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Macroscopic yield strength of reinforced soils: From homogenization theory to a multiphase approach

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ABSTRACT

This contribution presents a multiphase approach for ascertaining the macroscopic yield strength properties of soils reinforced by linear inclusions, conceived as an extended or improved homogenization procedure allowing one to capture scale and boundary effects, which may play an important role in the yield design of actual structures. The decisive element of such an extension is the introduction of a parameter characterizing the strength of the interaction between two continuous media ("phases") representing the soil and the reinforcing inclusions, respectively. A preliminary analysis suggests that such a parameter varies in direct proportion to the inverse of a scale factor.

1. Introduction

A large variety of soil reinforcement techniques has been developed in the last few decades, aimed at improving the stability of geotechnical structures, such as reinforced earth embankments, soil nailed retaining walls, piled raft foundations or slope stabilization systems. Despite obvious differences regarding their installation mode or construction method, such techniques present some important common features:

a) The reinforcing inclusions usually take the form of linear structural elements (metal or polymeric strips or bars, concrete piles, etc.) incorporated into the soil mass following a regular (periodic) arrangement and one or several preferential orientations, in much the same way as for industrial fiber composite materials, although at a quite different scale;
b) The reinforcing material displays much higher strength characteristics than the surrounding soil: concrete or steel yield strength is one to several tens of thousands times greater than that of a soft clay or a sand;
c) Conversely, the volume fraction of the reinforcing material remains quite small, rarely exceeding a few percents.

The extreme heterogeneity of the so-obtained composite reinforced soil, combined with the relatively large number of inclusions involved in such reinforcement techniques, precludes the use of design-oriented calculation methods in which the inclusions would be treated as individual elements embedded in the soil. Through the definition of a macroscopic strength condition for the reinforced soil, the homogenization method provides an attractive alternative for dealing with the stability analysis of reinforced soil structures.

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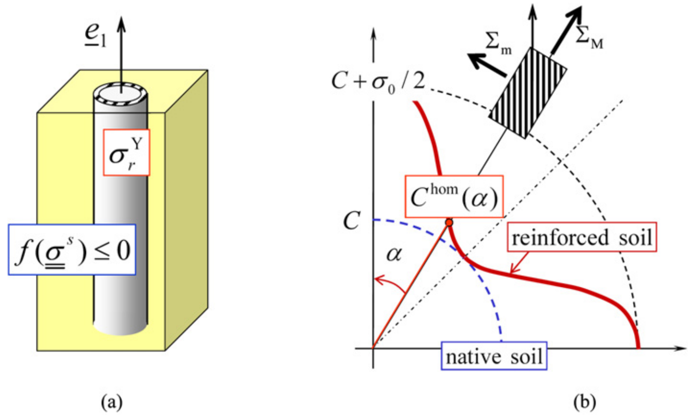

Fig. 1. (a) Representative unit cell of reinforced soil. (b) Polar diagram for a unidirectionally reinforced purely cohesive soil.

The objective of this paper is to point out the shortcomings of such a homogenization procedure and to show how a multiphase approach, perceived as an extension of the homogenization concept, is able to capture "scale" as well as "boundary effects", which may have important consequences in the yield design of reinforced soil structures.

2. Macroscopic strength condition of a unidirectionally reinforced soil

According to the homogenization theory for periodic media implemented in the context of yield design (limit analysis), the determination of the macroscopic strength condition of a reinforced soil relies upon the solution to a yield design boundary value problem defined over the reinforced soil's representative unit cell, sketched in Fig. 1(a) [1,2]. As previously mentioned, the reinforcement volume fraction η is very small whereas the strength of the reinforcing material is considerably higher than that of the soil. This situation can be mathematically obtained by making the volume fraction tend to zero while the product of this volume fraction by the reinforcing material's uniaxial yield strength σ_r^Y is kept constant:

$$\eta \to 0 \quad \text{as } \eta\sigma_r^Y = \sigma_0 = ct \tag{1}$$

where σ_0 may be interpreted as the tensile (compressive) resistance of the reinforcing inclusions per unit transverse area. Under such circumstances, it can be shown [3] that, under the assumption of perfect bonding at the interface between the inclusion and the surrounding soil, the macroscopic strength condition of the reinforced soil simply reduces to:

$$F(\underline{\underline{\Sigma}}) \leqslant 0 \quad \Leftrightarrow \quad \begin{cases} \underline{\underline{\Sigma}} = \underline{\underline{\sigma}}^s + \sigma \underline{e}_1 \otimes \underline{e}_1 \\ f(\underline{\underline{\sigma}}^s) \leqslant 0, \ |\sigma| \leqslant \sigma_0 \end{cases} \tag{2}$$

where $f(.)$ denotes the soil's strength condition. The above simplified criterion proves also valid for plane strain-loaded multilayered materials under the same condition as (1) [4,5]. For a purely cohesive soil (soft clay) characterized by a cohesion or undrained shear strength equal to C, the macroscopic strength condition, expressed under plane strain conditions parallel to the reinforcement direction, writes [2]:

$$F(\underline{\underline{\Sigma}}) \leqslant 0 \quad \Leftrightarrow \quad \Sigma_M - \Sigma_m \leqslant 2C(\alpha) \tag{3}$$

where Σ_M (resp. Σ_m) is the major (resp. minor) principal stress and α is its orientation with respect to the reinforcement direction. The reinforced soil thus appears to a purely cohesive anisotropic medium, with its cohesion, represented in Fig. 1(b) in the form of a polar diagram, varying from that of the native soil (C) for $\alpha = \pm 45°$ to a maximum value equal to $C + \sigma_0/2$ for $\alpha = 0°, 90°$.

3. A partial validation of the homogenization approach

The validity of such a homogenization procedure is now assessed on the illustrative problem of evaluating the compressive resistance of a transversely reinforced block shown in Fig. 2. This block, of height H and half-width L, is subject to a plane strain vertical compressive loading exerted by a vertically moving rigid plane in smooth contact with its upper section, while its bottom section is also in smooth contact with a fixed plane and its lateral sides are stress free.

The application of the homogenization procedure to the prediction of such a compressive strength is straightforward, since according to this procedure, the reinforced soil is modelled as a homogeneous anisotropic purely cohesive medium obeying the strength condition (3). Applying the lower bound static approach of yield design, as well as the upper bound kinematic one, yields immediately:

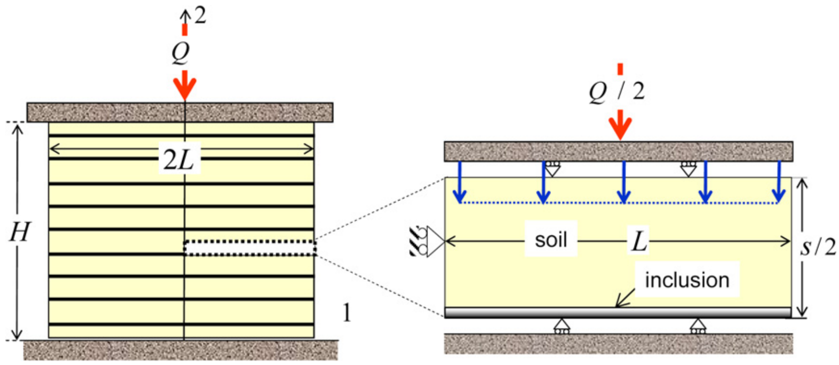

Fig. 2. Compressive strength of a purely cohesive reinforced block: initial and auxiliary problems.

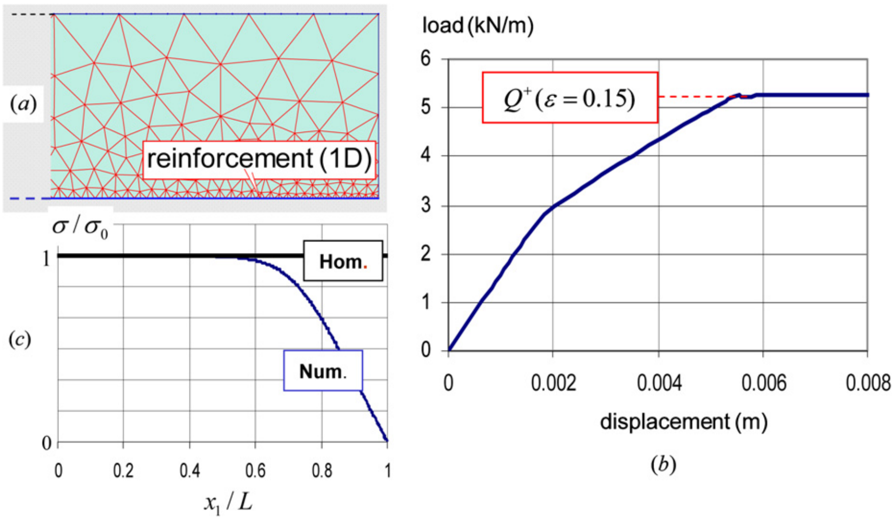

Fig. 3. Results of elastoplastic simulation for $\varepsilon = 0.15$: (a) finite element mesh; (b) computed load–displacement curve; (c) "at failure" stress distribution in the reinforcement.

$$Q^+_{\text{hom}} = 4CL[1 + \sigma_0/2C] \tag{4}$$

This value is to be compared with a direct numerical simulation of the same problem, where for the sake of simplicity, but without any loss of generality, the reinforced soil is modelled as a multilayered material in which the reinforcements are treated as 1D beam elements, equally spaced by a distance s throughout the block, so that a "scale factor" defined by the spacing to half-width ratio, may be introduced:

$$\varepsilon = s/L \tag{5}$$

Owing to the different symmetry and periodicity conditions, the initial problem can be reduced to the auxiliary problem displayed on the right-hand side of Fig. 2, involving a representative "slice" of reinforced soil with the appropriate boundary conditions. The limit loads have been evaluated by means of a standard f.e.m.-based elastoplastic computer code (namely the PLAXIS software [6]) for values of the scale factor ranging between 0.05 and 0.5. By way of example, the results of such a calculation have been displayed in Fig. 3 for a scale factor equal to 0.15. It is to be noted that the computational time for each elastoplastic calculation up to failure, represented by a load–displacement curve, does not exceed one minute on any standard PC.

All the results, which are presented in a non-dimensional form in Fig. 4, where the prediction of the homogenization method has also been reported, call for the following comments.

The comparison does confirm the well-known convergence result of the homogenization approach, which can be formulated as:

$$\lim_{\varepsilon \to 0} Q^+(\varepsilon) = Q^+_{\text{hom}} \tag{6}$$

but it clearly indicates at the same time that the homogenization method may significantly overestimate the actual value of the compressive resistance if the scale factor is not sufficiently small. Such a "scale effect" is obviously of no consequence

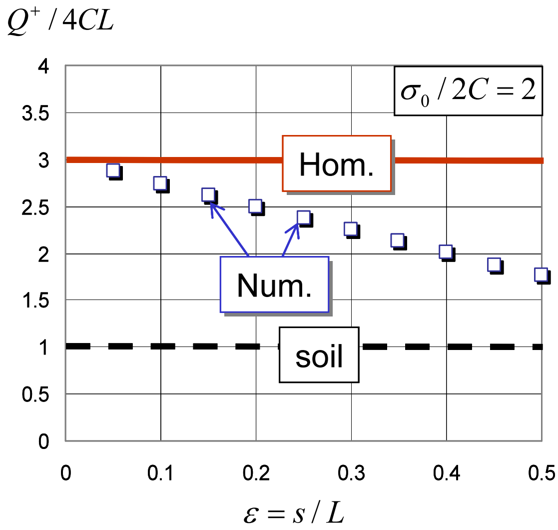

Fig. 4. Homogenization vs. numerical simulation.

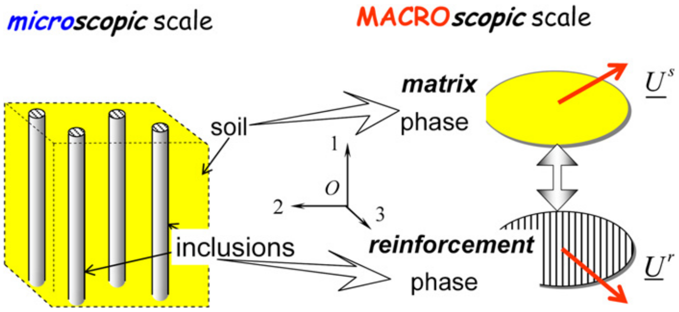

Fig. 5. Principle of the multiphase model for reinforced soils.

as far as industrial composite materials are concerned (leaving aside purely local effects associated with brittle failure, such as delamination phenomena), but remains a relevant question for reinforced soils, since the scale factor is generally of the order of 0.1–0.3 for this kind of composite material. Such a discrepancy is corroborated by the picture in the lower left-hand side of Fig. 3, representing the "at failure" tensile stress distribution in the reinforcement determined from the f.e.m. analysis. Indeed this stress is constant, equal to that predicted by the homogenization method in the central part of the block, but sharply decreases to zero when approaching its free end. In other terms the previously mentioned "scale effect" is closely connected with a "boundary effect".

4. Multiphase model as an extended homogenization method

This model is suggested by the expression (2) of the simplified macroscopic strength criterion, in which the "total" stress $\underline{\underline{\Sigma}}$ is additively decomposed into "partial" stresses relating to the soil and the reinforcement, with their own respective strength condition. Its basic principle, sketched in Fig. 5, stems from the intuitive idea that the soil on the one hand, the array of reinforcing inclusions on the other hand, can be homogenized separately, in such a way that the composite reinforced soil is described at the macroscopic scale by the superposition of two mutually interacting continuous media, called the "matrix" and "reinforcement phases".

Since the detailed developments of this model may be found in [7] or [8], its basic features will be briefly recalled here in the context of yield design. The equilibrium equations are written for each phase separately, that is in the absence of any external body force:

$$\operatorname{div} \underline{\underline{\sigma}}^s + I \underline{e}_1 = 0 \tag{7}$$

for the matrix phase, representing the soil, and:

$$\operatorname{div}(\sigma \underline{e}_1 \otimes \underline{e}_1) - I \underline{e}_1 = 0 \tag{8}$$

for the reinforcement phase, where I denotes the interaction body force density. These equations are completed by stress conditions defined on the boundary surface of each phase independently. Referring to a yield design boundary problem for any such two-phase system, it is necessary to specify the strength condition at any point of each phase, namely:

$$f\left(\underline{\underline{\sigma}}^s\right) \leqslant 0 \quad \text{and} \quad |\sigma| \leqslant \sigma_0 \tag{9}$$

along with an interaction strength condition of the form:

$$|I| \leqslant I_0 \tag{10}$$

In the situation of "perfect bonding", characterized by the fact that the interaction strength parameter I_0 takes an infinite value, it is quite apparent from summing up Eqs. (7) and (8), and thus eliminating the interaction force density, that the yield design homogenization method is recovered.

The above defined compressive strength problem is now revisited in the light of the multiphase model, the strength properties of which being defined by the soil's cohesion C, reinforcement uniaxial strength density σ_0, and a so far arbitrarily selected value of the interaction strength parameter I_0. The compressive strength of the reinforced block is thus defined as the maximum value of Q for which it is possible to exhibit a couple of stress fields, $\underline{\underline{\sigma}}^s$ in the matrix phase and σ in the reinforcement phase, along with an interaction force density I, satisfying both the equilibrium equations (7) and (8) along with the boundary conditions specified for each phase independently, and the respective strength conditions (9) and (10): see [8] for more details concerning the general formulation of a two-phase yield design problem.

Implementing the lower bound static approach, it can be easily shown that the generalized stress field defined on the reinforced block by:

$$\sigma(x_1) = -\sigma^s_{11}(x_1) = 2C - \sigma^s_{22}(x_1) = \begin{cases} I_0(L - x_1) & \text{if } \sigma_0 \geqslant I_0 L \\ \text{Min}\{I_0(L - x_1); \sigma_0\} & \text{if } \sigma_0 \leqslant I_0 L \end{cases} \tag{11}$$

complies with the above equilibrium and strength requirements, leading to the following lower bound for the reinforced block compressive resistance:

$$Q^+_{\text{mult.}} \geqslant 4CL + \begin{cases} I_0 L^2 & \text{if } \sigma_0 \geqslant I_0 L \\ 2\sigma_0 L\left[1 - \frac{\sigma_0}{2 I_0 L}\right] & \text{if } \sigma_0 \leqslant I_0 L \end{cases} \tag{12}$$

It can be proved that the same analytical expressions are obtained as upper bound values, making use of appropriate (virtual) velocity fields in the kinematic approach of yield design. Hence the exact value of the compressive resistance predicted by the multiphase model:

$$Q^+_{\text{mult.}}/4CL = 1 + \left| \begin{array}{ll} \chi R/2, & \chi = I_0 L/\sigma_0 \leqslant 1 \\ R(1 - (2\chi)^{-1}), & \chi \geqslant 1 \end{array} \right. \quad \text{where } R = \frac{\sigma_0}{2C} \tag{13}$$

In the above formula R could be labelled as the reinforcement factor, while the non-dimensional parameter χ can be interpreted as the relative resistance of the interaction as compared to that of the reinforcement phase. It can be observed in particular, that in the situation of perfect bonding the above expressions reduce to that derived from the homogenization approach:

$$Q^+_{\text{mult.}}(\chi \to \infty) = Q^+_{\text{hom}} = 4CL(1 + R) \tag{14}$$

5. Identification of the interaction strength parameter

The curve representing the variation of the (non-dimensional) compressive resistance predicted by the multiphase model has been drawn for $R = 2$ on the right-hand side of Fig. 6 as a function of χ, while the numerical predictions have been reported on the left-hand side of the same figure as a function of the scale factor ε. A simple procedure, shown in this figure and based on the identification of the value of the compressive resistance given by (13), with that resulting from the numerical calculation, makes it possible to establish the relationship between χ and ε, which is represented in Fig. 7 in the form of a series of points, which are best fitted by a curve have the following approximate equation:

$$\chi \cong 0.4\varepsilon^{-1} \tag{15}$$

which means that χ, and hence the interaction strength parameter I_0, is inversely proportional to the scale factor. The coefficient of proportionality (equal to 0.4 in the present case) can therefore be determined from one single numerical simulation. A more thorough and detailed analysis (which is beyond the scope of the present, rather short, Note) would certainly show that this coefficient of proportionality depends on the soil's cohesion, since no failure is considered at the soil–inclusion interface at the microscopic scale.

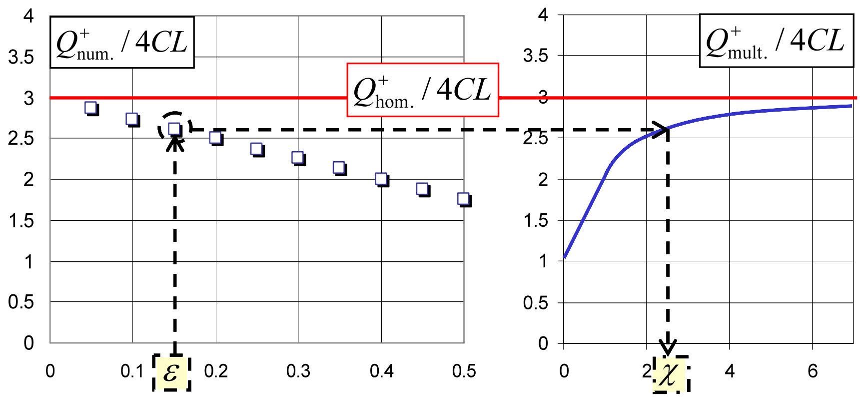

Fig. 6. Identification procedure for the interaction strength parameters ($R=2$).

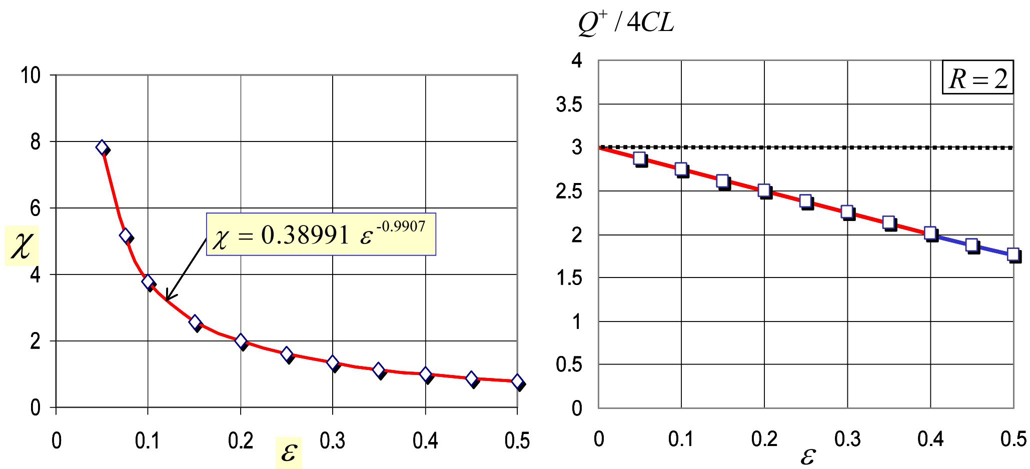

Fig. 7. Identification of the interaction strength parameter and comparison between the numerical results and the multiphase model-based predictions.

Introducing the relationship (15) into Eq. (13) finally leads to the following expression of the compressive strength as a function of the scale factor, predicted by the multiphase approach:

$$\frac{Q^+_{\text{mult.}}}{4CL} = \frac{Q^+_{\text{num.}}}{4CL} = \begin{cases} 3 - 2.5\varepsilon & \text{if } \varepsilon \leqslant 0.4 \\ 1 + 0.4/\varepsilon & \text{if } \varepsilon \geqslant 0.4 \end{cases} \tag{16}$$

which appears to linearly decrease until the value corresponding to the 0.4 scale factor (Fig. 7).

6. Concluding remarks

As it has been clearly shown in this contribution on an illustrative design example, the multiphase approach developed in the context of yield design makes it possible to overcome the limitations of a classical homogenization method and related macroscopic strength concept, namely its inherent inability to account for any scale/boundary effect. This is achieved through the introduction of an appropriately identified interaction strength parameter which tends to infinity, thus recovering the homogenization method as a particular case, when the scale factor tends to zero. It should be emphasized that such a limited interaction strength between phases, at the *macroscopic* scale, should be taken into account even in the case of perfect bonding at the *microscopic* scale, that is unlimited strength, at the soil–inclusion interface, thus assuming that the soil is perfectly adherent to the reinforcement.

This has of course important implications from an engineering design viewpoint, since the multiphase approach combines the decisive advantage of a homogenization method, notably in terms of computational efficiency, with its ability to assess the stability of reinforced soil structures in a more realistic way, as illustrated in [9] for reinforced earth retaining walls.

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