



Isotropic brittle damage and unilateral effect

Endommagement isotrope fragile et effet unilatéral

Hélène Welemene*, Cristina Goidescu

Université de Toulouse; INP/ENIT; LGP; 47, avenue d'Azereix, 65013 Tarbes, France

ARTICLE INFO

Article history:

Received 29 December 2009

Accepted after revision 12 April 2010

Available online 11 May 2010

Keywords:

Damage

Microcracks

Isotropic model

Unilateral effect

Mots-clés:

Endommagement

Microfissures

Modèle isotrope

Effet unilatéral

ABSTRACT

This Note investigates the isotropic version of a general macroscopic model for brittle damage accounting for unilateral effects proposed by Welemene and Cormery (H. Welemene, F. Cormery, An alternative 3D model for damage induced anisotropy and unilateral effect in microcracked materials, *J. Phys. IV* 105 (2003) 329–336). Built within a rigorous thermodynamic framework, the model uses a single scalar damage variable and accounts for the contribution of each set of parallel microcracks whether they are opened or closed. The consideration of unilateral effects allows to represent an anisotropic elastic behaviour induced by the closure of some microcracks and also the dissymmetric response in tension and compression which characterizes brittle materials.

© 2010 Académie des sciences. Published by Elsevier Masson SAS. All rights reserved.

R É S U M É

On s'intéresse dans cette Note à la version isotrope d'un modèle macroscopique d'endommagement fragile intégrant les effets unilatéraux proposé par Welemene et Cormery (H. Welemene, F. Cormery, An alternative 3D model for damage induced anisotropy and unilateral effect in microcracked materials, *J. Phys. IV* 105 (2003) 329–336). Etablie dans un cadre thermodynamique rigoureux, cette formulation utilise une seule variable scalaire d'endommagement et distingue la contribution de chaque ensemble de microfissures parallèles suivant qu'elles sont ouvertes ou fermées. L'introduction de ces effets unilatéraux permet de représenter une élasticité anisotrope lors de la fermeture d'une partie des défauts ainsi que la dissymétrie de réponse entre traction et compression typique des matériaux fragiles.

© 2010 Académie des sciences. Published by Elsevier Masson SAS. All rights reserved.

1. Introduction

Among specific features of brittle damage, the question of unilateral effect is still an open and difficult task for the constitutive modelling of these materials. Indeed, microcracks can be open or closed according to the applied loading and then affect the mechanical behaviour of the materials differently. As pointed out by many authors [1–3], the account of such aspect often leads to serious inconsistencies in macroscopic formulations.

Built within the consistent thermodynamics framework with internal variables, the new approach recently proposed by Welemene and Cormery [4,5] provides a rigorous, physical and continuum description of opening-closure effects. Salient

* Corresponding author.

E-mail addresses: Helene.Welemene@enit.fr (H. Welemene), Cristina.Goidescu@enit.fr (C. Goidescu).

features of the model are a damage representation referring to the microcracks density distribution and based on fabric tensors, a recovery condition at the closure of microcracks motivated by micromechanics and a damage evolution law written in the standard scheme. Moreover, the formulation exhibits a modular character since it can be developed for various configurations of damage anisotropy.

The present work focuses on the isotropic version of this modelling approach. Since isotropic damage models are still very used in literature for structural analysis [6,7], it seems indeed interesting to investigate such version which proposes an enriched description of damage effects. Contrary to existing formulations that use several damage parameters or discrete decompositions (for example [8–11]), this model deals with a single scalar damage variable while accounting for the unilateral behaviour of each set of parallel microcracks of the material.

2. Isotropic damage model with closure effects

In this section, we briefly present the isotropic version of the three-dimensional damage model by [4,5]. An objective of the constitutive model summarized below is to describe the process of microcrack-induced degradation and corresponding behaviour of quasi-brittle materials for rate independent, isothermal and small transformations. Considering a single dissipative mechanism, namely the generation and growth of non-interacting microcracks, it relies on a physical damage description and on micromechanically-based assumptions to account for the closure of microcracks. Special attention has been given also to the respect of mathematical conditions and thermodynamical principles.

Usual intrinsic notations are employed throughout. The formulation is based on the following hypotheses and developments:

(1) The damage state of a medium is described by its microcrack density distribution ρ such that the density $\rho(\mathbf{n})$ refers to a scalar and dimensionless measure of the extent of crack-like defects orthogonal to unit vector \mathbf{n} . Retaining an approximation of order zero of function ρ , the damage mechanism is described here by a single scalar variable d corresponding to the density of all microcracks within the representative volume:

$$d = \frac{1}{4\pi} \int_S \rho(\mathbf{n}) \, ds \quad (1)$$

with $S = \{\mathbf{n} \in \mathbb{R}^3, \mathbf{n} \cdot \mathbf{n} = 1\}$ the unit sphere of \mathbb{R}^3 and ds the infinitesimal surface element on S .

(2) In view of further applications to structural analysis, we have retained a strain-based formulation for the thermodynamic potential. We assume the existence of the free energy W per unit volume, isotropic invariant of the state variables $(\boldsymbol{\varepsilon}, d)$, of class C^1 and positively homogeneous of degree two with respect to $\boldsymbol{\varepsilon}$. Precisely, we propose the following form:

$$W(\boldsymbol{\varepsilon}, d) = W_0(\boldsymbol{\varepsilon}) + \frac{1}{4\pi} \int_S w(\boldsymbol{\varepsilon}, d, \mathbf{n}) \, ds \quad (2)$$

where $W_0(\boldsymbol{\varepsilon})$ denotes the free energy of the undamaged material (assumed to be isotropic and linear elastic with Lamé constants λ_0 and μ_0). For the sake of simplicity, the elementary energy function w that characterizes the energy modification induced by each set of parallel microcracks is taken linear in the density d :

$$w(\boldsymbol{\varepsilon}, d, \mathbf{n}) = dh(\boldsymbol{\varepsilon}, \mathbf{n}) \quad (3)$$

which implicitly corresponds to the small microcrack density assumption. Moreover, as microcracks affect the mechanical response of brittle materials differently whether they are open or closed, their contribution should differ according to this status. In this way, we introduce an opening-closure characteristic function g depending on $(\boldsymbol{\varepsilon}, \mathbf{n})$, of class C^1 , radially symmetric with respect to \mathbf{n} , that defines the status of microcracks between the open configuration (if $g(\boldsymbol{\varepsilon}, \mathbf{n}) > 0$) and the closed state (if $g(\boldsymbol{\varepsilon}, \mathbf{n}) \leq 0$). Accordingly, function h introduced in Eq. (3) is defined in the following way:

$$h(\boldsymbol{\varepsilon}, \mathbf{n}) = \begin{cases} h^{open}(\boldsymbol{\varepsilon}, \mathbf{n}), & \text{if } g(\boldsymbol{\varepsilon}, \mathbf{n}) > 0 \\ h^{clos}(\boldsymbol{\varepsilon}, \mathbf{n}), & \text{if } g(\boldsymbol{\varepsilon}, \mathbf{n}) \leq 0 \end{cases} \quad (4)$$

(3) General expressions of functions h^{open} and h^{clos} are given by representation theories of tensorial functions. In order to account for microcracks closure effects and related damage partial deactivation, function h^{clos} should additionally satisfy a recovery condition on elastic properties at the closure of defects. Following micromechanical results [12,13], we thus assume that a set of microcracks with normal \mathbf{n} does not contribute to the degradation of the elongation modulus related to their normal direction, neither to the material volumetric modulus related to any direction of the space (defined in [14]). Besides, for the mathematical admissibility of W , h^{open} and h^{clos} must also respect some continuity conditions that are derived from a rigorous analysis of multilinear functions with several variables ([15] extended by [4,16]). All these arguments lead finally to the expression of the thermodynamic potential (details are given in [4,5]):

$$W(\boldsymbol{\varepsilon}, d) = W_0(\boldsymbol{\varepsilon}) + d \left[\alpha \operatorname{tr}(\boldsymbol{\varepsilon} \cdot \boldsymbol{\varepsilon}) + \beta (\operatorname{tr} \boldsymbol{\varepsilon})^2 - \frac{\Delta}{8\pi} \int_{S^{clos}(\boldsymbol{\varepsilon})} g^2(\boldsymbol{\varepsilon}, \mathbf{n}) \, ds \right] \quad (5)$$

Table 1
Coefficients $(\Delta, \delta_1, \delta_2)$.

	$\alpha \neq \alpha'$	$\alpha = \alpha'$
Δ	$15(\alpha - \alpha')$	$\frac{2}{3}(\alpha + 3\beta)$
δ_1	1	0
δ_2	$\frac{1}{3}(\sqrt{\frac{2(\alpha+3\beta)}{5(\alpha-\alpha')}} - 1)$	1

where (α, β, Δ) are independent material constants and the closure domain $S^{clos}(\boldsymbol{\epsilon}) = \{\mathbf{n} \in \mathcal{S}, g(\boldsymbol{\epsilon}, \mathbf{n}) \leq 0\}$ represents the sets of normals to closed microcracks for the strain state $\boldsymbol{\epsilon}$ (computation on this domain is explained in [17]). Precisely, the microcrack opening-closure function g takes the following general form $(\delta_1, \delta_2$ two dependent material constants):

$$g(\boldsymbol{\epsilon}, \mathbf{n}) = \delta_1 \mathbf{n} \cdot \boldsymbol{\epsilon} \cdot \mathbf{n} + \delta_2 tr \boldsymbol{\epsilon} \tag{6}$$

In Eq. (5), terms in α and β define the damage contribution to energy when microcracks are all open while the integral term in Δ represents the modification induced by the possible closure of some defects according to the loading. Accordingly, the state equations, defining respectively the stress $\boldsymbol{\sigma}$ and the conjugate thermodynamic force F^d associated to damage (damage energy release rate) and determined by corresponding partial derivation of the energy (2), are also affected by unilateral effects [4,17].

(4) Damage growth is assumed to be progressive and rate independent. Its evolution law is expressed here within the standard thermodynamic framework which ensures the systematic satisfaction of the second thermodynamic principle. We postulate the existence of a scalar dissipation potential \mathcal{D} function of the damage rate \dot{d} and current damage state d in the form:

$$\mathcal{D}(\dot{d}, d) = \mathcal{R}(d)\dot{d} \tag{7}$$

In any evolution process, the dissipation $\Phi = F^d \dot{d} = \mathcal{D}(\dot{d}, d)$ is then proportional to the rate of the total density of microcracks in the medium and $\mathcal{R}(d)$ can be directly linked to the surface energy of fracture mechanics. Following Marigo [18], a linear form is adopted in this work, $\mathcal{R}(d) = k_0(1 + \eta d)$ with (k_0, η) two strictly positive material constants. Combining this damage function with the normality rule, the damage evolution law reads:

$$\begin{cases} \dot{d} = \dot{\lambda} \frac{\partial f}{\partial F^d}(F^d, d) = \dot{\lambda} \\ \dot{\lambda} = 0, & \text{if } f(F^d, d) \leq 0, \dot{f}(F^d, d) < 0 \\ \dot{\lambda} > 0, & \text{if } f(F^d, d) = 0, \dot{f}(F^d, d) = 0 \end{cases} \tag{8}$$

where the yield function f in the relevant F^d -space is given by:

$$f(F^d, d) = F^d - \mathcal{R}(d) \tag{9}$$

and the damage multiplier $\dot{\lambda}$ is determined by the consistency condition. If the criterion (9) together with evolution law (8) maintains the isotropic character of the microcrack distribution (scalar damage representation), note that the evolution yield and damage rate depend however on the amount and directions of closed microcracks.

(5) Concerning the model identification, one needs first the determination of the two Lamé coefficients of the virgin material (λ_0, μ_0) which is rather classical and of the constants related to the microcracks contribution to energy. The latter requires the indication at fixed damage state of both the damage density and effective compliance of the material when microcracks are all open (to define constants α and β), and when at least some defects are closed (to define constants Δ and parameters (δ_1, δ_2) of the opening-closure function g). If we express the material energy (5) in the two particular cases where microcracks are all open ($S^{clos}(\boldsymbol{\epsilon}) = \emptyset$) or all closed ($S^{clos}(\boldsymbol{\epsilon}) = \mathcal{S}$), we obtain:

$$W(\boldsymbol{\epsilon}, d) = \begin{cases} (\mu_0 + \alpha d) tr(\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}) + (\frac{\lambda_0}{2} + \beta d) tr^2 \boldsymbol{\epsilon}, & \text{if } S^{clos}(\boldsymbol{\epsilon}) = \emptyset \\ (\mu_0 + \alpha' d) tr(\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}) + (\frac{\lambda_0}{2} + \beta' d) tr^2 \boldsymbol{\epsilon}, & \text{if } S^{clos}(\boldsymbol{\epsilon}) = \mathcal{S} \end{cases} \tag{10}$$

with $\beta' = -\alpha'/3$. From this, one can demonstrate that parameters $(\Delta, \delta_1, \delta_2)$ characterizing closure effects on the energy can be determined only from the triplet (α, β, α') (see Table 1). As a consequence, the energetic part of the model can be identified from all open and all closed states of microcracks. Precisely, in view of the current lack of experimental data needed (damage distribution and elastic properties on the same material), a strategy based on micromechanics can be adopted to identify these parameters [4,16]. Finally, the last question concerns parameters (k_0, η) related to the damage evolution. Broadly speaking, k_0 characterizes the initial damage limit whereas η is linked to the variation of the elastic domain with the damage growth. In both cases, the determination of these two constants is performed by calibration from the nonlinear portion of experimental loading curves [4].

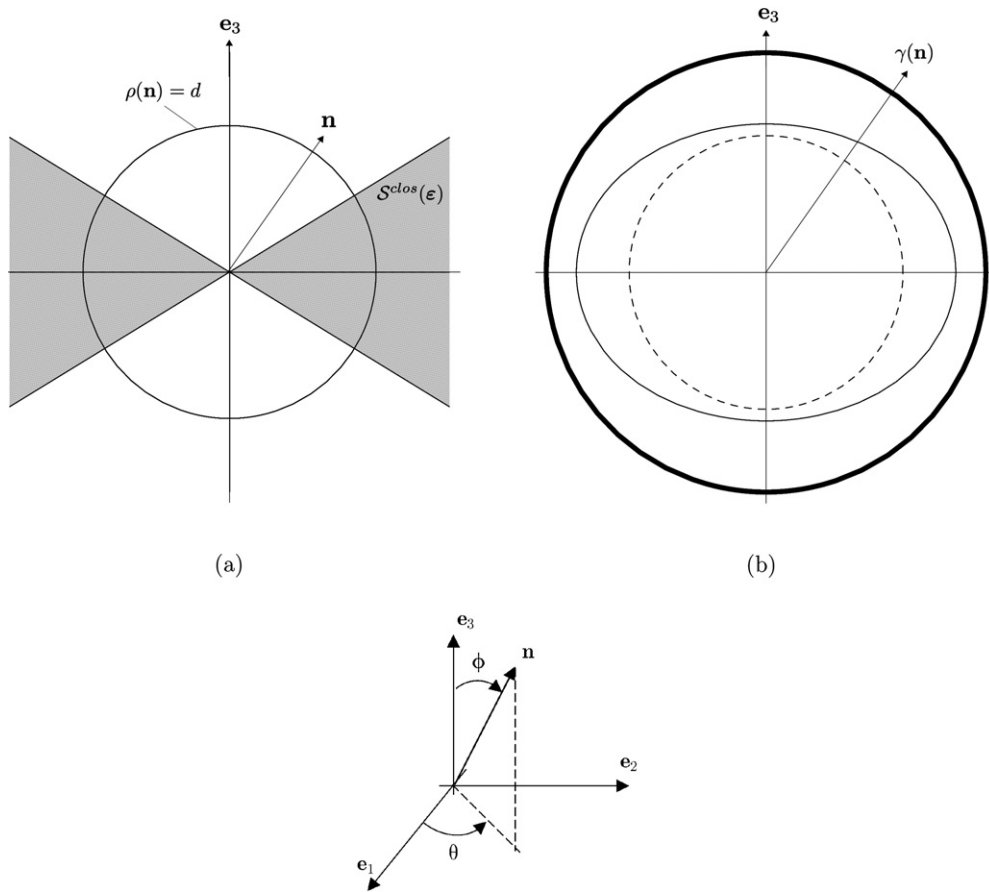


Fig. 1. Concrete weakened by a fixed isotropic damage distribution ($d = 0.2$): (a) Damage distribution and microcracks closure domain (in grey); (b) Elongation moduli (as a reference are given also isotropic cases of “all open” (---) and “all closed” (—) states of microcracks).

3. Applications and discussion

3.1. Elastic properties analysis

This section aims at investigating the model representation of the material elastic properties at fixed damage state. The integral formulation (2) of the energy leads to a continuum representation of unilateral effect, which accounts for all possible orientations of microdefects. However, the representation depends on the opening-closure characteristic function defined by (6). According to the identification, g may thus take two forms (the case $(\delta_i = 0)_{i=1,2}$ without unilateral effect is ignored):

- if $\delta_1 = 0$, the criterion reduces to a function of $\boldsymbol{\varepsilon}$, namely $g(\boldsymbol{\varepsilon}) = \text{tr } \boldsymbol{\varepsilon}$, and microcracks can be either all open (when $S^{\text{clos}}(\boldsymbol{\varepsilon}) = \emptyset$) either all closed (when $S^{\text{clos}}(\boldsymbol{\varepsilon}) = \mathcal{S}$). In this case, the representation of closure effects takes its simplest form: the material behaviour remains isotropic whatever the defects status even if damage degradation differs whether they are open or closed;
- if $\delta_1 \neq 0$, the criterion function depends explicitly of both the strain $\boldsymbol{\varepsilon}$ and the normal \mathbf{n} to each set of parallel microcracks, that is $g(\boldsymbol{\varepsilon}, \mathbf{n})$. This case allows to account for mixed states for which some microcracks are open and some are closed and the dependence in \mathbf{n} leads to a distinction of the microcracks contribution for each direction of the space. Even for an isotropic damage extent, the model may thus describe an anisotropic elastic behaviour induced by closure effects for configurations of mixed states with a resulting anisotropy corresponding to the symmetry of the closure domain $S^{\text{clos}}(\boldsymbol{\varepsilon})$.

Consider for example a concrete material ($\lambda_0 = 8965$ MPa, $\mu_0 = 13\,445$ MPa, $k_0 = 0.001$, $\eta = 200$) whose model constants relative to damage have been identified from the micromechanical analysis of a medium weakened by an isotropic distribution of flat penny-shaped microcracks ($\alpha = -20\,395$ MPa, $\beta = -18\,700$ MPa, $\alpha' = -12\,745$ MPa). Table 1 leads then to $\delta_1 = 1$ and $\delta_2 = \frac{\lambda_0}{2\mu_0}$ which corresponds to the second case detailed above. Accordingly, for the mixed state of opening-closure defined on Fig. 1(a), one obtains on Fig. 1(b) the corresponding anisotropic distribution of elongation moduli $\gamma(\mathbf{n})$.

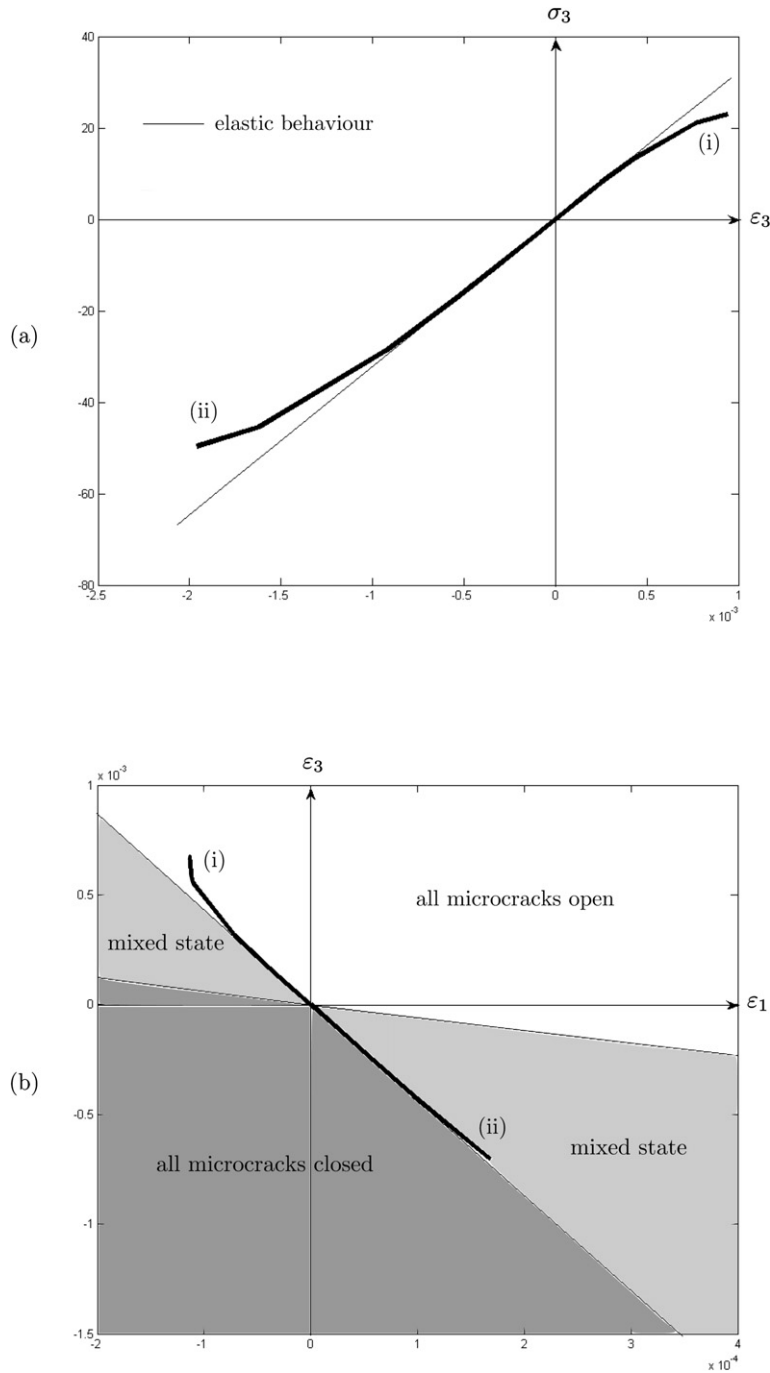


Fig. 2. Concrete submitted to uniaxial tension (i) and compression (ii) tests: (a) Axial strain-stress responses; (b) Loading paths and opening-closure domains of microcracks in the axisymmetric strain space.

3.2. Uniaxial tests

Let examine now the dissipative response provided by the model when the material is submitted to uniaxial stress $\sigma = \sigma_3 \mathbf{e}_3 \otimes \mathbf{e}_3$: (i) in tension ($\sigma_3 > 0$) and (ii) in compression ($\sigma_3 < 0$). Fig. 2(a) presents the axial stress-strain responses $\sigma_3 - \epsilon_3$ (with $\boldsymbol{\epsilon} = \epsilon_1(\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2) + \epsilon_3 \mathbf{e}_3 \otimes \mathbf{e}_3$) during the loading phase of each test. Since the elastic behaviour is represented as a reference, we can demonstrate first that the formulation accounts for the material nonlinear response induced by damage evolution. While dealing with a single scalar damage variable and keeping the simple standard thermodynamic framework, the formulation represents moreover the dissymmetric behaviour in tension and compression typical of the

brittle-like behaviour. This results also from closure effects as demonstrated by Fig. 2(b) of the opening and closure domains of microcracks: the tension loading path (i) remains indeed in the “all microcracks open” part whereas the compression loading path (ii) belongs to a mixed domain where some microcracks are open and some are closed (precisely those rather perpendicular to the compression axis). Accordingly, the microcracks contribution to the damage energy release rate F^d differs between these two situations and induces a different damage evolution according to the yield function (9). Note that the representation of some aspects (pronounced nonlinearity or dissymmetry) may be limited by simplified assumptions retained (especially for the damage evolution law) and can be obviously improved while keeping the same approach based on the introduction of closure effects.

4. Conclusion

We have presented an original approach of isotropic damage for brittle materials. Such formulation introduces new modelling arguments to account for the microcracks unilateral behaviour (damage representation, theory of multilinear functions, reference to micromechanical results) within the thermodynamic framework usually considered for standard isotropic models. Compared to existing studies, the model stands out by combining a single scalar damage variable with a continuous description of degrading effects that avoids any decomposition of state variables and related difficulties (mathematical or thermodynamical). The simple context adopted here (isotropic damage, basic standard evolution law) allows to highlight the consequences of the damage unilateral behaviour and, precisely, puts forward an alternative way of representing some specific features of the brittle behaviour (especially anisotropic elasticity, dissymmetry between tension and compression) by means of the introduction of microcracks opening-closure effects.

References

- [1] J.-L. Chaboche, Damage induced anisotropy: On the difficulties associated with the active/passive unilateral condition, *Int. J. Damage Mech.* 1 (1992) 148–171.
- [2] I. Carol, K. Willam, Spurious energy dissipation/generation in stiffness recovery models for elastic degradation and damage, *Int. J. Solids Structures* 33 (1996) 2939–2957.
- [3] F. Cormery, H. Welemane, A critical review of some damage models with unilateral effect, *Mech. Res. Comm.* 29 (2002) 391–395.
- [4] H. Welemane, Une modélisation des matériaux microfissurés – Application aux roches et aux bétons, PhD thesis, Lille, France, 2002.
- [5] H. Welemane, F. Cormery, An alternative 3D model for damage induced anisotropy and unilateral effect in microcracked materials, *J. Phys. IV* 105 (2003) 329–336.
- [6] C. Comi, Computational modelling of gradient-enhanced damage in quasi-brittle materials, *Mech. Cohes. Frict. Mater.* 4 (1999) 17–36.
- [7] E. Lorentz, P. Badel, J. Ghousoub-El Gharib, Simuler l'endommagement d'une structure : traitement numérique des instabilités, *Méca. Indus.* 3 (2002) 463–469.
- [8] P. Ladevèze, J. Lemaitre, Damage effective stress in quasi-unilateral conditions, in: *Proc. 16th Int. Cong. Theo. Appl. Mech.*, Lyngby, Denmark, 1984.
- [9] J. Mazars, Y. Berthaud, S. Ramtani, The unilateral behaviour of damaged concrete, *Engng. Fract. Mech.* 35 (1990) 629–635.
- [10] P. Ladevèze, Mechanics and mechanisms of damage in composites and multimaterials, in: D. Baptiste (Ed.), MEP, London, 1991.
- [11] R. Bargellini, D. Halm, A. Dragon, Modelling of anisotropic damage by microcracks: Towards a discrete approach, *Arch. Mech.* 58 (2006) 93–123.
- [12] H. Welemane, F. Cormery, Some remarks on the damage unilateral effect modelling for microcracked materials, *Int. J. Damage Mech.* 11 (2002) 65–86.
- [13] V. Pensée, D. Kondo, Une analyse micromécanique 3-D de l'endommagement par microfissuration, *C. R. Acad. Sci. Paris IIb* 329 (2001) 271–276.
- [14] Q.C. He, A. Curnier, A more fundamental approach to damaged elastic stress-strain relations, *Int. J. Solids Structures* 32 (1995) 1433–1457.
- [15] A. Curnier, Q.C. He, P. Zysset, Conewise linear elastic materials, *J. Elasticity* 37 (1995) 1–38.
- [16] F. Cormery, H. Welemane, A stress-based macroscopic approach for microcracks unilateral effect, *Comp. Mat. Sci.* 47 (2010) 727–738.
- [17] H. Welemane, C. Goidescu, Accounting for microcracks unilateral effect with isotropic damage, in: *Proc. Int. Conf. Struc. Analysis Adv. Mater.*, Tarbes, France, 2009.
- [18] J.J. Marigo, Modelling of brittle and fatigue damage for elastic material by growth of microvoids, *Engng. Fract. Mech.* 21 (1985) 861–874.