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Sequential Monte Carlo hydraulic state estimation of an irrigation canal

Estimation de l'état hydraulique d'un canal d'irrigation par une méthode Monte Carlo séquentielle

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ABSTRACT

The estimation in real time of the hydraulic state of irrigation canals is becoming one of the major concerns of network managers. With this end in view, this Note presents a new approach based on the combination of a numerical solution of the open channel Saint-Venant PDE with a sequential Monte Carlo state-space estimation. We shall show that discharges and elevations along the canal are successfully estimated, and also that, concurrently, model parameters identification, such as the Manning–Strickler friction coefficient, can be performed.

RÉSUMÉ

L'estimation en temps réel de l'état hydraulique de canaux d'irrigation devient l'une des préoccupations majeures des gestionnaires de réseaux. A cette fin, cette Note présente une nouvelle approche qui combine une solution numérique des EDP de Saint-Venant pour les écoulements à surface libre avec une estimation d'état basée sur une méthode Monte Carlo séquentielle. Nous montrons que les débits et côtes le long du canal sont reproduits, et aussi que l'identification simultanée de paramètres du modèle, comme le coefficient de frottement de Manning–Strickler, peut être réalisée.

1. Introduction

Irrigated lands contribute more that 40% of World food production with less than 20% of the cultivated area. Irrigation is also well known for being responsible of more than 70% of fresh water withdrawal. Recent FAO figures indicate that for 2030, food production will have to be increased by more that 80%, but with no more than 12% more water. Therefore, high levels of efficiency of water uses are increasingly expected from the managers of irrigation canals. In order to fulfill this task, detailed information on the hydraulic state of such systems must be available. Usually, the only known quantities are the measurements performed on the hydraulic system, in limited locations. At present, no implementation of a real time estimation of the complete hydraulic state of a canal has been realized, only data reconciliation for daily volumes, where dynamic effects can be neglected, has been carried out [1]. The main causes are the noisy character of the measurements

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and the high non-linearity of the open channel dynamics and cross-structure equations. The aim of this Note is to propose, in the open channel hydraulic context, a new approach based on a numerical treatment of the open channel equations associated with a Bayesian filtering using Monte Carlo methods [2,3]. These last methods have the great advantage of not being subject to any linearity assumption on the model and they deal properly with the stochastic feature of the problem.

This paper is organized as follows. Section 2 describes the open channel hydraulic model. The sequential Monte Carlo (SMC) state estimation is presented in Section 3. Section 4 presents the results of this approach on a classical irrigation test canal. Moreover, in this section, we show that model parameter estimation can be achieved together with that of the hydraulic state. Finally, Section 5 provides a general conclusion on the applicability of the method.

2. The open channel hydraulic model

In order to be well-founded, a state estimation must be based on a proper modeling of the physical system. Cemagref is developing methodologies and software tools dedicated to consultant companies and canal managers allowing accurate modeling of irrigation canals. The numerical tests carried out hereafter are run on the SIC hydrodynamic software developed by Cemagref [4]. This model is based on the 1D Saint-Venant equations discretized using the Preissmann implicit scheme. The equations are completed by external and internal boundary conditions such as inflows and gate equations. In order to match the typical field operating conditions, the SIC software will perform successive $\Delta t = 15$ min time step transitions, which is the typical observation time step used in modern automated irrigation canals [5]. The numerical time step used in these successive calculations is a smaller one (1 min in our example), so as to obtain an accurate enough description of the open channel flow. In other words, the SIC software is run for a 15 min simulation duration, comprising fifteen 1 min numerical time steps; observations of measured levels are then performed at the end of this 15 min observation time step, and the next observation time step is run, and so on. This is absolutely similar to what happens in the real world.

The hydraulic state of the canal at observation time step t is characterized by a state vector x_t, the elements of which are the elevations and discharges along the canal:

$$x_t = (z_1 \quad z_2 \quad \ldots \quad z_n \quad q_1 \quad q_2 \quad \ldots \quad q_n)'_t$$

This state vector is then of dimension $2n$, where n is the number of sections in the spatial discretization. The change of the hydraulic state variables for one observation time step can be formally written:

$$x_{t+1} = f(x_t, u_t) \tag{1}$$

In this state equation, the vector u_t stands for the vector of all external actions on the system: gate motion, offtake discharges, change in boundary conditions, seepage, etc. The f function, obtained as the result of a numerical calculation, is a complicated and highly non-linear quantity. As we shall see below, it is the strength of the Monte Carlo approach to be independent of such difficulties.

The state equation is accompanied by an observation equation. The observed quantities can be discharges and/or elevations, but are almost always elevations in the context of open channel hydraulic networks. This observation equation can be written:

$$y_t = Cx_t \tag{2}$$

where the observation matrix C has as many lines as sensors in the system and $2n$ columns. The elements of each line are zeros, except the element corresponding to the localization of the sensor where it is equal to 1.

The above equations must be completed by the noisy characteristics of the system, especially as we wish to be close to the field reality. Uncertainties of measurements lead to random variations added to the observation equation. In the same way, random variations must be added to the state equation. Indeed, even if the model is, in average, close to the physic of the system, random changes can take place in the course of time, since Saint-Venant equations describe the mean behavior and other unknown effects can occur. In the following, these random variations will be classically supposed to be white Gaussian. The complete physical equations for the canal system take then the form:

$$x_{t+1} = f(x_t, u_t) + \xi_t, \quad \xi_t \approx N(0, Q_t) \tag{3a}$$

$$y_t = Cx_t + \zeta_t, \quad \zeta_t \approx N(0, R_t) \tag{3b}$$

where Q_t and R_t are the covariance matrices of the respective noises. We must underline that a proper description of the system must include the uncertainties, otherwise essential features of the system dynamics are missed.

3. Sequential Monte Carlo state estimation

We recall briefly in the following the main elements of the sequential Monte Carlo estimation technique. The aim of this approach is the recursive estimation in time of the posterior $P(x_t|y_{0:t}u_{0:t-1})$.

The sequential Monte Carlo approach [2,3,6,7] is a technique for implementing the sequential Bayesian relations by Monte Carlo simulations, the required posterior density at time t being represented by a set of random state vector samples with

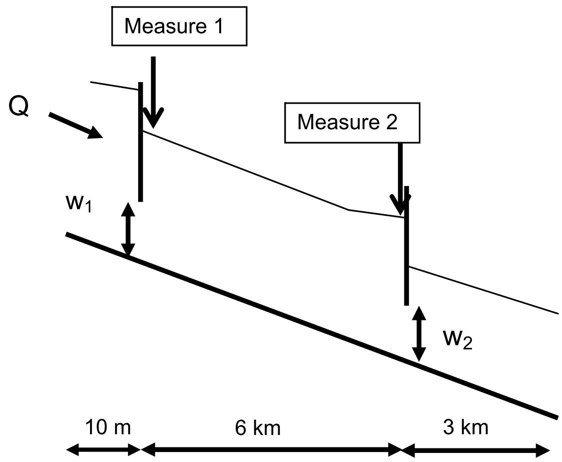


Fig. 1. Pool general scheme.

Table 1
Pool characteristics.

Bottom width	8 m
Bank slope	1.5
Bed slope	0.0008
Pool length	6000 m
Strickler friction coefficient	50 $m^{1/3}/s$
Steady state upstream discharge	20 m^3/s

associated normalized weights. On each time step, new samples are generated and the weights updated. In this application, with the model given by Eqs. (3), the partial Gaussian state space case [6] is a well-founded approximation. The updates of both the samples and the weights take then the following form:

$$\text{Sampling function at time } t\text{:} \quad P(x_t|x_{t-1}, u_{t-1}, y_t) \cong N(\mu_t, S_t)$$

$$\text{Weight update at time } t\text{:} \quad w_t^i = w_{t-1}^i P\big(y_t|x_{t-1}^i, u_{t-1}\big)$$

with $P(y_t|x_{t-1}^i, u_{t-1}) \cong N(Cf(x_{t-1}^i, u_{t-1}), R_t + CQ_tC')$.

The weights being afterwards normalized such that $\sum_i w_t^i = 1$.

S_t and μ_t are such that:

$$\begin{cases} S_t^{-1} = Q_t^{-1} + C'R_t^{-1}C \\ \mu_t = S_t Q_t^{-1} f(x_{t-1}, u_{t-1}) + C'R_t^{-1} y_t \end{cases}$$

Once the posterior $P(x_t|y_{0:t}u_{0:t-1})$ is being correctly sampled at time t, relevant quantities, in particular estimated mean state vector $\hat{x}_t$, can be calculated:

$$\hat{x}_t = \sum_i w_t^i x_t^i$$

4. Application of SMC to open channel estimation

The method is now applied on the Cemagref test canal type 5 as described in [8]. The test canal is shown in Fig. 1. In canals, for physical and technological reasons, measurement points are systematically located upstream and downstream of cross-structures. Indeed, these measurements provide also a mean to calculate flows using approximate gate or weir laws together with a supervision of the structure. Second, measurement points need electrical energy to work and network means to send measurement values to central supervision center. These resources are already present on cross-structure locations which are supervised, and are almost impossible to provide and maintain elsewhere on isolated and unwatched points for canal systems of hundreds kilometers long. Consequently, since we want to be close to the real case, the localization of the elevation measurements is downstream the first gate and upstream the second gate. Table 1 summarizes the geometry and physical parameters of the canal pool.

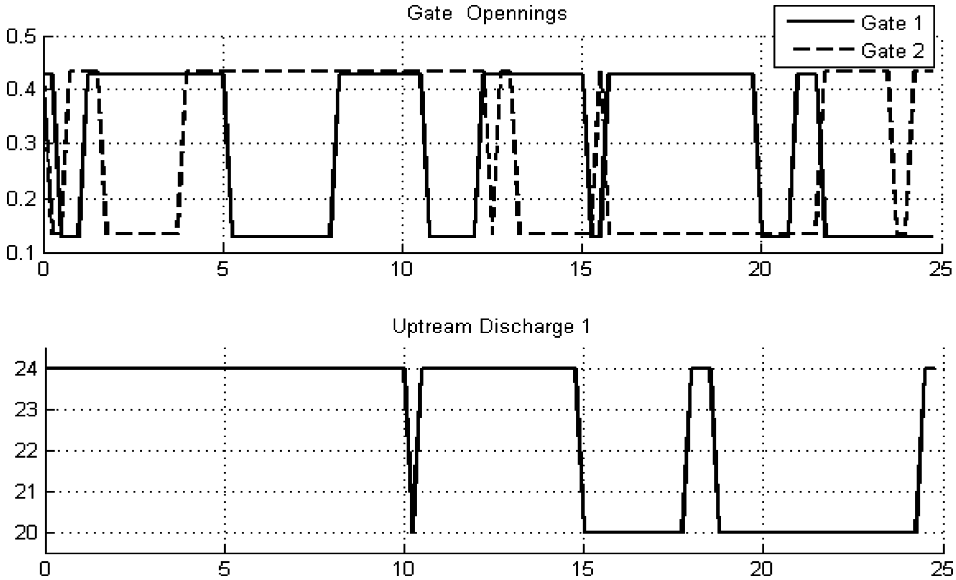


Fig. 2. Upstream discharge and gate openings scenario.

Boundary conditions:

- Upstream: Time varying discharge $Q(t)$.
- Downstream: Constant elevation $Z = 93.36$ m.
- Internal boundary conditions: Cemagref [4] gate equations, providing time varying gate discharge depending on gate opening, upstream and downstream gate depths.

Noisy parameters:

- $R = \mathrm{diag}(\sigma_z^2, \sigma_z^2)$ with $\sigma_z = 0.02$ m.
- $Q = \mathrm{diag}(V)$, with V, the vector $V = (\sigma_1^2\ \sigma_1^2\ \cdots\ \sigma_1^2\ \sigma_2^2\ \sigma_2^2\ \cdots\ \sigma_2^2)$ of dimension $2n$. $\sigma_1 = 0.001$ m and $\sigma_2 = 0.01$ m^3/s are the standard deviations associated respectively with the elevation and discharge parts of the space state vector.

The measurement uncertainty is of the order of magnitude of the real one on hydraulic networks, and even a little larger, in order to test the robustness of the method against measurement noise.

State noise has to account for small random changes from one observation step time to the next one. It has not to account for permanent model difference, for example when one model parameter has not the right value. This last problem will be treated in Section 4.2.

Noisy measurements are generated on a simulated hydraulic scenario on this canal. Starting from these measurements, the goal is then to estimate the hydraulic state and to compare it to the true one which is, of course, known in this case as calculated by the SIC hydrodynamic software. Three kinds of situations will be investigated:

- Normal hydraulic scenario: Estimation of the complete hydraulic state during a hydraulic simulation. In particular, one will see that estimation of discharge and elevation can be given on locations where no measurements are performed. The observer, in this case, plays the role of a virtual sensor;
- Occurrence of sensor failures, with reconstruction of faulty measurements;
- Estimation of model parameter: the Strickler coefficient is supposed unknown and must be estimated at the same time as the hydraulic state.

In all these scenarios, the initial state is supposed unknown, but we shall see that the observer is convergent and is asymptotically independent of the initial conditions.

4.1. Normal hydraulic scenario

The data are created as follows. Starting from a steady hydraulic state, large amplitude modifications of the upstream discharge and of gate openings are generated (see Fig. 2).

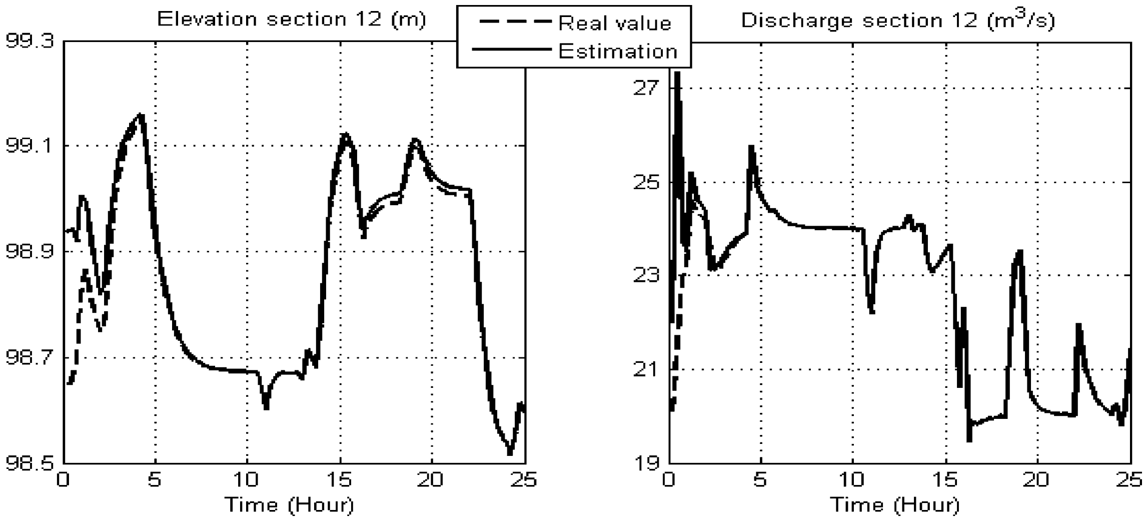


Fig. 3. Elevation and discharge estimation on section 12.

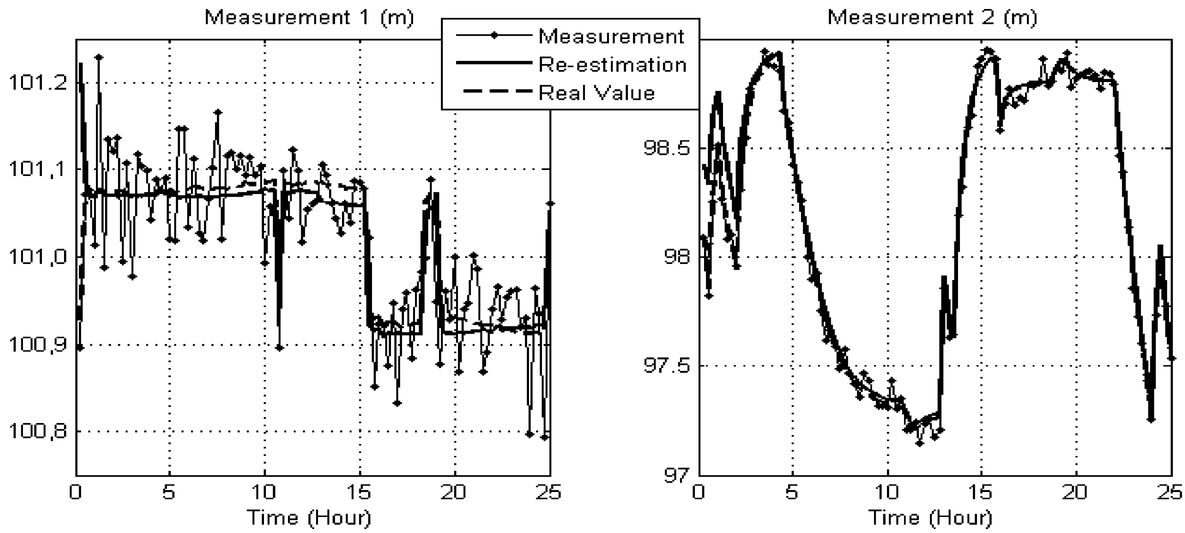


Fig. 4. Measurement re-estimations.

Sequential Monte Carlo estimation of the hydraulic state is then performed. Fig. 3 gives an example of estimated discharge and elevation with comparison to the real ones at longitudinal abscissa 3000 m (section 12), where no measurement is performed.

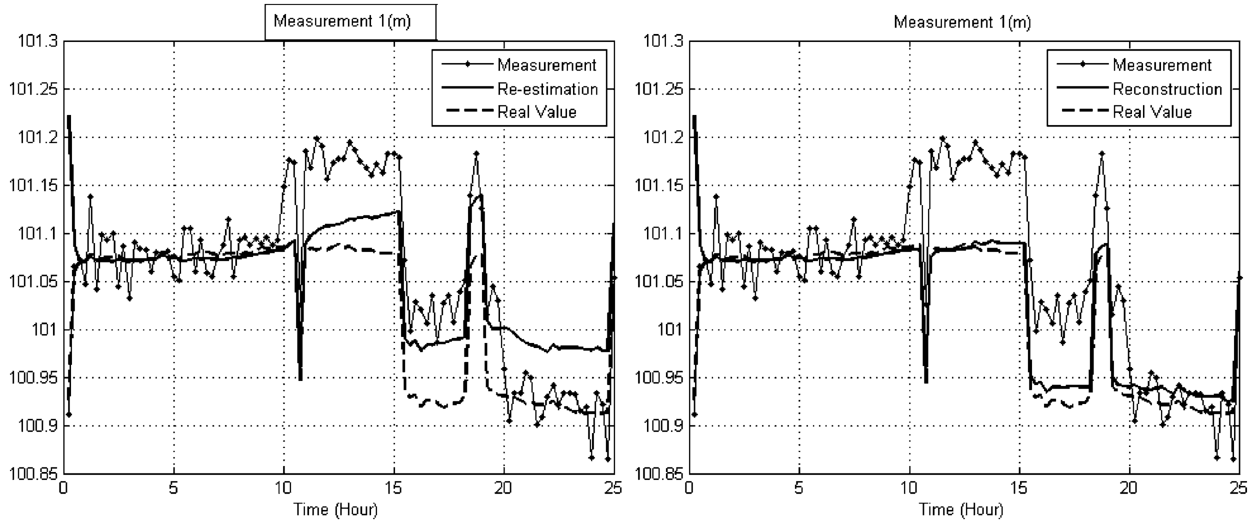
The insensitivity to initial conditions is clearly seen in these figures. The hydraulic simulation has been performed starting from an initial state different from the real one. In the two figures we observe, in the beginning, a transient period before the observer converges to the true real time state values.

Fig. 4 shows the re-estimated elevations at the measurement localizations, with comparison to the measurements and the real values. One can clearly observe that, after the initial transient period, the elevation re-estimations are closer to the real values than the original measurements.

4.2. Sensor failure

Sensor faults are major causes of water resource management deficiency. Since the SMC approach is able to re-estimate measurements, drift can be detected, the failed sensor removed from the measurements and the faulty measurement reconstructed.

In order to test the sensor fault detection and reconstruction ability, a drift of 0.1 m has been introduced in the measurements of the first sensor between 10 and 20 o'clock. The hydraulic state estimation, with re-estimation of measurements, is then performed as if all measurements were good. Fig. 5a shows the result for measurement point 1, where a clear discrepancy exists between measurements and their re-estimation. This fault detection being carried out, the correspond-



(a) (b)

Fig. 5. Measurement 1 re-estimation and reconstruction.

ing measurements are removed from the measurement set, and the estimation procedure performed again, using only the measurements from the other sensor. The result is shown in Fig. 5b, where clearly the missing measurements have been properly reconstructed. The case of out of order sensor is simpler: no received measurements, then no detection phase needed, and we are directly led to the reconstruction phase.

4.3. Estimation of model parameters

For model parameter calibration, the classical approach in canal systems is steady flow calibration. However, steady flow conditions can hardly be obtained on the system to be calibrated: it is difficult to ask a canal manager not to move any gate during couple of days. In any way, the corresponding calibration procedures do not use dynamical information (e.g.: time lags, oscillation modes) that we could thing useful in order to get a good hydrodynamic model. As a consequence, even if we can calibrate a model in steady state flow conditions very well, we have no proof that the model will also have good transient representation of the real system. We shall show in this paper that the estimation of model parameters can be performed together with that of the hydraulic state. The method will be described on the example of the particularly important roughness Strickler coefficient k. This can be achieved by augmenting the vector state, i.e. defining:

$$Z_t = \begin{pmatrix} x_t \\ k \end{pmatrix} \tag{4a}$$

and a new state equation:

$$Z_{t+1} = F(Z_t, u_t) = f(x_t, k, u_t) \tag{4b}$$

In other words, the Strickler coefficient appears as a component of a state vector of a new dynamical system described by Eqs. (4a) and (4b).

The diagonal of the state noise matrix $Q(t)$ must also been augmented by the variance σ_k^2 assigned to this new component of the state vector. This variance is a parameter of the method and governs the Strickler coefficient capability to change from one iteration step to the next one, until the convergence to the actual value is achieved. If this variance is chosen small, the convergence is slow. If it is chosen large, the convergence is faster, but with larger local variations due the larger influence of random measurements and system uncertainties. In the application, we have set $\sigma_k = 1.5\ \mathrm{m}^{1/3}/\mathrm{s}$.

The Monte Carlo method is then applied on this new formulation, achieving the sequential estimation of the Strickler coefficient. Fig. 6 shows the convergence to the real value ($k = 50\ \mathrm{m}^{1/3}/\mathrm{s}$) of the estimated coefficient. At the same time, the hydraulic part of the state vector is also assessed. Fig. 6 also shows, for example, the estimation of the elevation at longitudinal abscissa 3000 m (section 12), where no measurement is implemented.

We can see that the estimation converges towards the real value, but the transient duration for this convergence lasts longer and is equal to that of the Strickler coefficient.

The classical methods of parameter calibration, based on optimization procedure, suppose that model parameters have constant values. In our approach, model parameters are considered as dynamical variables, on the same footing as other components of state vector. Therefore, variation of model parameters can be detected and assessed, which is hardly possible in the classical optimization based methods. This is particularly suited to Strickler coefficient, which can vary in time due to

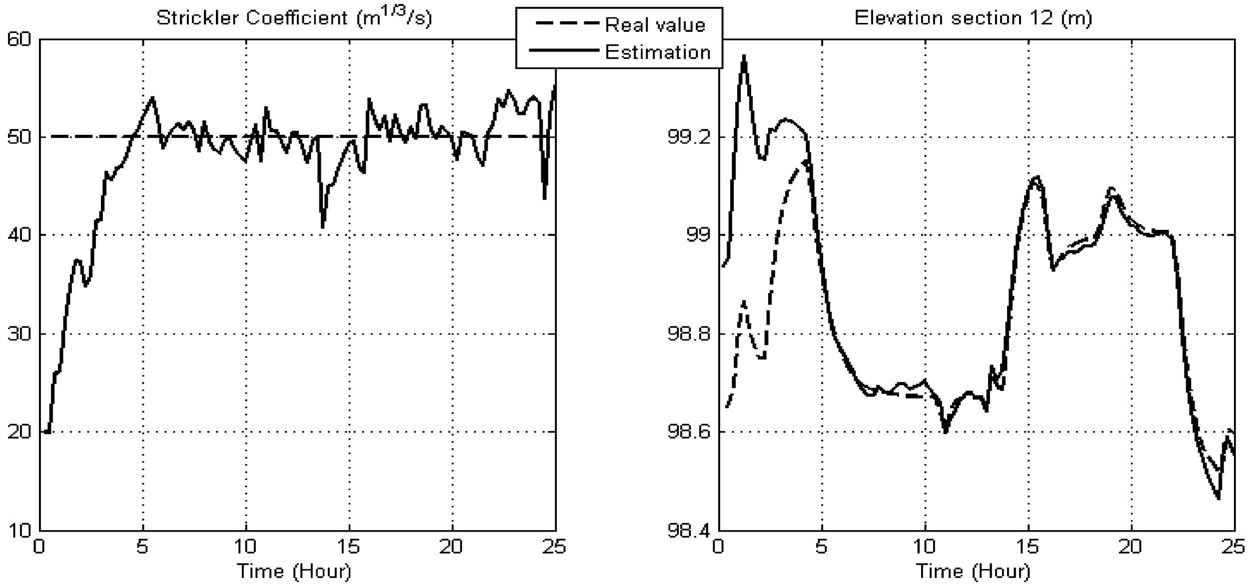


Fig. 6. Strickler coefficient and section 12 elevation estimations.

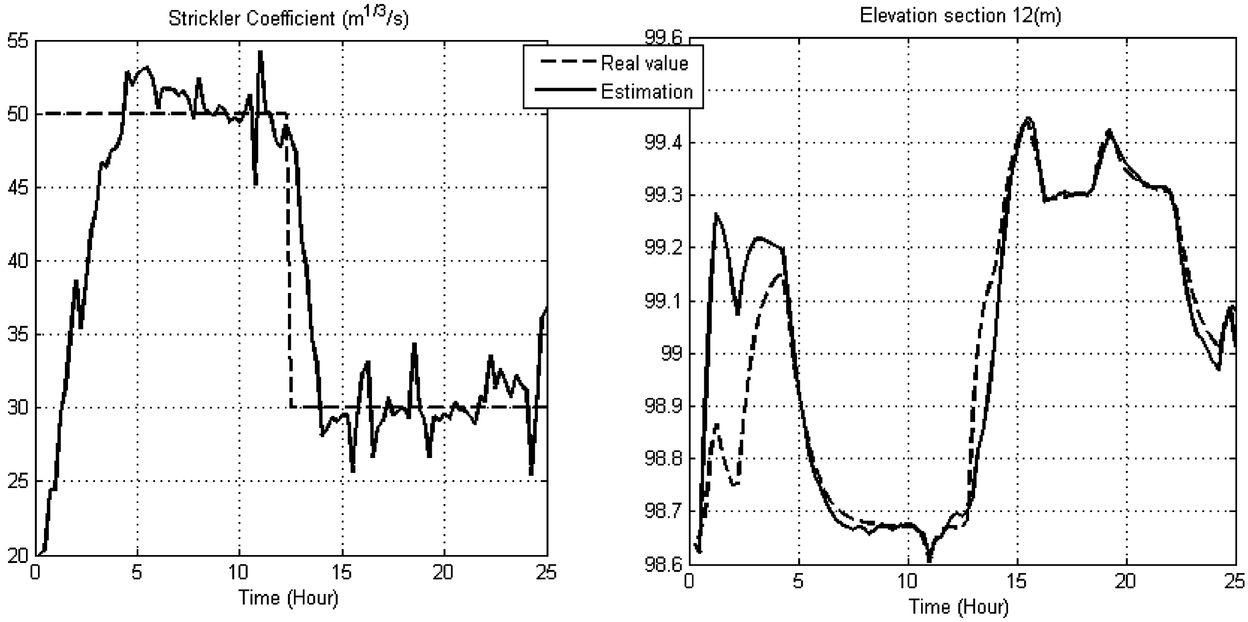


Fig. 7. Variable Strickler coefficient and section 12 elevation estimations.

external reasons like sediment, algae, etc. In order to test the ability of the method to perform a dynamical calibration, the Strickler parameter has been suddenly changed from 50 to 30 $m^{1/3}/s$ at 12.5 o'clock during the generation of measurements. Using then only the measured hydraulic variables, the method is able to perform at the same time the estimation of the hydraulic state of the canal and of the Strickler parameter. This is shown Fig. 7 where, starting from a wrong value, parameter estimation is satisfactory performed, with the drop of the value clearly localized and the hydraulic state correctly estimated, with a short transient length of time.

The above parameter sudden change event is the worst case. Roughness parameter usually changes progressively in time. We have then implemented a more realistic scenario where Strickler parameter undergoes a gradual exponential decrease beginning at 5 o'clock. Fig. 8 depicts the resulting parameter estimation, together with the concurrent estimation of the hydraulic state: it can be seen that the gradual decrease of the Strickler coefficient is satisfactorily well reproduced.

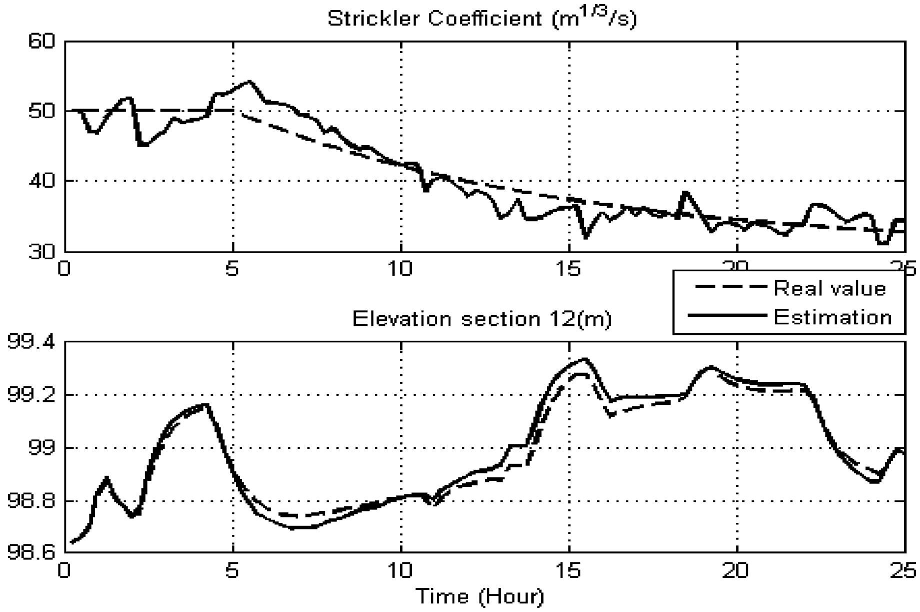


Fig. 8. Gradual Strickler coefficient variation and section 12 elevation estimations.

5. Conclusion

In this Note, we have proposed a way to estimate, in real time, the hydraulic state of an irrigation canal from measurements and a proper modeling of the system. One can easily understand that this knowledge of the hydraulic state is of first importance in the current context of water resources management. Moreover, the method can be extended to the estimation of physical parameters related to the current state of the canal.

In [9] the methods of water management have been presented and classified. Among them, there is the method of "volume regulation", which aims at controlling the volumes of the pools. This method is close to the objectives of operational operators and can be very efficient. There is no easy means to measure, in real time, the volume of a pool which is always in unsteady state. Empirical and very approximate methods have been implemented as in [5]. The approach we have proposed above is able to give an estimation of the volume since it is able to estimate the water profile along the pool. Then it can be used either in operational conditions or for the validation the empirical evaluations.

Finally, since the application is based on Monte Carlo simulations, particular attention should be focused on the choice of the random number generator. We used Ranlux, the high quality generator based on the work of M. Lüscher [10] and written by F. James [11]. This generator is asserted to have a period of 10^{171}. It is available at the scientific library Cernlib of the CERN (European Center of Nuclear Research).

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