad \Delta\sigma = \sigma_1 - \sigma_0; \qquad \Delta\varepsilon^e = \varepsilon_1^e - \varepsilon_0^e; \qquad \Delta\varepsilon^p = \Delta\tau.\dot{\varepsilon}^p \tag{14}$$

where the index 0 (resp. 1) is relative to beginning (resp. to the end) of the step, $\dot{\varepsilon}^p$ is the plastic strain rate given by the normality law ($\dot{\varepsilon}^p = \dot{\lambda}.\partial f/\partial\sigma$, f is the yield function and λ is plastic multiplier).

We use the concept of the inf-convolution to calculate the incremental elastoplastic superpotential $\Delta V(\Delta\varepsilon)$:

$$\Delta V(\Delta\varepsilon) = (\Delta V_e \otimes \Delta V_p)(\Delta\varepsilon) = \operatorname*{Inf}_{\Delta\varepsilon^p \text{ incompressible}} \big(\Delta V_e\big(\Delta\varepsilon - \Delta\varepsilon^p\big) + \Delta V_p\big(\Delta\varepsilon^p\big)\big) \tag{15}$$

where ΔV_e and ΔV_p are, respectively, the elastic and plastic incremental superpotentials.

We obtain finally the incremental elastoplastic superpotential in term of strain for a material obeying the Von-Mises criterion by the following algorithm:

$$\begin{cases} \text{If } \|\Delta \mathrm{e}\| \geqslant \frac{\sigma_Y}{G\sqrt{6}} \quad \text{then} \begin{cases} \|\Delta e^p\| = \|\Delta e\| - \frac{\sigma_Y}{G\sqrt{6}} \\ \Delta V(\Delta\varepsilon) = \frac{1}{2} K_c (\Delta e_m)^2 + G\big(\|\Delta e\|^2 - \|\Delta e^p\|^2\big) \end{cases} \\ \text{Else } \|\Delta e^p\| = 0 \text{ and } \Delta V(\Delta\varepsilon) = \frac{1}{2} K_c (\Delta e_m)^2 + G\|\Delta e\|^2 \end{cases} \tag{16}$$

where $\|.\|$ denotes the Euclidean norm, K_c is the factor of compressibility, G is the Coulomb's shear modulus, σ_Y is the yield stress of material considered, Δe and Δe_m are, respectively, deviatoric and spherical parts of the tensor of elastoplastic strain $\Delta\varepsilon$ and $\|\Delta e^p\|$ is the Euclidean norm of the plastic strain deviator.

Finally, the incremental law of material is defined by:

$$\Delta\sigma \in \partial_{\Delta\varepsilon}\Delta V(\Delta\varepsilon); \quad \Delta\varepsilon \in \partial_{\Delta\sigma}\Delta W(\Delta\sigma) \tag{17}$$

where $\Delta W(\Delta\sigma)$ represents the dual incremental superpotential of $\Delta V(\Delta\varepsilon)$.

3.2. Frictional contact analysis

The incremental formulation of the contact law with Coulomb's dry friction is expressed by the incremental bipotential proposed by De Saxce and Feng (more details can be seen in [11]):

$$\Delta b_c(-\Delta u, \Delta t) = t_{n0}\Delta u_n + t_{t0}\Delta u_t + \mu(t_{n0} + \Delta t_n)\|\Delta u_t\| \tag{18}$$

where Δu_n and Δu_t denote, respectively, normal and tangential components of the displacement increment Δu; t_{n0} and t_{t0} indicate initially normal and tangential components of contact traction t and μ is friction coefficient.

The corresponding incremental contact laws take the form

$$-\Delta u \in \partial_{\Delta t}\Delta b_c(-\Delta u, \Delta t); \quad \Delta t \in \partial_{-\Delta u}\Delta b_c(-\Delta u, \Delta t) \tag{19}$$

4. Variational formulation

Let $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) be union of the contacting bodies with a regular boundary $\Gamma = \partial\Omega$; submitted to a traction increments $\Delta\bar{t}$ to a portion Γ_t; imposed displacement increments $\Delta\bar{u}$ to a portion Γ_u; and on the part $\Gamma_c = \Gamma - \Gamma_u \cup \Gamma_t$ of boundary such as $\Gamma_c \cap \Gamma_t \cap \Gamma_u = \{\emptyset\}$, contact may occur. A displacement increment field is called kinematically admissible (*K.A.*) if the following compatibility conditions are fulfilled:

$$\Delta\varepsilon(\Delta u^k) = \nabla_s \Delta u^k \quad \text{in } \Omega; \qquad \Delta u^k = \Delta\bar{u} \quad \text{on the essential boundary } \Gamma_u \tag{20}$$

A stress increment field is said to be statically admissible (*S.A.*) if the following equilibrium equations are satisfied:

$$\operatorname{div}(\Delta\sigma^s) = 0 \quad \text{in } \Omega; \qquad \Delta t(\Delta\sigma^s) = \Delta\sigma^s n = \Delta\bar{t} \quad \text{on the natural boundary } \Gamma_t \tag{21}$$

in which n is the outward unit normal to domain Ω.

The use the incremental formulation with the bipotential method leads to the following bifunctional:

$$\Delta\beta(\Delta u, \Delta\sigma) = \int_\Omega \big(\Delta V(\Delta\varepsilon(u)) + \Delta W(\Delta\sigma)\big)\, d\Omega - \int_{\Gamma_u} \Delta t(\Delta\sigma)\Delta\bar{u}\, d\Gamma - \int_{\Gamma_t} \Delta\bar{t}\Delta u\, d\Gamma + \int_{\Gamma_c} \Delta b_c(-\Delta u, \Delta t)\, d\Gamma \tag{22}$$

We prove that a field couple $(\Delta u, \Delta\sigma)$, the exact solution of boundary value problem, defined by Eqs. (20), (21) and the constitutive laws (17), (19), is also a solution to the following variational principles:

$$\operatorname*{Inf}_{\Delta u^k\, KA} \Delta\beta(\Delta u^k, \Delta\sigma); \qquad \operatorname*{Inf}_{\Delta\sigma^s\, SA} \Delta\beta(\Delta u, \Delta\sigma^s) \tag{23}$$

For the variational formulation in terms of displacements, the terms which do not depend on the incremental field Δu disappear and Eq. (22) is reduced to

$$\Delta\Psi(\Delta u) = \int_\Omega \Delta V(\Delta\varepsilon(u))\, d\Omega - \int_{\Gamma_t} \Delta\bar{t}\Delta u\, d\Gamma + \int_{\Gamma_c} \Delta b_c(-\Delta u, \Delta t)\, d\Gamma \tag{24}$$

Therefore, the kinematical variational principle becomes

$$\operatorname*{Inf}_{\Delta u^k\, KA} \Delta\Psi(\Delta u^k) \tag{25}$$

5. Least squares discretization

The displacement and strain increment fields are expressed with respect to an unknown nodal displacement increment vector ΔU as

$$\Delta u(x) = \bar{\phi}(x)\Delta U; \qquad \Delta\varepsilon = \bar{B}(x)\Delta U \tag{26}$$

where $\bar{B}(x) = \nabla_s(\bar{\phi}(x))$ and ∇_s is the symmetric gradient operator.

The discretized form of Eq. (24) is then a set of nonlinear equations:

$$\Delta\Psi(\Delta U) = \int_{\Omega} \Delta V(\bar{B}\Delta U)\,d\Omega - \int_{\Gamma_t} \bar{\phi}^T \Delta\bar{t}\,d\Gamma + \int_{\Gamma_c} \Delta b_c(-\bar{\phi}\Delta U, \Delta t)\,d\Gamma \tag{27}$$

The bipotential of the contact with friction isn't differentiable everywhere which poses problems at the mathematical programming level. In order to overcome this difficulty, we suggest using the regularization method. For this purpose, we can introduce the following differentiable function, which will be added, by using the inf-convolution concept, to the incremental bipotential Δb_c.

$$\Delta b' = \frac{K_t}{2}\left(-\Delta u_t + \Delta u_t^f\right)^2 + \frac{K_n}{2}\left(-\Delta u_n + \Delta u_n^f\right)^2 \tag{28}$$

where K_t and K_n are the penalization factors, Δu_n^f and Δu_t^f are the fictitious increments computed from the actual displacement increment Δu and the previous contact forces increments Δt, so that

$$\Delta u_n = \Delta u_n^f + \Delta t_n/K_n; \qquad \Delta u_t = \Delta u_t^f + \Delta t_t/K_t \tag{29}$$

We show that Δb_c can be written as follows: $\Delta b_c = \Delta b_n + \Delta b_t$ with

$$\begin{aligned}
\Delta b_n &= \operatorname*{Inf}_{-\Delta u_n^f}\left(-t_{n0}\left(-\Delta u_n^f\right) + \frac{K_n}{2}\left(-\Delta u_n + \Delta u_n^f\right)^2\right)\\
\Delta b_t &= \operatorname*{Inf}_{-\Delta u_t^f}\left(-t_{t0}\left(-\Delta u_t^f\right) + \mu(t_{n0} + \Delta t_n)\left\|-\Delta u_t^f\right\| + \frac{K_t}{2}\left(-\Delta u_t + \Delta u_t^f\right)^2\right)
\end{aligned} \tag{30}$$

In this case, the increments of stresses are not discretized like the principal stresses, but can be deduced starting from the value from the increments from displacements by the equation:

$$\Delta t \in \partial_{-\Delta u}\Delta b_c(-\bar{\phi}\Delta U, \Delta t) \tag{31}$$

In addition, the problem of coupling of traction increments with those of displacements is solved by using an iterative procedure based on the fixed point method. The mathematical programming is made by the optimisation code MINOS [12].

6. Numerical results

We consider the frictional contact between the workpiece and the fixed rigid tool (see Fig. 1). The problem is solved as a plane strain state. The thermal effects are not taken into account. The parameters for this problem are as follows: Young's modulus of 210 000 MPa, Poisson's ratio of 0.3 and yield stress of 500 MPa. The other parameters of the cutting process are: friction coefficient of 0.3, cutting angle $\gamma = 0°$ and cutting depth $h = 0.1$ mm. The workpiece is discretized by 68 nodes and rectangular background cells, with 4 Gauss integration on each cell (Fig. 1 right). The distribution of the cutting forces is illustrated in Fig. 2.

6.1. Influence cutting angle

The mechanical properties of material are: Young's modulus $E = 210\,000$ MPa, Poisson's ratio $\nu = 0.3$ and yield stress $\sigma_Y = 300$ MPa. The other parameters of the cutting process considered here are: friction coefficient $\mu = 0.1$, cutting depth $h = 8$ mm and cutting angle $\gamma = 0°$, $5°$ and $10°$. The displacement field for different values of the cutting angle is shown on Fig. 3.

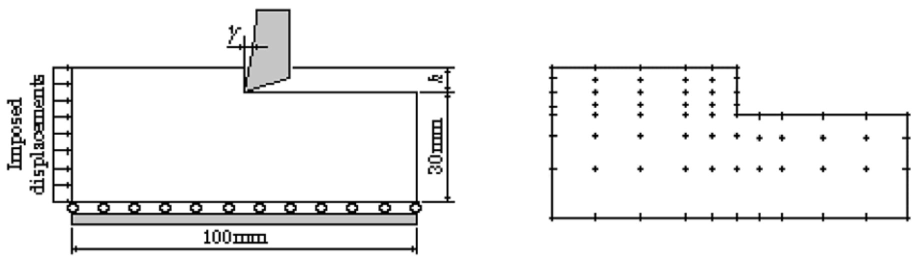

Fig. 1. Geometry with boundary conditions (left); Irregular nodal arrangement (right).

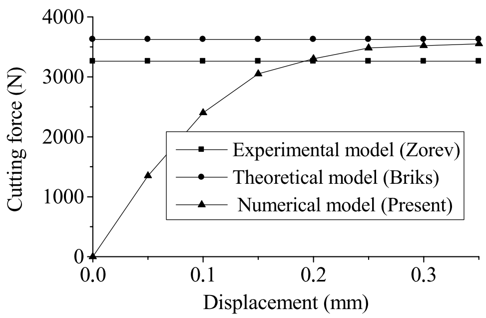

Fig. 2. Distribution of the cutting forces.

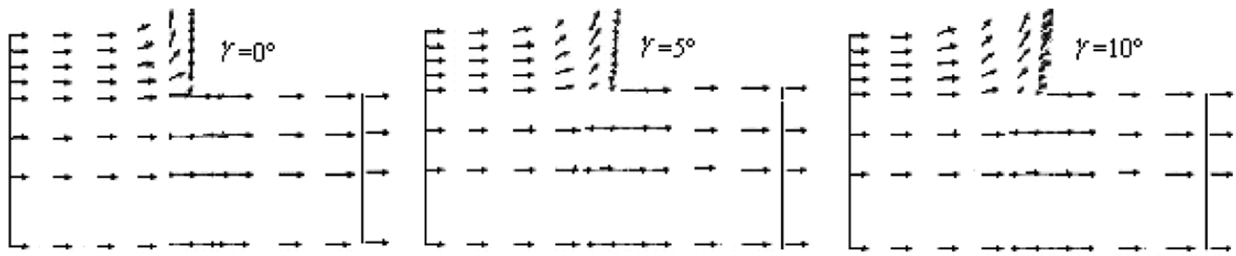

Fig. 3. Displacement field for different values of the cutting angle.

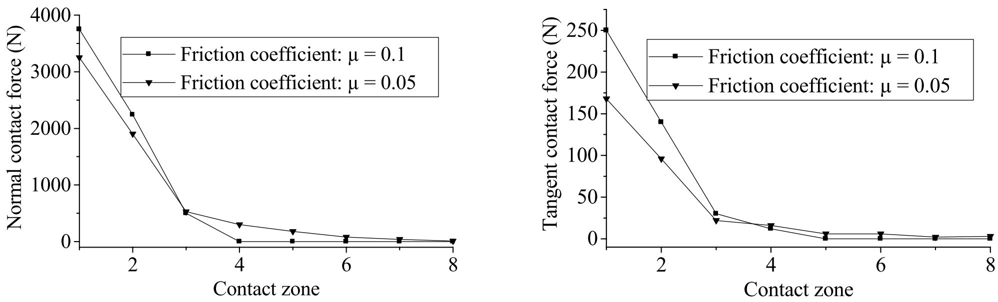

Fig. 4. Variation of contact forces along the contact surface for different values of the friction coefficient.

6.2. *Influence coefficient of friction*

Here, we keep the same mechanical properties indicated in the preceding section, but we take other parameters of the cutting process: cutting depth $h = 8$ mm, cutting angle $\gamma = 0°$ and friction coefficient $\mu = 0.05$ and 0.1 (Fig. 4).

7. Conclusion

The elastoplastic meshless formulation is presented for simulation of cutting process. Special emphasis is placed on the treatments of essential boundary conditions and friction boundaries. However, the MLS method presents some issues to impose Dirichlet boundary conditions when using shape functions that do not satisfy the Kronecker delta property. To solve this problem, this paper proposes the transformation method. In addition, the non-differentiable of the bipotential representing the contact with friction is surmounted by the use of the regularization procedure by penalization. A second difficulty which does not miss importance is the presence of a term of coupling between the contact and friction in the bipotential function. This problem of coupling is solved by the use of an iterative procedure based on the fixed point method. The numerical example is successfully analyzed.

This work can be extended in the future by taking into account other parameters as hardening, temperature and large deformations.

References

[1] T. Belytschko, Y.Y. Lu, L. Gu, Element-free Galerkin methods, Inter. J. Numer. Meth. Eng. 37 (1994) 229–256.

[2] Y.Y. Lu, T. Belytschko, L. Gu, A new implementation of the element free Galerkin method, Comput. Meth. Appl. Mech. Eng. 113 (1994) 397–414.

[3] T. Belytschko, Y.Y. Lu, L. Gu, Fracture and crack growth by element-free Galerkin methods, Model. Simul. Mater. Sci. Eng. 2 (1994) 519–534.

[4] Y. Krongauz, T. Belytschko, Enforcement of essential boundary conditions in meshless approximations using finite elements, Comput. Meth. Appl. Mech. Eng. 131 (1996) 133–145.

[5] I.A. Time, Resistance of Metal and Wood to Cutting (Soprotivleniye Metallov i Dereva Rezaniyu), 1870.

[6] A.A. Briks, Cutting of Metals, 1896.

[7] W. Liu, S. Jun, Y. Zhang, Reproducing kernel particle methods, Int. J. Numer. Meth. Fluids 20 (1995) 1081–1106.

[8] J.S. Chen, C.M.O.L. Roque, C. Pan, S.T. Button, Analysis of metal forming process based on meshfree method, J. Mater. Process. Technol. 80–81 (1998) 642–646.

[9] J.S. Chen, H.P. Wang, New boundary condition treatments in meshfree computation of contact problems, Comput. Meth. Appl. Mech. Eng. 187 (2000) 441–468.

[10] P. Lancaster, K. Salkauskas, Surfaces generated by moving least squares methods, Math. Comput. 37 (1981) 141–158.

[11] G. De Saxce, Z.Q. Feng, The bipotential method: A constructive approach to design the complete contact law friction and improved numerical algorithms, Math. Comput. Model. 28 (1998) 225–245.

[12] B.A. Murtagh, M.A. Saunders, Minos 5.1 User's Guide, Standford University, 1987.