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# Non-saturating nonlinear kinematic hardening laws

## *Lois d'écrouissage cinématique non linéaire sans saturation*

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ABSTRACT

To compensate the drawback of most kinematic hardening laws who exhibit hardening saturation, a solution is proposed by replacing the accumulated plastic strain rate in the springback term by a rate related to the kinematic hardening variable itself. The proposed approach defines a power-law counterpart to the linear (Prager) and exponential (Armstrong–Frederick) laws.

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RÉSUMÉ

Pour pallier le défaut de la plupart des lois d'écrouissage cinématique qui présentent une saturation de l'écrouissage, une solution est proposée en remplaçant le taux de déformation plastique cumulée $\dot{p}$ du terme de rappel par un taux relié à la variable d'écrouissage cinématique elle-même. L'approche proposée définit une loi puissance pour l'écrouissage cinématique, complétant ainsi le panel des lois linéaire (Prager) et exponentielle (Armstrong–Frederick) disponibles en plasticité.

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## 1. Introduction

Most modern nonlinear kinematic hardening laws in metal plasticity have the generic form

$$\dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\epsilon}}^p - \boldsymbol{\mathcal{B}}(\boldsymbol{X}, p, \boldsymbol{\sigma})\dot{\mathcal{P}}\big(\boldsymbol{X}, \boldsymbol{\sigma}, \dot{\boldsymbol{\epsilon}}^p\big) \tag{1}$$

where $\boldsymbol{X}$ is the kinematic hardening, $p$ is the accumulated plastic strain, $\boldsymbol{\sigma}$ is the stress, $\dot{\boldsymbol{\epsilon}}^p$ is the plastic strain rate, $C$ is a material parameter, and the springback term $\boldsymbol{\mathcal{B}}\dot{\mathcal{P}}$ is sometimes replaced by a sum $\sum \boldsymbol{\mathcal{B}}_k\dot{\mathcal{P}}_k$. The scalar function $\dot{\mathcal{P}}$ (as $\dot{\mathcal{P}}_k$) is a homogeneous function of degree 1 in $\dot{\boldsymbol{\epsilon}}^p$, such as $\dot{\mathcal{P}}(\boldsymbol{X}, \boldsymbol{\sigma}, \lambda\dot{\boldsymbol{\epsilon}}^p) = \lambda\dot{\mathcal{P}}(\boldsymbol{X}, \boldsymbol{\sigma}, \dot{\boldsymbol{\epsilon}}^p)$ $\forall\lambda \geqslant 0$. The tensorial function $\boldsymbol{\mathcal{B}}$ has usually the sign of $\boldsymbol{X}$ and $\|\boldsymbol{\mathcal{B}}\|$ increases when the loading increases (in norm). This last feature gives back the concave shape of stress–strain curves for metals.

For instance, this is the form of Armstrong–Frederick law [1],

$$\dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\epsilon}}^p - \gamma\boldsymbol{X}\dot{p} \tag{2}$$

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with a material parameter $\gamma$, but also of Burlet–Cailletaud law [2], of Chaboche law [3] and of Ohno–Wang law [4]. The normal $\boldsymbol{n} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$ of the yield surface $f = 0$ is used in some models,[1] it is a function of the stress $\boldsymbol{\sigma}$ and of the kinematic hardening $\boldsymbol{X}$, the plastic strain rate reading then $\dot{\boldsymbol{\epsilon}}^p = \dot{p}\boldsymbol{n}(\boldsymbol{\sigma}, \boldsymbol{X})$.

In uniaxial monotonic tension the generic law (1) simplifies in $\dot{X} = (C - B)\dot{\epsilon}_p$, with a positive increasing nonlinear function $B$, rate independent. One observes then that a saturation $\dot{X} = 0$, $X = X_\infty = Const$, is reached for all laws ensuring $B \to C$ at high loading.

Different possibilities to avoid such a saturation of the kinematic hardening exist: make $\gamma = \gamma(p)$ a decreasing (to zero) function of the accumulated plastic strain as in [5], make $C$ dependent of the plastic strain amplitude, through an index function written in the strain space, as in [6]. None recover the power law shape at high plastic strains. One proposes in present note simple ways to naturally gain the non-saturation of the kinematic hardening, but also to define for kinematic hardening a power law counterpart to the usual exponential law.

## 2. A first family of non-saturating kinematic hardening laws

Kinematic hardening $\boldsymbol{X}$ is a thermodynamics force associated with a tensorial internal state variable denoted $\boldsymbol{\alpha}$, homogeneous to a strain. It is often derived from a quadratic thermodynamics potential as [7]

$$\boldsymbol{X} = \frac{2}{3}C(T)\boldsymbol{\alpha} \tag{3}$$

where $C$ is the hardening parameter previously introduced, temperature dependent. Initially isotropic and plastically incompressible materials are considered next, with then the expression $p = \int (\frac{2}{3}\dot{\boldsymbol{\epsilon}}^p : \dot{\boldsymbol{\epsilon}}^p)^{1/2}\,\mathrm{d}t$ for the accumulated plastic strain and with the deviatoric plastic strain rate $\boldsymbol{\epsilon}^p = \boldsymbol{\epsilon}^{p\prime}$. In Prager law of linear hardening the internal variable $\boldsymbol{\alpha}$ is equal to $\boldsymbol{\epsilon}^p$. In case of (anisothermal) Armstrong–Frederick law it is given by the evolution law $\dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\epsilon}}^p - \gamma\boldsymbol{\alpha}\dot{p}$. It is almost equal to the plastic strain either when $\gamma$ is small or when the plastic strain remains limited.

Among others, a law avoiding kinematic hardening saturation is the following, valid for anisothermal cases (see Section 4 for thermodynamics considerations):

$$\dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\epsilon}}^p - \frac{3\Gamma}{2}\boldsymbol{X}\dot{a}, \quad \dot{a} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\alpha}} : \dot{\boldsymbol{\alpha}}} \tag{4}$$

in which the back stress is now governed by von Mises norm $\dot{a}$ of the rate $\dot{\boldsymbol{\alpha}}$ and with $\Gamma$ as material parameter.

In case of isothermal loading, $C$ is constant, and law (4) reads

$$\dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\epsilon}}^p - \Gamma\boldsymbol{X}\dot{x}, \quad \dot{x} = \sqrt{\frac{3}{2}\dot{\boldsymbol{X}} : \dot{\boldsymbol{X}}} \tag{5}$$

and leads to a non-vanishing rate $\dot{\boldsymbol{X}}$ solution of the separate variables differential equation $\dot{\boldsymbol{X}} + \Gamma\boldsymbol{X}\dot{x} = \frac{2}{3}C\dot{\boldsymbol{\epsilon}}^p$.

In order to recover a power-law like response in monotonic loading, the law (4) can be generalized as

$$\dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\epsilon}}^p - \frac{3\Gamma}{2}X_{eq}^{M-2}\boldsymbol{X}\dot{a}, \quad X_{eq} = \sqrt{\frac{3}{2}\boldsymbol{X} : \boldsymbol{X}} \tag{6}$$

or (isothermal case):

$$\dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\epsilon}}^p - \Gamma X_{eq}^{M-2}\boldsymbol{X}\dot{x} \tag{7}$$

with an additional parameter $M \geqslant 2$ (already introduced in [3] in another context).

In uniaxial tension–compression (along 1), $\boldsymbol{\epsilon}^p = \mathrm{diag}[\epsilon_p, -\frac{1}{2}\epsilon_p, -\frac{1}{2}\epsilon_p]$, $\boldsymbol{X} = \mathrm{diag}[\frac{2}{3}X, -\frac{1}{3}X, -\frac{1}{3}X]$ so that $X_{eq} = |X|$, $\dot{x} = |\dot{X}|$. The proposed law reduces to the scalar expression

$$\dot{X} + \Gamma|X|^{M-2}X|\dot{X}| = C\dot{\epsilon}_p \quad \text{(1D)} \tag{8}$$

- *In case of monotonic tension*, $X$ and $\dot{X}$ are positive and Eq. (8) reduces to $(1 + \Gamma X^{M-1})\dot{X} = C\dot{\epsilon}_p$ therefore to the kinematic hardening solution of

$$X + \frac{1}{M}\Gamma X^M = C\epsilon_p \tag{9}$$

At large plastic strains $X$ is unbounded and behaves in $\epsilon_p^{1/M}$

$$X \approx K\epsilon_p^{1/M}, \quad K = \left(\frac{MC}{\Gamma}\right)^{1/M} \tag{10}$$

[1] Often $f = (\boldsymbol{\sigma} - \boldsymbol{X})_{eq} - R - \sigma_y$ in von Mises plasticity, with the isotropic hardening $R$ and the yield stress $\sigma_y$.

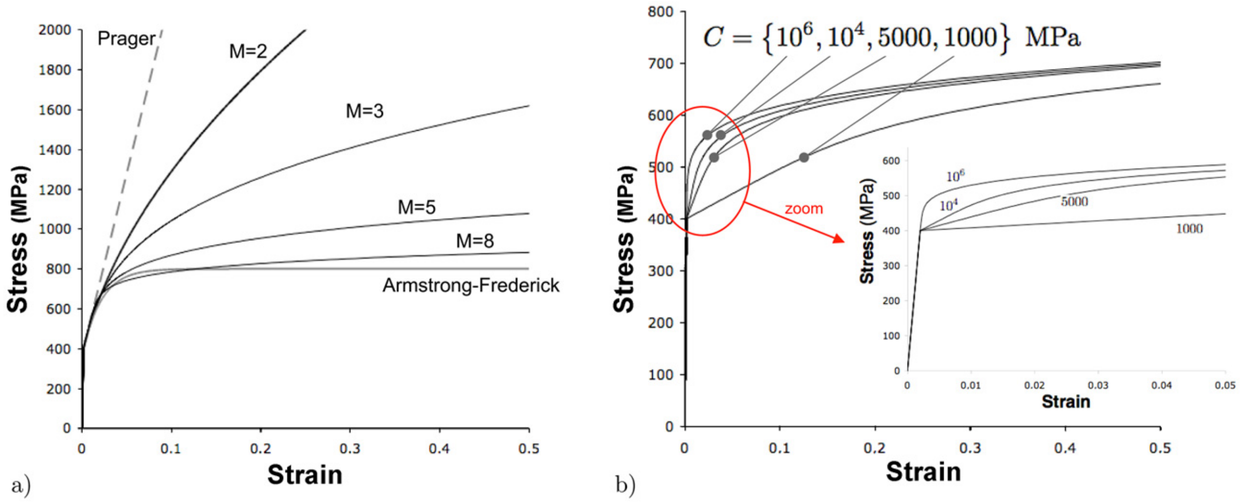


**Fig. 1.** Tensile stress–strain response from proposed non-saturating kinematic hardening law: a) compared to linear Prager law and Armstrong–Frederick saturating law ($C = 20\,000$ MPa, $\gamma = 50$) at given $C$ for different exponents $M$, b) at given $K = (MC/\Gamma)^{1/M}$ and $M$ for different values of parameter $C$ ($K = 347$ MPa, $M = 5$).

– *In case of symmetric cyclic loading,* $X$ ranges between $X_{\text{Max}}$ and $X_{\min} = -X_{\text{Max}}$, the same calculation now with $\dot{X} > 0$ in tension and $\dot{X} < 0$ in compression ends up to cycle stabilization and to the maximum kinematic hardening solution of

$$X_{\text{Max}} + \frac{1}{M}\Gamma X_{\text{Max}}^{M} = C\frac{\Delta\epsilon_p}{2} \tag{11}$$

and then to a cyclic hardening law[2] $\frac{\Delta\sigma}{2} = k + X_{\text{Max}}$ linear in plastic strain amplitude at small $\Delta\epsilon_p$ and asymptotically a power function at large $\Delta\epsilon_p$ with then

$$X_{\text{Max}} \approx K\left(\frac{\Delta\epsilon_p}{2}\right)^{1/M} \tag{12}$$

Again it is unbounded and no saturation is reached.

The tensile responses obtained for different sets of parameters are given in Fig. 1. Young's modulus is taken as $E = 200\,000$ MPa and $k = 400$ MPa is set. For the comparison with Prager and Armstrong–Frederick laws (Fig. 1a), the same constant $C = 20\,000$ MPa is used for all models and the chosen value for $\Gamma(M = 2)$ is $2.5 \times 10^{-3}$ MPa$^{-1}$ and corresponds to the same first $\frac{d\sigma}{d\epsilon}$ and second $\frac{d^2\sigma}{d\epsilon^2}$ derivatives at yielding onset than with Armstrong–Frederick law (for which $C = 20\,000$ MPa still and $\gamma = 50$). Parameters $\Gamma$ for other $M$ are chosen such as all the curves meet at point ($\epsilon = 0.02$, $\sigma = 655$ MPa).

Fig. 1b shows a feature specific to the present law: the possibility with large modulus $C$ ($10^6$ MPa in the example) to model very steep stress increase at low plastic strain. In the figure all stress–strain curves are plotted with the same value for modulus $K$, i.e. for the same power law limit at large plastic strains.

In cyclic loading a (classical) modeling flaw is encountered if the value of the kinematic hardening obtained in tension reaches the critical value $X_{\text{Max}} = \Gamma^{\frac{1}{1-M}}$. For $X_{\text{Max}} = \Gamma^{\frac{1}{1-M}}$, the slope $\frac{dX}{d\epsilon_p}$ becomes negative (!) right after load reversal. Such a flaw has been pointed out and solved in [4] simply by making linear the kinematic hardening after load reversal. The law proposed next uses this remedy.

## 3. Proposal of a non-saturating kinematic hardening law

In order to avoid kinematic hardening saturation, one proposes instead of Eq. (6) the following law, *this time with no flaw at large plastic strain amplitudes,*

$$\begin{cases} \boldsymbol{X} = \dfrac{2}{3}C\boldsymbol{\alpha} \\ \dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\epsilon}}^p - \dfrac{3\Gamma}{2C}X_{eq}^{M-2}\boldsymbol{X}\langle\dot{X}_{eq}\rangle \end{cases} \quad \text{or (isothermal)} \quad \dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\epsilon}}^p - \Gamma X_{eq}^{M-2}\boldsymbol{X}\langle\dot{X}_{eq}\rangle \tag{13}$$

[2] The constant $k = \sigma_y + R_\infty$ is the sum of the yield stress and of the (assumed) saturated isotropic hardening $R_\infty$.

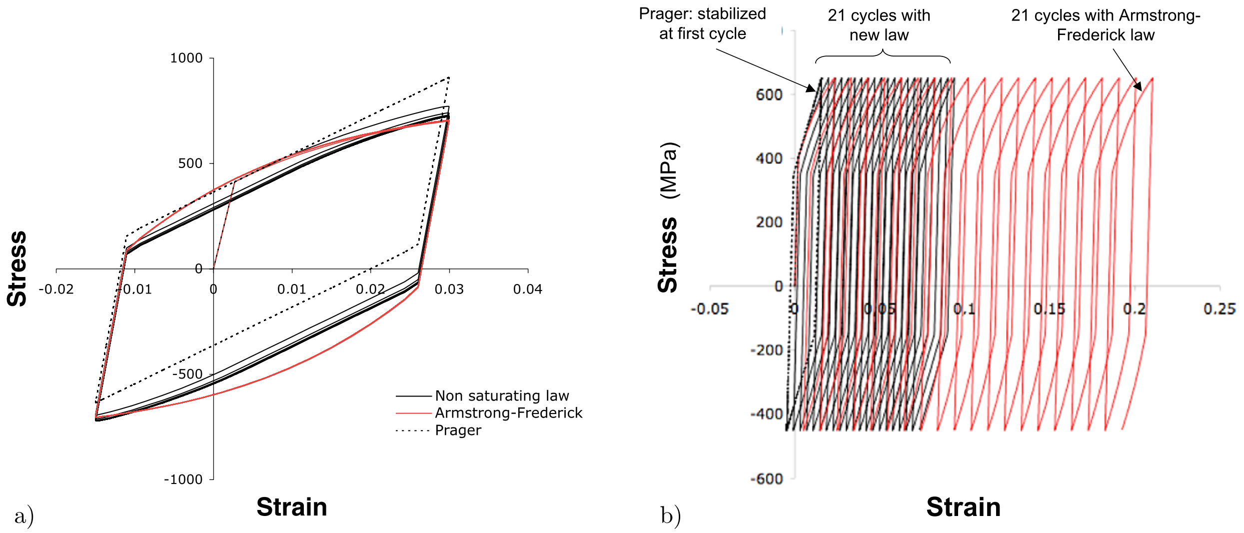


**Fig. 2.** Cyclic stress–strain responses at given $C$: a) strain control, stress in MPa, b) stress control – new law (Eq. (13), case $M = 2$, $C = 20\,000$ MPa, $\Gamma = 2.5 \times 10^{-3}$ $\text{MPa}^{-1}$) compared to linear Prager law and Armstrong–Frederick saturating law ($\gamma = 50$).

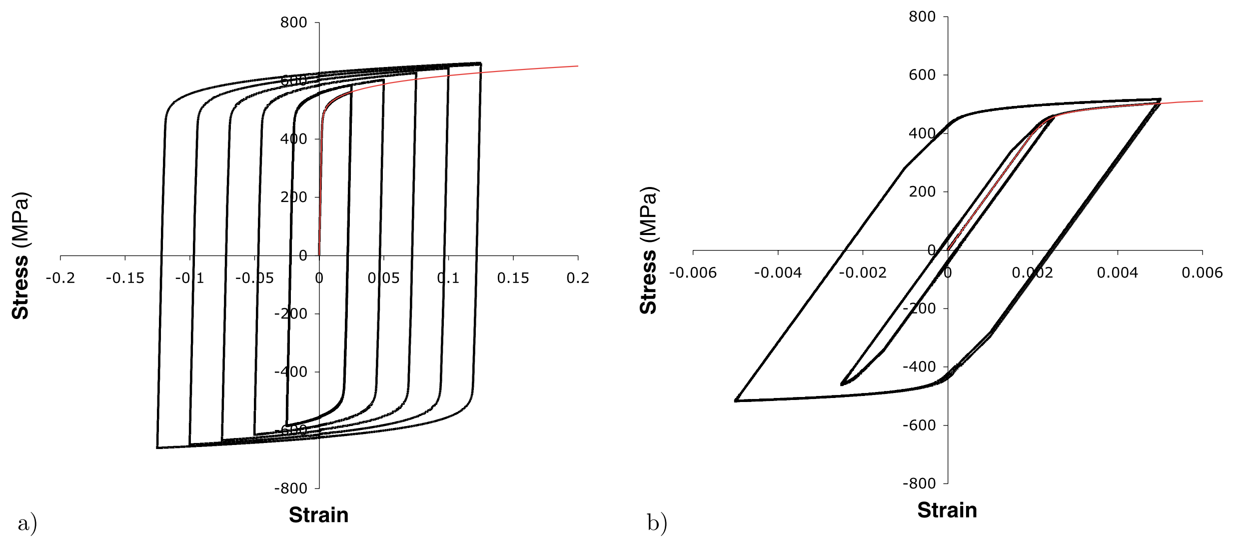


**Fig. 3.** Cyclic response obtained for increasing stress amplitudes (with $k = 400$ MPa, $C = 5 \times 10^5$ MPa, $M = 5$, $\Gamma = 5 \times 10^{-7}$ $\text{MPa}^{1-M}$): a) $\Delta\epsilon = 0.05$, 0.1, 0.15, 0.2, 0.25, b) $\Delta\epsilon = 0.0025$, 0.005.

where $\langle . \rangle$ stands for positive part, i.e. $\langle \dot{X}_{eq} \rangle = \dot{X}_{eq} = \frac{\mathrm{d}}{\mathrm{d}t}(\frac{3}{2}\boldsymbol{X} : \boldsymbol{X})^{1/2}$ when positive, $\langle \dot{X}_{eq} \rangle = 0$ else. The tensile response is unchanged compared to previous law. But a linear kinematic hardening is now obtained in the cycle parts at decreasing (in norm) kinematic hardening, i.e. at re-yielding just after load reversal (note that this feature is encountered in Ohno–Wang model). This is shown in Fig. 2a, in which a quite low value for $C$ is considered. Both the monotonic and cyclic features of the new kinematic hardening law (13) are illustrated in Fig. 1 (again with constant isotropic hardening), still with $E = 200\,000$ MPa and $k = 400$ MPa.

Cycle stabilization is obtained in case of symmetric (immediate, Fig. 3) and of non-symmetric periodic applied strains (cyclic softening up to stabilization, Fig. 1b).

Fig. 3 illustrates the main model feature for large values of $C$: the possibility to represent very steep stress increase at the onset of plasticity (with no visible elasticity/plasticity slope discontinuity), also then in case of cyclic loading. The stress-amplitude is increased after each two cycles (first loading case in Fig. 3a starting with $\Delta\epsilon = 5 \times 10^{-2}$, second loading case in Fig. 3b starting with $\Delta\epsilon = 5 \times 10^{-3}$). Such a smooth shape of cyclic strain–stress curves, very steep just out from elasticity domain and decreasing rapidly when yielding (but with no saturation), cannot be represented by means of a single Armstrong–Frederick law. As the value for $C$ is large, the linear part after load reversal is barely noticeable (Fig. 3b). The monotonic tensile model response is reported in the figures.

For the case $C = 20\,000$ MPa, the ratcheting behavior with the new law is found at given $C$ intermediate between linear Prager modeling (no ratcheting at all) and Armstrong–Frederick modeling (which usually overestimates ratcheting). This is shown in Fig. 2b where 21 cycles are presented. This is only a qualitative illustration of the model ratcheting feature, and due to quite large plastic strains failure would occur rapidly for the cyclic load considered. The ratchet step – i.e. the plastic strain increment over an hysteresis loop – for a stress varying cyclically between $\sigma_{\min} > -k$ and $\sigma_{\text{Max}} > k$ (with $\sigma_{\text{Max}} - \sigma_{\min} > 2k$) is gained in a closed form as

$$\delta\epsilon_p = \frac{(\sigma_{\text{Max}} - k)^M - (\sigma_{\min} + k)^M}{K^M} \tag{14}$$

It is found constant and related to the value of exponent $M$ and modulus $K$ governing the non-saturation of the kinematic hardening (and to the size of elasticity domain through $k$). Note that ratcheting is often modeled by the introduction of several kinematic hardening variables $\boldsymbol{X}_i$, setting $\boldsymbol{X} = \sum \boldsymbol{X}_i$ and taking for $k$ a relatively small value. According to the corresponding different plasticity mechanisms at the microscopic scale, it seems judicious to consider different laws, i.e. laws of different nature, of different mathematical expression for each $\boldsymbol{X}_i$, including laws of Armstrong–Frederick type, including law (13).

Let us end this section by a remark indirectly related to the implementation in a finite element code: the form given by Eq. (13) is implicit since the rate of $\boldsymbol{\alpha}$ (therefore of $\boldsymbol{X}$) depends on the rate of $X_{eq}$. Recalling the definition of von Mises norm gives $\langle \dot{X}_{eq} \rangle = \frac{3}{2} \langle \boldsymbol{X} : \dot{\boldsymbol{X}} \rangle / X_{eq}$. Altogether with Eq. (13), this allows to show that $\boldsymbol{X} : \dot{\boldsymbol{X}}$ is of same sign than $\boldsymbol{X} : \dot{\boldsymbol{\epsilon}}^p$, at least in the isothermal case. After some algebraic work, the following alternative (nevertheless fully equivalent) expression for $\dot{\boldsymbol{X}}$ to isothermal law (13) is derived,

$$\dot{\boldsymbol{X}} = \frac{2}{3} C \dot{\boldsymbol{\epsilon}}^p - \frac{C \Gamma X_{eq}^{M-3}}{1 + \Gamma X_{eq}^{M-1}} \langle \boldsymbol{X} : \dot{\boldsymbol{\epsilon}}^p \rangle \, \boldsymbol{X} \tag{15}$$

more classical to implement.

## 4. Positivity of the intrinsic dissipation

A full plasticity model using the proposed kinematic hardening laws is a non-standard model, the new springback terms not deriving from an evolution potential. One must then prove the positivity of the intrinsic dissipation $\mathcal{D} = \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^p - R\dot{p} - \boldsymbol{X} : \dot{\boldsymbol{\alpha}}$ [7]. Isotropic hardening is introduced as the couple of variables $(R, p)$. The criterion function is the classical $f = (\boldsymbol{\sigma} - \boldsymbol{X})_{eq} - R - \sigma_y$ such as $f < 0 \to$ elasticity. Also classically, the plastic strain rate is derived by normality: $\dot{\boldsymbol{\epsilon}}^p = \dot{p} \frac{3}{2} \frac{\boldsymbol{\sigma}' - \boldsymbol{X}}{(\boldsymbol{\sigma} - \boldsymbol{X})_{eq}}$. Plasticity is incompressible ($\operatorname{tr} \dot{\boldsymbol{\epsilon}}^p = 0$) and kinematic hardening is deviatoric ($\boldsymbol{X} = \boldsymbol{X}'$), as announced.

After some algebraic work, the dissipation takes the form

$$\begin{aligned}
&\text{Law (4):} \quad \mathcal{D} = \big[(\boldsymbol{\sigma} - \boldsymbol{X})_{eq} - R\big]\dot{p} + \frac{3\Gamma}{2} \boldsymbol{X} : \boldsymbol{X}\, \dot{a} = \sigma_y \dot{p} + \Gamma X_{eq}^2 \dot{a} \geqslant 0 \\
&\text{Law (6):} \quad \mathcal{D} = \big[(\boldsymbol{\sigma} - \boldsymbol{X})_{eq} - R\big]\dot{p} + \frac{3\Gamma}{2} X_{eq}^{M-2} \boldsymbol{X} : \boldsymbol{X}\, \dot{a} = \sigma_y \dot{p} + \Gamma X_{eq}^M \dot{a} \geqslant 0 \\
&\text{Law (13):} \quad \mathcal{D} = \big[(\boldsymbol{\sigma} - \boldsymbol{X})_{eq} - R\big]\dot{p} + \frac{3\Gamma}{2C} X_{eq}^{M-2} \boldsymbol{X} : \boldsymbol{X}\, \langle \dot{X}_{eq} \rangle = \sigma_y \dot{p} + \frac{\Gamma}{C} X_{eq}^M \langle \dot{X}_{eq} \rangle \geqslant 0
\end{aligned} \tag{16}$$

and is therefore positive for any loading, proportional or not, isothermal or not ($\dot{p}$, $\dot{a}$ and $\langle \dot{X}_{eq} \rangle$ are positive by definition).

## 5. Conclusion

Families of non-saturating kinematic hardening laws have been proposed. In order to gain non-saturation of the kinematic hardening, the springback term $\boldsymbol{\mathcal{B}}\mathcal{\dot{P}}$ in Eq. (1) is not assumed linear in $\dot{p}$ anymore but in $\dot{a} = (\frac{2}{3} \dot{\boldsymbol{\alpha}} : \dot{\boldsymbol{\alpha}})^{1/2}$ or, better, in the positive part $\langle \dot{X}_{eq} \rangle$, with $X_{eq}$ the von Mises norm of kinematic hardening $\boldsymbol{X}$. By use of this replacement, any existing law $\dot{\boldsymbol{X}} = \frac{2}{3} C \dot{\boldsymbol{\epsilon}}^p - \boldsymbol{\mathcal{B}} \dot{p}$ can then easily gain the non-saturation property by changing it into $\dot{\boldsymbol{X}} = \frac{2}{3} C \dot{\boldsymbol{\epsilon}}^p - \boldsymbol{\mathcal{B}} \langle \dot{X}_{eq} \rangle$. Proposed law (13) is the power-law counterpart for kinematic hardening, fully complementary to Armstrong–Frederick saturating law. Its properties have been illustrated on qualitative examples.

General plasticity modeling, including ratcheting, often introduces several kinematic hardening variables $\boldsymbol{X}_i$. Considering laws of different nature for each $\boldsymbol{X}_i$ can help to extend the validity domain of the plasticity models, setting for example $\boldsymbol{X} = \boldsymbol{X}_{\text{AF}} + \boldsymbol{X}_{\text{NSat}} + \cdots$, with $\boldsymbol{X}_{\text{AF}}$ following Armstrong–Frederick law (2), with $\boldsymbol{X}_{\text{NSat}}$ following the non-saturating law (13).

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