Modeling of Rayleigh–Bénard natural convection heat transfer in nanofluids

Modélisation des transferts de chaleur en configuration de Rayleigh–Bénard dans les nanofluides

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A R T I C L E   I N F O

A B S T R A C T

Heat transfer in Rayleigh–Bénard convection is investigated for three types of nanofluid. Instead of using the expressions commonly found in the literature for specific heat capacity and thermal expansion coefficient, we used two relations that are in agreement with the laws of thermodynamics. The influence of nanoparticles on conductive and convective heat transfer is studied. It is shown that adding nanoparticles in a fluid delays the onset of convection. Contrary to what is argued by many authors, we prove by direct numerical simulations that the use of nanofluids can reduce heat transfer instead of increasing it.

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R É S U M É

Le transfert de chaleur en configuration de Rayleigh–Bénard est analysé pour trois nanofluides différents. Au lieu de recourir aux relations que l'on trouve habituellement dans la littérature pour la capacité calorifique et le coefficient d'expansion thermique, nous avons utilisé deux relations en adéquation avec les lois de la thermodynamique. On étudie l'influence de la concentration en nanoparticules sur le transfert conductif et convectif. On a montré que la naissance de la convection est retardée par l'ajout de nanoparticules. Contrairement à ce qui a été obtenu par plusieurs auteurs, nous avons montré à partir de simulations numériques directes que la présence de nanoparticules dans un fluide peut réduire le transfert de chaleur au lieu de l'augmenter.

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1. Introduction

Nanofluids consist of uniformly dispersed and suspended nanometer-sized particles (10–50 nm) with very low concentration in a base fluid. A review of the state of knowledge on nanofluids was published by Wang and Mujumdar [1]. During

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this last decade, many authors claim that the presence of nanoparticles in a fluid alters the flow structure and increases the natural convection heat transfer. The results presented by Khanafer et al. [2] illustrated that the suspended nanoparticles substantially increased the heat transfer rate for any given Grashof number. In addition they showed that the heat transfer rate in nanofluids increased when increasing the volume fraction of nanoparticles. Abu-Nada et al. [3] studied the effects of inclination angle on natural convection in enclosures filled with Cu-water nanofluid. They indicated that the addition of nanoparticles produced a noticeable enhancement of heat transfer and that the heat transfer rate increased with the Rayleigh number. Many experimental studies are also available in the literature. Putra et al. [4] presented an experimental investigation of the natural convection of water-based Al_2O_3 nanofluids inside horizontal cylinder heated from one end and cooled from the other. An apparently paradoxical behavior of heat transfer deterioration was observed in their experimental study. They reported that the presence of nanoparticles of Al_2O_3 in base fluid reduced the natural convective heat transfer. The degradation of natural convection heat transfer when increasing the particle concentration for a given Rayleigh number was experimentally observed. However, Putra et al. [4] did not clearly explain why natural convective heat transfer is decreased with an increase in volume fraction of nanoparticles. Our study shows that the experimental results obtained by Putra et al. [4] are not at all paradoxical. Our numerical simulations, in the case of the Rayleigh–Bénard problem, provide an explanation for their experimental result. Kim et al. [5] showed that, for natural convection in nanofluids, the Nusselt number increased with the volume fraction of nanoparticles. These results are not consistent with the experimental results presented by Putra et al. [4] and with the numerical results presented in this paper.

It appears from this literature review, that the use of nanofluid to improve natural convection heat transfer remains an open problem.

The purpose of the present study is to examine the effect of adding nanoparticles to the base fluid, near the onset of convection, on the conduction and convection heat transfer rate in a rectangular cavity heated from below (Rayleigh–Bénard configuration). In this work we show that the relations expressing the specific heat and the volumetric thermal expansion coefficients which are still used by many authors [3–8] lead to results which differ significantly from the ones obtained with thermodynamically valid expression.

2. Mathematical formulation

We consider a rectangular cavity filled with a nanofluid. The impermeable horizontal walls are maintained at different and constant temperatures T_1 for $z = 0$ and T_2 for $z = H$ with $T_1 > T_2$. The vertical walls are impermeable and insulated. All the boundaries are assumed rigid. We assume that the Oberbeck–Boussinesq approximation is valid. The thermophysical properties of the nanofluid are considered constant except the density in the buoyancy term which varies linearly with the temperature: $\rho_{nf}(T) = \rho_{0,nf}[1 - \beta_{nf}(T - T_0)]$. Here the subscript 0 denotes the reference state, the subscript (nf) is relative to the nanofluids, β_{nf} is the thermal expansion coefficient of nanofluid and T is the dimensional temperature.

The governing equations for the problem can be written in dimensionless form as follows:

$$\vec{\nabla} \cdot \vec{V} = 0 \tag{1}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V} = -\vec{\nabla}P + Pr_{nf}Ra_{nf}T\vec{e}_z + Pr_{nf}\nabla^2\vec{V} \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{V}.\vec{\nabla}T = \nabla^2 T \tag{3}$$

where $\vec{V}$ is the velocity, P is the pressure and T is the temperature. The dimensionless parameters governing this problem are the Prandtl number and the Rayleigh number for the nanofluid defined by:

$$Ra_{nf} = \frac{g\beta_{nf}H^3(T_1 - T_2)}{\alpha_{nf}\nu_{nf}} \quad \text{and} \quad Pr_{nf} = \frac{\mu_{nf}(Cp)_{nf}}{k_{nf}} \tag{4}$$

where g is the gravitational acceleration, α_{nf} is the thermal diffusivity, ν_{nf} is the kinematic viscosity, μ_{nf} is the dynamic viscosity, $(Cp)_{nf}$ is the specific heat capacity and k_{nf} is the thermal conductivity.

The dimensionless boundary conditions are:

$$\vec{V} = \vec{0} \quad \text{on the boundaries} \tag{5}$$

$$T = 1 \quad \text{for } z = 0 \text{ and } T = 0 \quad \text{for } z = 1 \ \forall x \in [0, A] \tag{6}$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{for } x = 0, A \ \forall z \in [0, 1] \tag{7}$$

where A is the aspect ratio of the cavity.

We assume that the nanoparticles and the base fluid are in thermal equilibrium and that the nanoparticles are uniformly dispersed within the base fluid. The thermophysical properties of the nanofluid, namely the density, specific heat capacity and volumetric thermal expansion coefficient, were calculated from nanoparticles and base fluid properties at ambient

temperature using the following formulas which are compatible with the thermodynamic laws. Introducing the particle volume fraction φ, the density of the mixture is given by the following equation:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s \tag{8}$$

where the subscripts (s) and (f), denote respectively the nanoparticles and the base fluid. The volumetric thermal expansion coefficient β_{nf} of a nanofluid is given by:

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s \tag{9}$$

The specific heat capacity of nanofluid $(Cp)_{nf}$ is given by:

$$(\rho Cp)_{nf} = (1-\varphi)(\rho Cp)_f + \varphi(\rho Cp)_s \tag{10}$$

To obtain β_{nf}, most of the authors previously mentioned [5,6,9] used relations similar to the relation (8) which is only valid for ρ_{nf}:

$$\beta_{nf} = (1-\varphi)\beta_f + \varphi\beta_s \tag{11}$$

The specific heat capacity of nanofluid $(Cp)_{nf}$ was calculated, by many authors [6,9] using the approximate relation:

$$(Cp)_{nf} = (1-\varphi)(Cp)_f + \varphi(Cp)_s \tag{12}$$

where $(Cp)_f$ and $(Cp)_s$ are respectively the specific heat capacity of the base fluid and the nanoparticles. Polidori et al. [9] mentioned that several authors preferred to use this simpler relation to determine the specific heat and they used this relation to be consistent with them. We verified that these relations (11) and (12) did not lead to a good approximation of the nanofluid volumetric thermal expansion coefficient β_{nf} and of the specific heat capacity $(Cp)_{nf}$ for the three varieties of nanofluids studied.

The presence of nanoparticles in a fluid increases significantly the effective thermal conductivity of the fluid and consequently enhances the conductive heat transfer characteristics. Hamilton and Crosser [10] proposed a model to calculate the effective thermal conductivity of nanofluids:

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f - (n-1)(k_f - k_s)\varphi}{k_s + (n-1)k_f + (k_f - k_s)\varphi} \tag{13}$$

where k_f and k_s are respectively the thermal conductivity of the base fluid and the nanoparticles, φ is the volume fraction of nanoparticles and n is the empirical shape factor. Their experimental research showed satisfactory agreement between the theoretical predictions and the experimental results for many nanofluids in a range of volume fraction up to 30%.

The ratio of effective viscosity of nanofluids to that of their base fluid is calculated by Brinkman's model [11] currently used in literature [2,5,6]:

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\varphi)^{2.5}} \tag{14}$$

3. Heat transfer rate

Rayleigh–Bénard (RB) convection represents a fundamental process for heat transfer in fluids. The classical RB configuration corresponds to a horizontal infinite mono-constituent fluid layer heated from below. The linear stability analysis for the case of rigid–rigid infinite horizontal boundary condition shows that the conductive solution loses its stability when the Rayleigh number Ra_f reaches the critical value: $Ra_c = 1707.76$ associated with the critical wave number $k_c = 3.116$.

In this study, we focus only on heat transfer by natural convection in the case where the nanofluid is assimilated to mono-constituent fluid with specific thermophysical proprieties defined above.

To study the heat transfer by natural convection in nanofluids, we introduce the Nusselt number defined by:

$$Nu = \frac{Q_{conv}}{Q_{cond}} = \frac{h\Delta T}{(k\Delta T)/H} = \frac{hH}{k} \tag{15}$$

Here H is the thickness of the cell, k is the thermal conductivity of the fluid and h is the convection heat transfer coefficient.

In order to investigate the effects of volume fraction φ on the onset of convection, the Rayleigh number for nanofluids is expressed as follows:

$$Ra_{nf} = \frac{(\rho\beta)_{nf}}{(\rho\beta)_f} \frac{k_f}{k_{nf}} \frac{(\rho Cp)_{nf}}{(\rho Cp)_f} \frac{\mu_f}{\mu_{nf}} Ra_f \tag{16}$$

The Rayleigh number for this investigation varied from 0 to 10^4, and the range of the volume fraction φ varied between 0% and 8%.

Table 1
Grid convergence test for $Ra_f = 5000$ and $\phi = 0.08$, the global Nusselt number Nu at the bottom wall.

Grid system	20×100	22×110	25×125	28×140	30×150	32×160
Nu	1.5198	1.5194	1.5190	1.5188	1.5186	1.5186

Using Eq. (16) we show that $Ra_{nf} < Ra_f$ for all values of $\varphi \in [0, 0.08]$ and for water as base fluid with three nanoparticles (Al$_2$O$_3$, CuO and Cu). The ratio of the nanofluid Rayleigh number to the base fluid Rayleigh number decreases with the volume fraction of nanoparticles. For a fixed value of Ra_f, the value of Ra_{nf} decreases when adding nanoparticles, thus the onset of convection inside the nanofluid is delayed.

As shown in Eq. (16), in order to evaluate the nanofluid Rayleigh number function of the base fluid one, we need to understand the relations between the density, the thermal conductivity, the specific heat, the viscosity and the thermal expansion coefficient of the nanofluids and the base fluids.

For $Ra_{nf} < 1707.76$, the nanofluid assimilated to a mono-constituent fluid is quiescent and the temperature decreases linearly from T_1 to T_2.

The heat transfer rate across the fluid layer occurs by pure conduction, therefore $Nu = 1$.

Kim et al. [5] and Hwang et al. [6] used classical correlations giving the Nusselt number function of Ra in the case of natural convection in a cavity heated from below. Kim et al. [5] used the following correlations of heat transfer for the free convection driven by buoyancy:

$$Nu = 0.012Ra^{0.6} \quad (1700 < Ra \leqslant 6000)$$
$$Nu = 0.375Ra^{0.2} \quad (6000 < Ra < 37\,000)$$
(17)

These authors did not perform direct numerical simulations that would provide the variations of h_{nf}/h_f with Ra_f. In our paper direct numerical simulations are used to determine the Nusselt number in the case of base fluids and nanofluids. Eqs. (1), (2) and (3) with the associated boundary conditions ((5), (6) and (7)) were solved numerically using a collocation spectral method [12] and a finite element method (the industrial code Comsol) with a rectangular grid system, better suited to the rectangular shape of the cell used. For the spectral method, the time scheme was a second order Adams–Bashforth–Euler backward scheme and the spatial resolution was 100×20 collocation points along the horizontal and vertical axes, respectively, and for Comsol the spatial resolution was 150×30. A grid convergence test is given for the industrial code Comsol in Table 1.

The ratio of the nanofluid heat transfer rate to the base fluid one is defined as follows:

$$\frac{Q_{nf}}{Q_f} = \frac{h_{nf}}{h_f} = \frac{k_{nf}Nu_{nf}}{k_f Nu_f}$$
(18)

For Ra_{nf} and $Ra_f < 1707.76$, $Nu = 1$ and the variation of heat transfer can be calculated by the ratio:

$$\frac{Q_{nf}}{Q_f} = \frac{h_{nf}}{h_f} = \frac{k_{nf}}{k_f}$$
(19)

For a given value of φ and for each value of Rayleigh number we calculated the Nusselt number for the base fluid by direct numerical simulations. Then the corresponding Rayleigh number for the nanofluid was calculated using Eq. (16). Finally the Nusselt number was determined for the nanofluid. For $Ra_{nf} < Ra_f < 1707.76$, there is no convection in the nanofluid and the base fluid, and the heat transfer occurs by pure conduction, so the ratio of heat transfer is equal to the ratio of thermal conductivities and is constant and independent of the Rayleigh number (cf. Fig. 1). For $Ra_{nf} < 1707.76$ and $Ra_f > 1707.76$ the nanofluid remains in the conductive regime while convection appears in the base fluid. The heat transfer is more important in the base fluid than in the nanofluid and the ratio $Q_{nf}/Q_f = h_{nf}/h_f$ decreases until $Ra_f = 3000$ for $\varphi = 0.08$ in the case of water Al$_2$O$_3$ nanofluids (Fig. 1). We also note, in this figure that for $1707.76 < Ra_{nf} < Ra_f$ and for $Ra_f > 2000$ or 3000 depending on the nanofluid and the values of φ, the ratio $Q_{nf}/Q_f = h_{nf}/h_f$ increases and its value becomes higher than one but remains lower than the ratio k_{nf}/k_f of the two conductivities which is obtained both for the nanofluid and the base fluid in conductive regime. When the ratio Q_{nf}/Q_f becomes higher than 1, the heat transfer rate in the nanofluid becomes higher than that in the base fluid.

Fig. 1 shows the effect of Rayleigh number on the ratio of heat transfer rate for water-based Al$_2$O$_3$ nanofluid for different values of the volume fraction. This variation of the ratio of heat transfer coefficients has not been observed by any author before.

Kim et al. [5] and Hwang et al. [6] showed that the heat transfer rate ratio always increases with the volume fraction without indicating the influence of the Rayleigh number.

In Fig. 1, we observe that the heat transfer can decrease or increase depending on the value of the Rayleigh number. For a water-based Al$_2$O$_3$ nanofluid and for $\varphi = 0.08$, the ratio of heat transfer rate becomes higher than 1 when the Rayleigh number reaches values around 5000 so we obtain an enhancement of heat transfer only after this value ($Ra_f = 5000$).

So adding nanoparticles increases the heat transfer only for given values of the width cavity and the temperature difference.
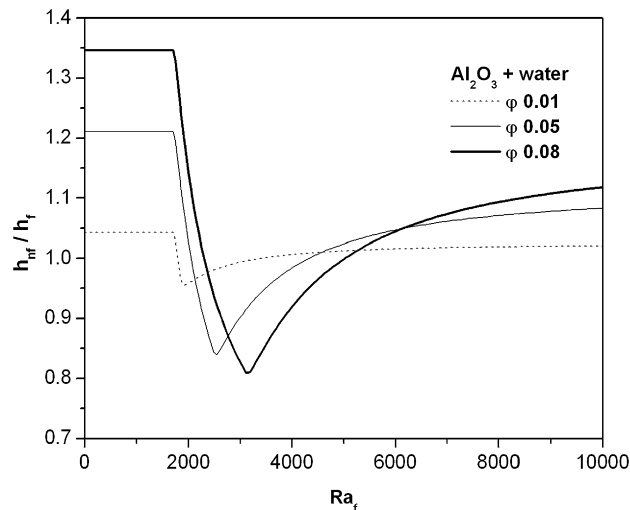
Fig. 1. Effect of the fluid Rayleigh number on the ratio of heat transfer coefficient for Al_2O_3 nanoparticles obtained for different volume fractions.

4. Conclusion

In summary, the heat transfer by natural convection inside a horizontal cell heated from below (Rayleigh–Bénard problem) was investigated. We used three nanofluids (water-based Al_2O_3, CuO and Cu). We supposed that nanofluids are mono-constituent fluids. We used the relations giving the specific heat capacity and the thermal expansion coefficient in accordance with the laws of thermodynamics. These relations are different from the ones used by many authors. The two relations used led to significantly different results from the ones obtained by the authors aforementioned.

It is true that the addition of nanoparticles of Al_2O_3, CuO and Cu in water increases its thermal conductivity and therefore improves the conductive heat transfer in the nanofluid compared to conductive heat transfer in the base fluid. Contrary to what many authors mentioned, we showed in this work that, in the classical Rayleigh–Bénard configuration, just after the onset of convection, there is more heat transfer in the base fluid than in the nanofluid. For a fixed value of the fluid Rayleigh number, the nanofluid Rayleigh number decreases with the volume fraction of nanoparticles. Thus the nanoparticles delay the onset of convection.

Recent experimental results obtained by Donzelli et al. [13] showed the important role played by the thermodiffusion in nanofluids with suspension of highly thermophilic nanoparticles. They observed that the first bifurcation, corresponding to the onset of natural convection in Rayleigh–Bénard problem, is not a stationary fork one but oscillatory, similar to that observed in a horizontal porous layer saturated by binary solution heated from below with negative separation ratio [12,14].

So it is necessary to verify if we obtain similar experimental results in the case of commonly used nanofluids such as Al_2O_3-water, CuO and Cu. Further theoretical and experimental research is necessary in order to be able to use nanofluids for natural convective heat transfer enhancement.

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