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Taylor series to solve friction problems

Technique de perturbations pour les problèmes de frottement

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ABSTRACT

Thin metallic sheet transportation appears in numerous manufacturing processes such as continuous annealing, levelling or galvanization. It involves various nonlinear phenomena and, in particular, contact with friction. We develop a numerical method to solve this kind of mechanical problem, using shell finite elements and the Asymptotic Numerical Method (ANM). This article focuses on the treatment of the friction equations with ANM.

RÉSUMÉ

Le transport de bandes minces métalliques apparaît dans de nombreux procédés industriels tels que le recuit continu, le planage ou la galvanisation. Au cours de celui-ci, différents phénomènes non linéaires entrent en jeu et en particulier le contact avec frottement. Nous développons un logiciel Eléments Finis permettant de modéliser ce processus, les non linéarités étant traitées avec la Méthode Asymptotique Numérique (MAN). Nous présentons ici comment traiter les équations de frottement avec la MAN.

1. Introduction

In steelworks, the conveying of thin metallic sheets is generally performed with a set of rollers which guide the strip in an up and down vertical motion. This process implies different kinds of nonlinear phenomena such as large rotations, contact with friction and elasto-plasticity. Within the ANR project Instabande, we have developed an FE program to simulate this kind of process. The aim of this project is to predict the occurrence of plastic creases called wrinkles, which may appear during the conveying [1], and make the sheet unusable. These wrinkles are caused by a buckling which occurs when the sheet is put under tension. It was shown that the friction arising between the sheet and the roller plays an important role in this mechanical process [2]. This Note will focus on the simulation of friction problems. In such processes, inertia effects can be neglected. Consequently, only quasi-static problems will be addressed.

The nonlinearities involved are handled with a high order continuation procedure, based on the Asymptotic Numerical Method (ANM) [3,4]. At each step of the continuation procedure, the variables of the problem are represented by truncated power series whose terms are solutions of a series of well-posed linear problems having the same tangent matrix. The computation of high order derivatives can be performed with an automatic differentiation algorithm [5]. All the results presented are obtained with truncature order of 19 and a precision of 10^{-4}.

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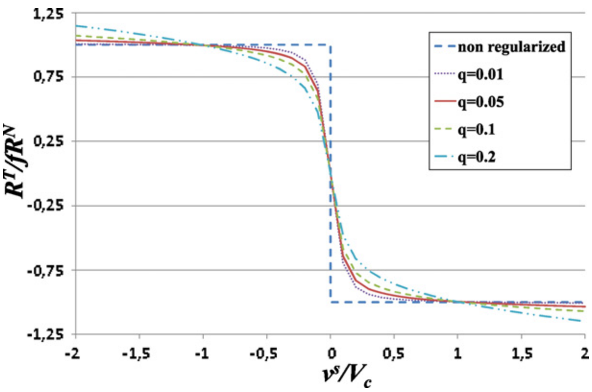

Fig. 1. Friction law for different values of q ($\omega = 0.1$).

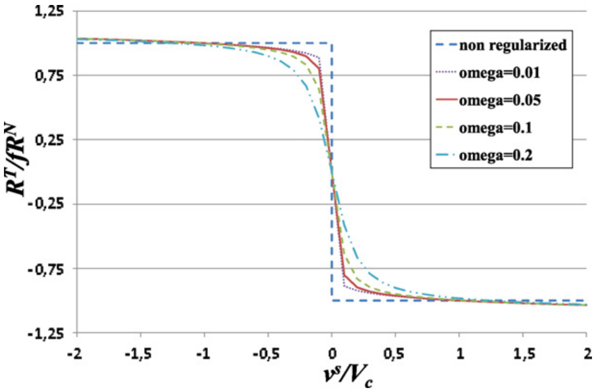

Fig. 2. Friction law for different values of ω ($q = 0.05$).

As the ANM can only be applied to smooth problems, one has first to regularize the contact and friction equations [3]. It is known that frictionless contact and elasticity lead to an algebraic equation for the displacement [6], while friction and perfect viscoplasticity lead to an algebraic equation for the velocity [7]. We consider here an elastic behavior coupled with friction so that the quasi-static approach leads to a Differential Algebraic Equation. In this Note, we briefly discuss the regularization technique and the computation algorithm to solve this DAE.

2. Regularization of the friction law

We consider a deformable solid (with a velocity $\boldsymbol{v}$) which can be put into contact with a rigid solid with a velocity $\boldsymbol{V}$. The contact pressure between the two solids is decomposed into a normal part $R^N\boldsymbol{n}$ and a tangential part $\boldsymbol{R}^T$, $\boldsymbol{n}$ being the normal to both solids at the contact point. The Coulomb's law classically writes:

$$\boldsymbol{R}^T = -fR^N \frac{\boldsymbol{v}^s}{|\boldsymbol{v}^s|} \quad \text{or} \quad \boldsymbol{v}^s = 0 \tag{1}$$

where f is the friction coefficient and $\boldsymbol{v}^s = (\boldsymbol{v} - \boldsymbol{V}) - ((\boldsymbol{v} - \boldsymbol{V}) \cdot \boldsymbol{n})\boldsymbol{n}$ denotes the tangential relative velocity. As said before, the use of an ANM algorithm requires regular laws. This is not the case of the law (1) which is singular in $\boldsymbol{v}^s = \boldsymbol{0}$ and not invertible. Several regularizations can be used, for instance a regularization with tanh [8]. In the sequel, we will use a "Norton–Hoff"-like law [9] which is of the form:

$$\boldsymbol{R}^T = -fR^N \frac{|\boldsymbol{v}^s|_{reg}^{q-1}}{{V_c}^q} \boldsymbol{v}^s \quad \text{with } |\boldsymbol{v}^s|_{reg} = \sqrt{\boldsymbol{v}^s.\boldsymbol{v}^s + \omega^2 {V_c}^2} \tag{2}$$

ω and q being regularization parameters (between 0 and 1) and V_c a characteristic velocity of the problem.

For $q \neq 0$, $\boldsymbol{R}^T$ depends on the modulus of the tangential relative velocity. Consequently, we have a one-to-one relation between $\boldsymbol{R}^T/fR^n$ and $\boldsymbol{v}^s$ and the friction law is analytic. Moreover, this law being invertible, it is possible to compute $\boldsymbol{v}^s$ for a given $\boldsymbol{R}^T/fR^n$. Note that with this law, the contact forces are allowed to be outside the friction cone (in the sense of Coulomb) when the tangential relative velocity is greater than V_c (see Figs. 1 and 2). For $q = \omega = 0$, we recover Coulomb's law.

3. Solving a Differential Algebraic Equation using ANM

3.1. Contact with a prescribed normal force

We consider here a simple one-dimensional problem including friction: a spring mass system is put on a rigid plane moving at the velocity V (Fig. 3). The normal pressure $R^N > 0$ is a assumed to be given.

This regularized problem is equivalent to the following nonlinear first order ordinary differential equation in u^x (displacement in the direction x):

$$ku^x + fR^N \frac{|v^x - V|_{reg}^{q-1}}{V_c^q}\left(v^x - V\right) = 0, \quad \text{where } \frac{\mathrm{d}u^x}{\mathrm{d}t} = v^x \tag{3}$$

Developing u^x and its derivative v^x in power series with respect to the variable t, we obtain at order p a problem of the following form:

$$Ku_{p+1}^x = F^{nl}\left(u_0^x, \ldots, u_p^x\right) \tag{4}$$

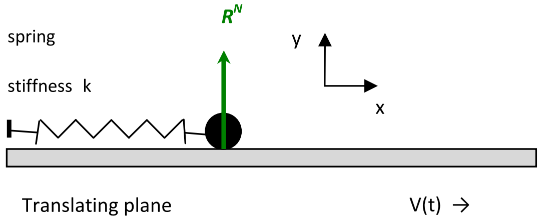

Fig. 3. Mass put on a rigid plane.

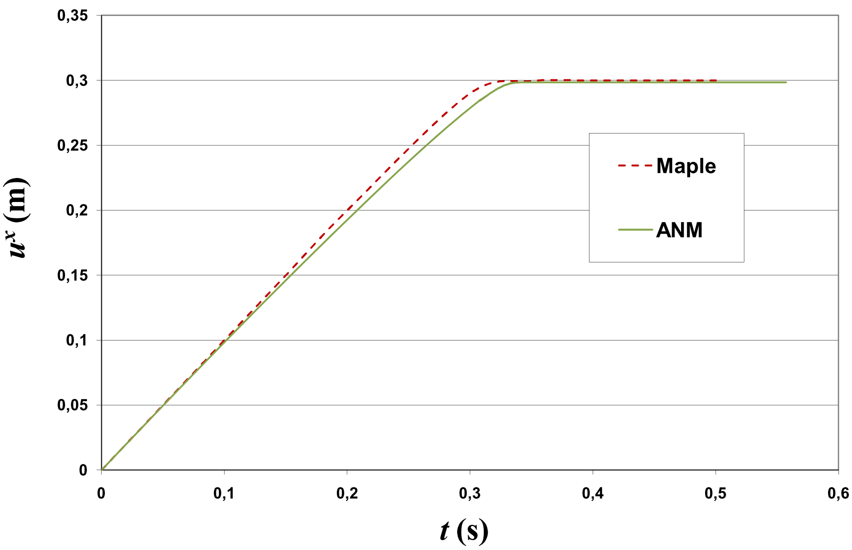

Fig. 4. Displacement u^x.

where F_p^{nl} is a function depending only on the previous orders. We have then a recurrence formula which traduces the fact that (3) can be solved in v^x but that supposes that $K \neq 0$ and consequently $q > 0$, $\omega > 0$ and $R^N > 0$.

At order $p = 0$, we have to solve a nonlinear equation to determine v_0^x. This can be done using a Newton method. However, if the initial tangential pressure vanishes, which is generally the case, the initial velocity is simply $v_0^x = V$. On Fig. 4, the result obtained with the ANM algorithm for $R^N = 1$ N, $k = 1$ N m^{-1}, $V = V_c = 1$ m s^{-1}, $\omega = 0.1$ and $q = 0.05$ (34 steps) is compared with a MAPLE solution found for $\omega = 0$.

We can see the influence of the regularization of the velocity when the norm of the tangential relative velocity increases from zero to a non-zero value. That is the case between $t = 0.1$ s and $t = 0.3$ s where we can see a small gap between the MAPLE curve (obtained for $\omega = 0$) and the ANM results. The final position corresponds to the equilibrium position where the tangential force is balanced by the spring force.

3.2. Contact with a mobile rigid plane

The aim of this part is to address a contact problem with friction in the simple case when normal and tangent parts are naturally split. To this end, we consider now the two-dimensional problem which is represented in Fig. 5.

A mass is affixed to two springs inclined of 45° with respect to the vertical. They are supposed very long compared to the other dimensions of the problem so that we can use a linear approximation of the geometry (the springs keeps the same inclination). At the initial state, the mass is placed at the distance δ of a rigid plane moving horizontally at the velocity V. At $t = 0$, a force proportional to the time is applied. The problem satisfies the system:

$$-ku^x + R^T = 0 \tag{5a}$$

$$-ku^y + R^N = Ft \tag{5b}$$

$$R^N\left(\frac{R^N}{K} + h\right) = \eta(\delta - h) \quad \text{with } h = \delta + u^y \tag{5c}$$

$$R^T = -f\left|R^N\right|_{reg}|v^s|_{reg}^{q-1}v^s \quad \text{with } \left|R^N\right|_{reg} = \sqrt{\left(R^N\right)^2 + \zeta^2\left(R_C^N\right)^2} \tag{5d}$$

ζ and R_C^N being, respectively, a regularization parameter and a characteristic force of the problem.

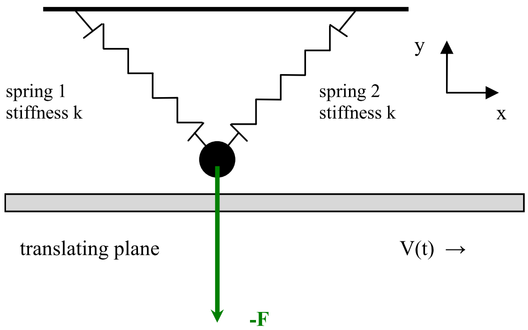

Fig. 5. Mass in contact with a mobile plane.

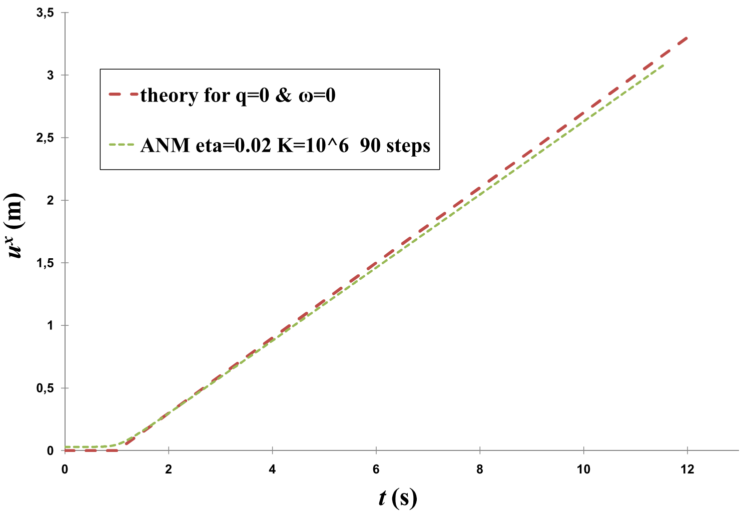

Fig. 6. Displacement u^x.

Eqs. (5a) and (5b) are the equilibrium equations of the mass. Eq. (5c) is a regularization of the Signorini condition for unilateral contact where η is a regularization parameter and K a kind of rigidity of the obstacle. This law allows a penetration in the obstacle so that h can be negative.

The system (5) is a Differential Algebraic Equation. The normal problem (5b), (5c) is of elastic type and is an algebraic equation that defines the normal quantities u^y and R^N as functions of time, independently of the rest of system (5). When this problem is solved at order p, this gives these unknowns at order p, u^y_p and R^N_p. The two other equations (5a), (5d) lead to a first order differential equation similar to (3). When they are solved at order p, this yields $u^x_{p+1} = g(u^x_0, \ldots, u^x_p, R^N_0, \ldots, R^N_p)$ which requires the results of the normal part.

Thus, at a given order p, we solve the normal part giving u^y_p and R^N_p, and then the ODE with a technique similar to that of Section 3.1. However, as R^N vanishes at the beginning (which implies that $K = 0$ in (4)), it was necessary to regularize the friction equation (2) again otherwise the first step could not be solved. The final regularization is given by Eq. (5d).

The displacement u_x is presented in Fig. 6 (obtained with 90 steps) and compared to an analytical result of the non-regularized problem. The influence of the regularization is perceptible when the contact occurs at about $t = 1$ s. After $t = 1$ s, there is contact and friction: the mass is carried away further and further because of the increasing force. The slope of the curve is smaller than that of the theoretical solution (0.29 against $0.3 = f$) because of the regularization ($q \neq 0$).

3.3. Contact with a circular rigid obstacle in rotation

In this section, we consider the same mass spring system as in the previous system but we now take a circular rigid obstacle of radius r which turns around its axis $(0, \boldsymbol{e}_z)$ at Ω (Fig. 7). A force proportional to the time is applied again.

In the Cartesian coordinate system $(\boldsymbol{e}_x, \boldsymbol{e}_y)$, the problem is no longer uncoupled, algebraic and differential parts are mixed together. To handle this difficulty, we write the equilibrium equations and decompose the displacement in the local coordinate system at the beginning of the step $(\boldsymbol{n}_J, \boldsymbol{t}_J)$.

This enables us to have then an uncoupled formulation at order p. Indeed, all the coupled terms in the equations are known from the previous orders. Thus, more generally at order p, we can solve this problem as in Section 3.2 where normal

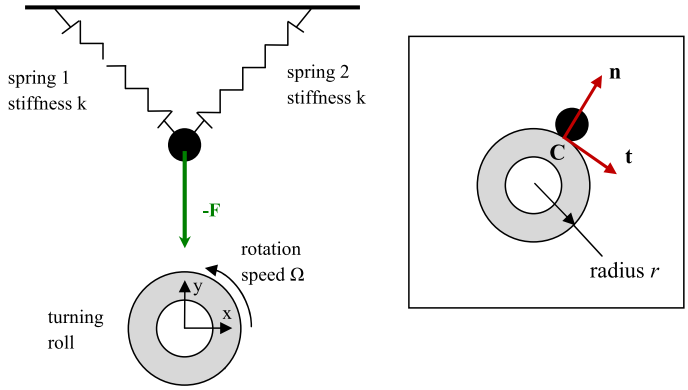

Fig. 7. Mass in contact with a rotating circular obstacle.

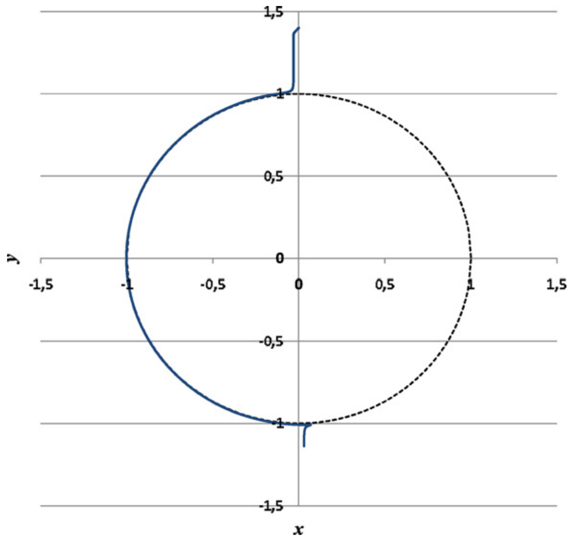

Fig. 8. Trajectory of the mass.

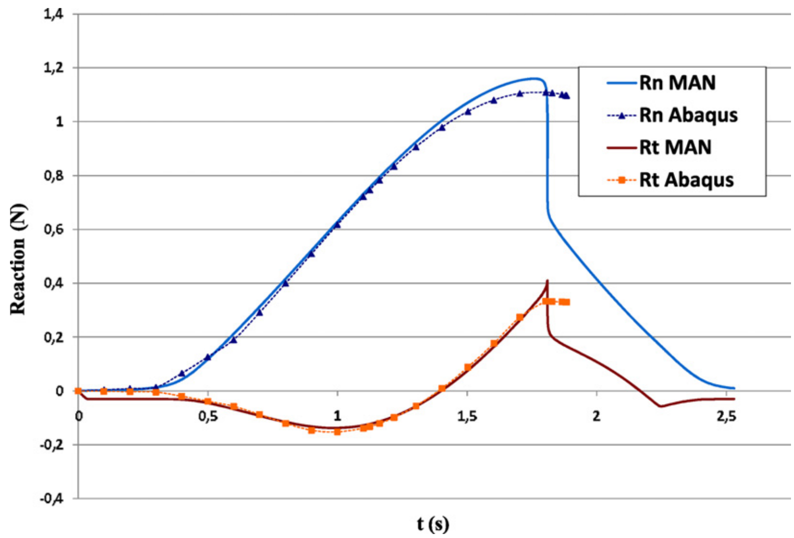

Fig. 9. Contact reactions.

and tangent parts are fully uncoupled. The main difference is that we require now the solution of the tangential problem at orders $k < p$ to solve the normal part at order p whereas it was fully independent in the previous case.

In Fig. 8, the trajectory of the mass (obtained with 90 steps) is plotted: the gap with respect to the line $x = 0$ at the beginning and at the end is due to the regularization of R^N in the friction equation (2).

The contact reactions are very close to the results obtained by an Abaqus model until the time $t = 1.8$ s (Fig. 9). At this time, the reactions are outside the friction cone and there is no solution for the non-regularized friction equation. It was not possible to get a numerical solution with Abaqus but also with our algorithm when using a friction law regularized with tanh. With the law (5d) and $q > 0$, the quasi-static process exists and it can be computed numerically, but the non-existence property is "detected" by a very rapid variation just after $t = 1.8$ s (Fig. 9). An alternative way would be to consider the dynamical problem. Thus, it would not be necessary to regularize the friction law to get a numerical solution [10].

4. Conclusion

In this Note, we presented some algorithms allowing to solve friction problems with the Asymptotic Numerical Method. This method requiring regular laws, we had to regularize Coulomb's law. We then proposed a method to solve mass spring system problems, first for a plane obstacle and then for a curved one. Difficulties come from uncoupling equations in normal and tangential parts and then solving a DAE with an automated solver. The next step consists in including this kind of algorithm in a shell finite element model based on the ANM. In this case, uncoupling and series convergence are not as obvious as in the academic case presented in this Note.

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