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Note on the POD-based time interpolation from successive PIV images

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Abstract

To enhance the temporal resolution of the PIV measurements of pseudo-periodic turbulent flows, Proper Orthogonal Decomposition (POD) has been previously used to time interpolate PIV database. In this note, it is demonstrated that such POD interpolation is equivalent to the classical mathematical interpolation when dealing with the whole POD eigenfunctions, since POD is a linear transform. In fact, the POD-based time interpolation is only valid for the large scale structures of the flow. The advantage of using POD procedure resides in its efficiency in extracting the dominant flow structures. In this sense, other interpolation methods such as turbulent filtering procedures could provide similar results. ***To cite this article: E. Bouhoubeiny, P. Druault, C. R. Mecanique 337 (2009).***

1. Introduction

Understanding the dynamics of the transport of momentum, heat, mass and noise in turbulent flows is of fundamental and practical significance in many branches of engineering involving fluid flow. For such investigation, the characterization of the three-dimensional structures present in turbulent flows remains a great challenge due to their influence on mixing processes, noise emission, heat transfer processes, etc. Experimentally, if one wants to access the dynamics of large scale structures, not only a satisfactory spatial resolution of the instantaneous flow field is required but also a satisfactory temporal resolution of the flow pattern. Furthermore, space- and time-resolved experimental database are more and more needed for numerical simulations of turbulent flow. Indeed, such experimental database could be used not only to validate numerical database but also to provide initial and/or inflow boundary conditions [1]. However, a high space- and time-resolved measurements of the three velocity components in a three-dimensional complex turbulent flow field remains still difficult. For instance, concerning the time resolution, even if the recent development of Time Resolved PIV system with a repetition rate superior to 10 kHz has a great future potential for unsteady turbulent flow analysis, its implementation remains still complex to be fully generalized. Com-

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mercial TRPIV system with several hundred Hz time resolution is then an alternative solution. However with such equipment, to access a high time flow resolution, it is necessary to develop post-processing tools which reconstruct the lack of available experimental information.

To enhance the spatial resolution of measurements, a lot of methodologies including Linear Stochastic Estimation [2,3], Proper Orthogonal Decomposition (POD) [4,5], kriging methods [5,6], classic mathematical interpolations (linear, splines, ...), have already been developed to reconstruct the lack of spatial data. For instance, specific applications of POD called *gappy POD* have been developed for reconstructing missing data [4]. These last methodologies are based on the minimization of the error between the valid data and its POD expansion using a small subset of the available eigenfunctions. Such mathematical tools have been applied with success to reconstruct gappy data from Particle Image Velocimetry (PIV) measurements [6,7]. Recently, some new reconstruction methodologies based on the kriging approach have also demonstrated their potential in accurately reconstructing spatial missing PIV data [6].

Conversely, the time interpolation from successive PIV images has not been extensively studied in the past. Druault et al. [8] have developed a new method for interpolating the time velocity information between two consecutive PIV vector field measurements in a spark ignition pseudo-periodic engine flow. Indeed, by performing a POD over the whole available Time Resolved PIV (TRPIV) velocity field snapshots, temporal POD coefficients were cubic spline interpolated in time permitting to recover the time information between two consecutive PIV snapshots. With such time interpolation, an analysis of the dynamics of the large scale flow structures occurring in such a flow was then possible.

Based on this preliminary work, we propose in this note to compare this POD-based interpolation method to the *direct* cubic spline interpolation. Then, a discussion of the PIV database time interpolation is provided.

2. Mathematical description of PIV database and POD application

Assume that TRPIV measurements have been performed in a pseudo-periodic turbulent flow such as a spark ignition engine flow. In such a flow, to access to the time-space flow dynamics, it is necessary to try to synchronize the engine speed with the Time Resolved PIV system. Based on a classical TRPIV system of several hundred Hz, it is possible to perform several vector field measurements per engine cycle for selected engine speeds. We then assume that such TRPIV measurements provide N_t instantaneous two-dimensional velocity vector fields $V(t_k, X_{ij})$; with k varying from 1 to N_t and (i, j) varying from 1 to n_x and n_y respectively (where $n_x \times n_y$ corresponds to the total number of PIV measurement locations). These measurements are supposed to be performed during a great number of engine cycles. The following developments will be performed only for one velocity component called u. Similar developments can be done for the other velocity components.

Based on the PIV velocity snapshots, a snapshot POD [10] leads to the following flow decomposition:

$$u(t_k, X_{ij}) = \sum_{n=1}^{N_{mod}} a^{(n)}(t_k) \Phi_u^{(n)}(X_{ij}) \tag{1}$$

where $a^{(n)}$ are the POD temporal coefficients and $\Phi_u^{(n)}$ are the spatial POD eigenfunctions of projection relative to u velocity component [11]. N_{mod} is the total mode number of the POD decomposition and in our case, $N_{mod} = N_t$.

Details of the implementation of the snapshot POD procedure can be found in Holmes et al. [9] or in Druault et al. [8].

We propose to interpolate velocity field in time from successive instantaneous PIV vector fields, for instance between $u(t_{k_1}, X_{ij})$ and $u(t_{k_1+1}, X_{ij})$ with $k_1 \in [1, N_t - 1]$. For such a purpose, two procedures are considered: (i) the *direct* time interpolation of raw PIV database $u(t_k, X_{ij})$, and (ii) the time interpolation of POD eigenfunctions $a^{(n)}(t_k)$ computed from raw PIV database. For a sake of clarity, the spatial variable X_{ij} will be omitted in the following formulae: $u(t_{k_1})$ and $\Phi_u^{(n)}$ will be used instead of $u(t_{k_1}, X_{ij})$ and $\Phi_u^{(n)}(X_{ij})$ respectively.

3. Mathematical cubic spline interpolation

Based on the knowledge of $u(t_{k_1})$ and $u(t_{k_1+1})$, $k_1 \in [1, N_t - 1]$, the cubic spline time interpolation is implemented to estimate $u_{int}(t)$, with t in $[t_{k_1}, t_{k_1+1}]$. Such interpolation provides a polynomial of the third order of the following shape:

$$u_{int}(t) = C_{k_1}^0 + C_{k_1}^1(t - t_{k_1}) + C_{k_1}^2(t - t_{k_1})^2 + C_{k_1}^3(t - t_{k_1})^3 \tag{2}$$

with $C_{k_1}^l$ ($l = 1, 4$) coefficients are expressed as follows:

$$C_{k_1}^0 = u(t_{k_1}) \tag{3}$$

$$C_{k_1}^1 = \frac{u(t_{k_1+1}) - u(t_{k_1})}{t_{k_1+1} - t_{k_1}} - \frac{t_{k_1+1} - t_{k_1}}{6}\big[2u''(t_{k_1}) + u''(t_{k_1+1})\big] \tag{4}$$

$$C_{k_1}^2 = \frac{u''(t_{k_1})}{2} \tag{5}$$

$$C_{k_1}^3 = \frac{1}{6(t_{k_1+1} - t_{k_1})}\big(u''(t_{k_1+1}) - u''(t_{k_1})\big) \tag{6}$$

where $'$ and $''$ indicate the partial and second-order partial derivative in the t direction respectively.

4. Use of proper orthogonal decomposition for time interpolation

Another way for interpolating TRPIV database in time consists in using the POD decomposition [8]. Such flow decomposition leads to the separation of time and space variables and instantaneous velocity component $u(t_{k_1})$ can be expressed with Eq. (1). The time interpolation of the TRPIV database is then reduced to the interpolation of POD temporal coefficients. The cubic spline time interpolation in the $[t_{k_1}, t_{k_1+1}]$ interval of the nth POD eigenfunction leads to the following formula:

$$a_{int}^{(n)}(t) = D_{k_1}^{0(n)} + D_{k_1}^{1(n)}(t - t_{k_1}) + D_{k_1}^{2(n)}(t - t_{k_1})^2 + D_{k_1}^{3(n)}(t - t_{k_1})^3 \tag{7}$$

where $D_{k_1}^l$ ($l = 1, 4$) coefficients have similar expressions as the $C_{k_1}^l$ ones (see Eqs. (3)–(6)) replacing u with $a^{(n)}$:

$$D_{k_1}^{0(n)} = a^{(n)}(t_{k_1}) \tag{8}$$

$$D_{k_1}^{1(n)} = \frac{a^{(n)}(t_{k_1+1}) - a^{(n)}(t_{k_1})}{t_{k_1+1} - t_{k_1}} - \frac{t_{k_1+1} - t_{k_1}}{6}\big[2a^{(n)''}(t_{k_1}) + a^{(n)''}(t_{k_1+1})\big] \tag{9}$$

$$D_{k_1}^{2(n)} = \frac{a^{(n)''}(t_{k_1})}{2} \tag{10}$$

$$D_{k_1}^{3(n)} = \frac{1}{6(t_{k_1+1} - t_{k_1})}\big(a^{(n)''}(t_{k_1+1}) - a^{(n)''}(t_{k_1})\big) \tag{11}$$

Based on this time interpolation, instantaneous velocity field is reconstructed as follows (see Eq. (1))

$$u_{int.pod}(t) = \sum_{n=1}^{N_{mod}} a_{int}^{(n)}(t)\Phi_u^{(n)} \tag{12}$$

Spatial eigenfunctions $\phi_u^{(n)}$ remain the same as the ones initially computed.

5. Comparative analysis of both interpolation procedures

A comparative analysis of both interpolated results is now performed. Using the fact that instantaneous velocity field can be expressed in the POD basis, $C_{k_1}^l$ coefficients are re-arranged into the following equivalent equations:

$$C_{k_1}^0 = u(t_{k_1}) = \sum_{n=1}^{N_{mod}} a^{(n)}(t_{k_1})\phi_u^{(n)} = \sum_{n=1}^{N_{mod}} D_{k_1}^{0(n)}\phi_u^{(n)} \tag{13}$$

Doing the same thing with the coefficient $C_{k_1}^1$ (Eq. (4)), we obtain:

$$C_{k_1}^1 = \sum_{n=1}^{N_{mod}} \left(\frac{a^{(n)}(t_{k_1+1}) - a^{(n)}(t_{k_1})}{t_{k_1+1} - t_{k_1}} - \frac{t_{k_1+1} - t_{k_1}}{6}\big[2a^{(n)''}(t_{k_1}) + a^{(n)''}(t_{k_1+1})\big] \right)\phi_u^{(n)} \tag{14}$$

Using Eq. (9), it becomes: $C_{k_1}^1 = \sum_{n=1}^{N_{mod}} D_{k_1}^{1(n)} \phi_u^{(n)}$. In a similar way, the following relations are obtained:

$$C_{k_1}^2 = \sum_{n=1}^{N_{mod}} D_{k_1}^{2(n)} \phi_u^{(n)} \quad \text{and} \quad C_{k_1}^3 = \sum_{n=1}^{N_{mod}} D_{k_1}^{3(n)} \phi_u^{(n)} \tag{15}$$

Based on these last equations, Eq. (2) becomes:

$$u_{int}(t) = \sum_{n=1}^{N_{mod}} \big(D_{k_1}^{0(n)} + D_{k_1}^{1(n)}(t - t_{k_1}) + D_{k_1}^{2(n)}(t - t_{k_1})^2 + D_{k_1}^{3(n)}(t - t_{k_1})^3\big) \phi_u^{(n)} \tag{16}$$

which corresponds exactly to Eq. (12) that is whatever t in $[t_{k_1}, t_{k_1+1}]$, $u_{int}(t) = u_{int.pod}(t)$. It is then demonstrated that when interpolating the whole available POD eigenfunctions, such time interpolation is exactly the same as the *direct* interpolation of raw available data.

6. Discussion

In this note, it is demonstrated that based on TRPIV database, the time cubic spline interpolation of the whole POD eigenfunctions is reduced to the time cubic spline interpolation of the raw available PIV database. This result is directly linked to a property of the POD procedure which is a linear transform allowing the separation of time and space variables of the velocity field. When dealing with the whole available POD temporal coefficients, using POD is favourable only in the sense that we have to interpolate $N_{mod} = N_t$ scalar functions corresponding to the POD coefficients instead of performing $n_x \times n_y$ time interpolations for each velocity component.

In fact, as it has been previously noticed [8], the cubic spline interpolation of POD coefficients is only satisfactory for the first POD coefficients. The determination of the number of the first POD modes which can be accurately modelled with cubic spline interpolation is directly correlated to the sampling frequency of the TRPIV system and to the in-cylinder engine turbulent flow under consideration. Conversely, the mathematical time interpolation of high POD mode numbers may be questionable. Indeed, the PIV sampling frequency is smaller than the one associated with the corresponding small scale structures which might not then be accurately time interpolated.

As a first conclusion, performing a time interpolation from successive PIV images is only valid for the large scale flow structures. The advantage of using POD flow decomposition for such interpolation resides in its efficiency in extracting the dominant modes corresponding to the large scale energetic structures. Such time reconstruction procedure can be related to other ones based on *gappy* POD which are used for the space interpolation of PIV data [7]. Indeed, *gappy* POD procedure takes into account also a small subset of the available POD modes.

Otherwise, other mathematical procedures could be used for performing a similar time interpolation from successive PIV images obtained in such a pseudo-periodic flow. Indeed, it seems that filtering methods such as turbulence filtering procedures [12] and/or wavelet flow decomposition or others can be similarly applied to TRPIV database in order to extract the velocity data information associated with the large scale flow structures. Resulted filtering PIV database could be then time interpolated with the aid of classical mathematical procedures. We attempt that the implementation of these other methods will provide similar results as the ones deduced from POD-based time interpolation.

As a second conclusion, when dealing with pseudo-periodic flows, any mathematical tool allowing the extraction of the large scale flow structures from successive PIV images, can be used for time interpolating these structures.

Implementating mathematical post-processing tools aiming at time reconstructing the large scale flow dynamics of a pseudo-periodic flow is of great interest for many reasons. First, such time reconstruction leads to an accurate flow dynamic description of the large scale flow structures during an engine cycle. Based on this available time resolved database, new analyses can then be performed like the cyclic variability analysis [13] which is of great importance for the engine efficiency and for reducing the exhaust emission. Second, having a two-dimensional experimental database with a high time resolution may offer some new possible applications related to the analysis and comparison with Large Eddy Simulation database. Third, knowing that TRPIV system at the repetition rates required for in-cylinder flows (superior to 10 kHz) is not routine, commercial lower speed TRPIV system ($\approx$500 Hz) may be an alternative solution. This last TRPIV system allows to access vector fields of greater consistency and quality that we expect could be produced when using a 10 kHz repetition rate. The implementation of the time reconstruction procedure is then of interest in that high quality vector fields can be obtained at lower repetition rate from direct experimental measurements and vector fields at a higher temporal sampling rate are then deduced.

Finally, these mathematical developments can also have some experimental measurement applications in other flows exhibiting quasi-periodic phenomena associated with a *regular* passage of large scale flow structures.

Acknowledgements

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